

EXERCISE 3: EXCHANGE

Timing of Tutorial Chapter 8 is crucial to the understanding of this tutorial.

Purpose of Tutorial To work through in detail the implications of a particular form of a general equilibrium story as told in the lectures; in so doing to understand more fully the implications of general equilibrium; to show how the desire for exchange and the ability to exchange affect the outcome of the exchange process and the resulting welfare properties.

Prior Preparation You should tackle parts (1) through (10) before the tutorial, write down your answers and hand them in to your tutor in advance of the tutorial; your tutor will use these answers to guide the running of the tutorial.

Written Work after Tutorial You should complete the parts of the tutorial you did not complete before the tutorial and hand your answers in to your tutor by the beginning of Tutorial 4.

Relevance to Examination Very relevant. You may well be asked a question which explores your understanding of the importance of exchange and the conditions for mutually advantageous exchange.

The story concerns a very simple economy in which there are just two individuals and two goods. We call the individuals Individual A and Individual B and the two goods Good 1 and Good 2.

The two individuals are similar in the sense that they both like both goods – though, as we shall see, their preferences are not identical. The two individuals differ in that Individual A is endowed only with Good 1 while Individual B is endowed only with Good 2. Let us denote the initial endowment of Good 1 given to Individual A by e_1 , and let us denote the initial endowment of Good 2 given to Individual B by e_2 .

As we have already remarked both individuals like both goods. Specifically both have Cobb-Douglas preferences over the two goods, Individual A with parameter a and Individual B with parameter b . Hence we can write the utility function of Individual A as

$$U_A(c_1, c_2) = a \log c_1 + (1 - a) \log c_2$$

and the utility function of Individual B as

$$U_B(c_1, c_2) = b \log c_1 + (1 - b) \log c_2$$

where c_1 denotes the quantity of Good 1 consumed by the individual and c_2 the quantity of Good 2 consumed by the individual. Note that if the consumption of either good is *zero* then the utility of either individual is *minus infinity*. You might like to think of this as death. It is clear that utility can be made finite by indulging in trade: Individual A has an obvious incentive to get some of Good 2 and Individual B has an obvious incentive to get some of Good 1. They do this by trading.

Suppose the prices of Goods 1 and 2 are denoted by p_1 and p_2 respectively. Suppose that we start by seeing if we can find the competitive equilibrium in this simple economy. That is, can we find prices p_1 and p_2 for which A is willing to sell a part of his or her endowment of Good 1 in exchange for some of B's endowment of Good 2, and for which B is willing to agree to the same exchange?

In what follows there is a little algebraic manipulation – but you are not required to do any mathematics. The one result that you do need - the demands of individuals with Cobb-Douglas preferences - can be found in the text. In any case, we remind you here of these demands.

(1) Argue that the budget constraint facing Individual A is $p_1 c_1 + p_2 c_2 = p_1 e_1$

and that the budget constraint facing Individual B is $p_1 c_1 + p_2 c_2 = p_2 e_2$.

(2) Using standard results concerning Cobb-Douglas demands, argue that the gross demands for Good 1 and Good 2 by Individual A are:

$$c_1 = \alpha e_1 \quad \text{and} \quad c_2 = (1-\alpha) p_1 e_1 / p_2$$

whilst those for Individual B are:

$$c_1 = b p_2 e_2 / p_1 \quad \text{and} \quad c_2 = (1-b) e_2$$

Comment: the standard result concerning Cobb-Douglas demands is that an individual with Cobb-Douglas preferences with parameter α always spends a fraction α of his or her income on Good 1 and spends a fraction $(1-\alpha)$ of his or her income on Good 2. Now Individual A's income is e_1 units of Good 1 – which is worth $p_1 e_1$ if the price of Good 1 is p_1 - so Individual A spends a fraction α of this on Good 1. Look at the demand for Good 1 by Individual A above. Obviously this is his or her gross demand – but he or she starts with e_1 units of Good 1 – so Individual A keeps a fraction α of his or her initial endowment of Good 1, sells the rest, and uses the proceeds to buy as much of Good 2 as possible. A similar argument applies to Individual B – he or she keeps a fraction $(1-b)$ of his or her initial endowment of Good 2, sells the rest and uses the proceeds to buy as much of Good 1 as possible.

