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The loss from uncertainty on policy targets

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Abstract

What is the welfare loss arising from uncertainty about true policy targets? We quantify these effects in a DSGE model where private agents are unable to distinguish between temporary shocks to potential output and to the inflation target. Agents use optimal filtering techniques to construct estimates of the unknown variables. We find that the welfare costs of not observing the inflation target and potential output are relevant even in the case of a small measurement error. We also show that, in our framework, uncertainty about the inflation target is more costly than uncertainty about potential output.

Keywords: Monetary Policy; Kalman filter; Potential Output

JEL: E5, E37, E52, E58

1 Introduction

In the last decade, most central banks have opted for increasing transparency of monetary policy and communication of their targets and strategies to the public. Implicit or explicit inflation targeting has gained consensus among central banks, requiring mechanisms to anchor private sector's expectations.

This approach survived, mostly everywhere, the turmoil caused by the financial crisis. The new important target of financial stability has been

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added to inflation targeting and output stabilization. However, given the difficulty in defining and measuring financial stability, its role remains pretty vague and has not been fully understood in its interactions with traditional macroeconomic stabilization. Hence, central banks actions are still mostly guided by the desire to stabilize inflation and output gap. The notion of output gap hinges on potential output, a non observable economic variable that is key to determine the target level of output. The central bank estimates and updates its measure of the potential output when conducting monetary policy. Given the difficulties inherent in real-time estimation of potential output, policy actions might not be immediately recognizable because of signal extraction errors arising under imperfect information. This feature is central for Orphanides (2001, 2003a, b), who reports evidence of a significant (real time) overestimation of potential output during the oil shocks of the 1970s. Cukierman and Lippi (2005) showed that, even if the policymakers efficiently estimate potential output, this does not avoid persistent *retrospective* policy errors.

Di Giorgio and Traficante (2010) investigate whether central bank's political transparency is desirable in presence of incomplete information about the state of the economy. In that paper, incomplete information was modeled by assuming an asymmetric information set. The central bank and the private sector share the same incomplete information on potential output, but the private sector does not observe the inflation target and cannot exactly infer the policy reaction function, linking the choice of the policy instrument to the final objectives. The authors show that full political transparency is socially beneficial in a model with monopolistic competition and Calvo pricing.

This paper evaluates how uncertainty about policy goals, namely potential output and the inflation target, affects welfare. Information is asymmetric: the central bank has complete information and follows a discretionary non-inertial policy. The private sector, on the other hand, observes the interest rate set by the central bank but it faces a signal-extraction problem involving potential output and inflation target because it does not know if the central bank is reacting to shocks to the inflation target or to shocks to potential output. The private sector uses a signal based on the policy function to estimate the unobservable variables. In their learning process, agents revise and update their estimates attaching less weight to older information.

Our results suggest that the macroeconomic benefits of credibly announcing the current level of the time-varying policy targets are consistent. Moreover, we show that uncertainty about the inflation target entails a higher welfare loss than uncertainty about potential output.

From a methodological point of view, this paper contributes to the literature on the role of incomplete information in DSGE models. Most papers assume a larger information set for the private sector¹. Here we depart from this assumption. Policy objectives and intentions are not always revealed explicitly and truthfully to the public. Hence, we assume asymmetric information about policy targets in favor of the central bank.

The paper is organized as follows: we describe the model setup and our parameterization in section 2 to 5. Section 3 presents the information structure and the private sector's learning process. In section 4 we study the effects of incomplete information on inflation and output dynamics, while welfare analysis is developed in section 5. Section 6 summarizes and concludes.

2 Setup

2.1 The model

Let us consider a representative household whose objective is to maximize an expected utility function defined on consumption and leisure, subject to a sequence of standard budget constraints. In our specification, the household's problem can be written as

$$\max E_o \sum_{t=0}^{\infty} \beta^t a_t \left[\ln(C_t) - \frac{1}{1+\psi} h_t^{1+\psi} \right] \quad (1)$$

$$s.t. \quad B_{t-1} + W_t h_t - T_t \geq P_t C_t + \frac{B_t}{R_t} \quad (2)$$

where: a_t is an AR(1) preference shock, h_t are units of labor needed to get nominal salary W_t , ψ is the inverse Frisch elasticity of labor supply, T_t are nominal taxes levied by the government and B_t are one-period zero-coupon

¹See Svensson and Woodford (2004), who assume that the central bank possesses less information about the state of the economy than the private sector. In a different setup, Cukierman and Lippi (2005) assume that the central bank does not observe potential output and this leads to *unavoidable* policy errors.

bonds returning gross interest rate R_t . β is the intertemporal discount factor.