(3) Hence argue that the condition for competitive equilibrium for Good 1 is given by

$$e_1 = (\alpha e_1) + (b p_2 e_2 / p_1)$$

Comment: the left-hand side is the gross supply of Good 1; the first term on the right-hand side is A's gross demand for Good 1 and the second term is B's gross demand for Good 1. This equation simply says that the gross supply must equal the aggregate gross demand.

(4) Hence show that the condition on p_1/p_2 for equilibrium in the market for Good 1 is given by:

$$p_1/p_2 = b e_2 / ((1-\alpha) e_1)$$

Comment: this is simply the equation in (3) above re-arranged to give an expression for p_1/p_2 .

(5) Write down the corresponding equilibrium condition for Good 2, and derive a corresponding condition on p_1/p_2 .

(6) Compare these two conditions on p_1/p_2 and comment.

Comment: they are the same. Why? Because if the relative prices of the two goods are such that A is happy to give to B the quantity of Good 1 that he or she wants in exchange for a quantity of Good 2, then it must also be true that B is happy to give to A that quantity of Good 2 that he or she wants in exchange for the given quantity of Good 1.

(7) Examine the relationship between the competitive equilibrium relative price p_1/p_2 and the parameters of the model (e_1, e_2, a, b). Note the following: the competitive equilibrium relative price p_1/p_2 rises when a or b rise, when e_2 rises and when e_1 falls. Why?

(8) Now draw an Edgeworth Box to show what is going on. Draw a box of length e_1 and height e_2 . We draw A's preferences and endowment measured from the left-hand bottom corner and B's preferences and endowment measured from the right-hand top corner. Note that the initial endowment point is the bottom right-hand corner of the box. You can, if you want, draw in some indifference curves - but this is a bit tedious - so you are not advised to do it. However, you can draw in the *price-offer curves* of the two individuals. Argue that the price-offer curve of Individual A is a vertical line a distance $a e_1$ from the left hand side, and a distance $(1-a) e_1$ from the right-hand side. Why? Look at the demand for Good 1 by Individual A as given in the first left-hand equation in (2) above. Similarly argue that the price-offer curve of Individual B is a horizontal line a distance $b e_2$ from the bottom, and a distance $(1-b) e_2$ from the top. Why? Look at the demand for Good 2 by Individual B as given in the second right-hand equation in (2) above.

(9) Argue that the competitive equilibrium is at the intersection of these two price-offer curves. Note that this intersection point is a distance $a e_1$ from the left hand side, and a distance $(1-a) e_1$ from the right-hand side and a distance $b e_2$ from the bottom, and a distance $(1-b) e_2$ from the top. Now argue that the budget line facing both individuals in this competitive equilibrium is the line from the bottom right-hand corner to this competitive equilibrium. What is the slope of this line? Yes: $b e_2 / ((1-a) e_1)$ - this looks familiar! WHY?

(10) Use this box to show graphically the results derived in (7).

(11) Suppose we now change the rules of the game. Suppose we let Individual A set the price - that is, act like a monopolist in the market for Good 1 and as a monopsonist in the market for Good 2. What relative price p_1/p_2 will he or she choose?

Comment: you should be able to answer this using the Edgeworth Box and the price-offer curves that you have drawn.

Comments as to what you should take away from this tutorial. You should realise that the competitive equilibrium is dependent on both preferences and endowments of the individuals in society. This is crucial. Note what we have shown: if the initial endowment of Good 1 rises then the relative price of Good 1 in the competitive equilibrium falls. The same is true for Good 2. So we get the – perhaps obvious – result – that the more of a good there is, the lower is its price. Moreover we can see that if either ***a*** or ***b*** rise then the price of Good 1 rises too. You should ask yourself why.