First order conditions with respect to consumption, hours worked and bonds respectively can be written as:

$$C_t : \quad \Lambda_t = \frac{a_t}{C_t} \quad (3)$$

$$h_t : \quad a_t = \Lambda_t \frac{W_t}{P_t} \quad (4)$$

$$B_t : \quad \Lambda_t = \beta R_t E_t \left(\frac{\Lambda_{t+1}}{\Pi_{t+1}} \right) \quad (5)$$

where Λ_t is the Lagrange multiplier on the budget constraint and $\Pi_t = \frac{P_t}{P_{t-1}}$ is the gross inflation rate at time t .

By combining first-order conditions and log-linearising the model around a zero inflation steady state equilibrium, we get the following IS relationship:

$$y_t = E_t y_{t+1} - r_t + E_t \pi_{t+1} + a_t - E_t a_{t+1} \quad (6)$$

where r_t and π_t are the nominal interest and inflation rates and y_t is real output.

Production is conducted by monopolistic competitive firms according to a stochastic constant-returns-to-scale linear technology using labor. More specifically

$$Y_t(i) = Z_t h_t(i) \quad (7)$$

where Z_t is a technology shock whose log z_t is an AR(1). Real marginal cost, in log-linearised terms, can be expressed in equilibrium as

$$mc_t = w_t - p_t - z_t = y_t - z_t \quad (8)$$

We assume price setting *à la* Rotemberg, where the degree of price stickiness is measured by the parameter $\phi > 1$. Let us also define ε as the price elasticity of demand, $\mathcal{F}_{t,t+1}$ as the stochastic discount factor and π_t^* as a stochastic AR(1) inflation target (along the lines of Ireland (2007)). Consequently, the firm's pricing problem can be written as:

$$\max_{P_t(i)} E_0 \sum_{t=0}^{\infty} \mathcal{F}_{t,t+1} \left[\left(\frac{P_t(i)}{P_t} \right)^{1-\varepsilon} Y_t - \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} Y_t MC_t - \frac{\phi}{2} \left(\frac{P_t(i)}{P_{t-1}(i)(1+\pi_t^*)} - 1 \right)^2 Y_t \right] \quad (9)$$

The first order condition for this problem is:

$$(1-\varepsilon)\frac{Y_t}{P_t}\left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} + \varepsilon Y_t \frac{MC_t}{P_t}\left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon-1} - \phi \frac{Y_t}{(1+\pi_t^*)_t P_t(i)} \left(\frac{P_t(i)}{(1+\pi_t^*)_t P_{t-1}(i)} - 1\right) \\ + \phi E_t \left\{ \mathcal{F}_{t,t+1} \left(\frac{P_{t+1}(i)}{(1+\pi_{t+1}^*)_t P_t(i)} - 1\right) \frac{P_{t+1}(i)}{(1+\pi_{t+1}^*)_t P_t(i)} \frac{Y_t}{P_t(i)} \right\} = 0$$

In equilibrium, all firms charge the same price, $\frac{P_t(i)}{P_t} = 1$. Exploiting the fact that in equilibrium $\mathcal{F}_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{-1}$, it is possible to show that the pricing decision leads to the following log-linear Phillips curve

$$\pi_t - \pi_t^* = \frac{(\varepsilon - 1)}{\phi} m c_t + \beta E_t (\pi_{t+1}^* - \pi_{t+1}^*) + s_t \quad (10)$$

where s_t is an AR(1) cost-push shock.

As real marginal costs are linearly related to an output gap measure, we can write a standard new keynesian Phillips curve:

$$\pi_t - \pi_t^* = \kappa (y_t - \bar{y}_t) + \beta E_t (\pi_{t+1}^* - \pi_{t+1}^*) + s_t \quad (11)$$

where $\kappa \equiv \frac{(\varepsilon-1)}{\phi} (1 + \psi)$

Finally, under Rotemberg pricing, the relevant welfare-based loss can be written as a function of inflation and output gap².

$$\mathcal{L}_t = \frac{1}{2} \left[\phi (\pi_t - \pi_t^*)^2 + (1 + \psi) (y_t - \bar{y}_t)^2 \right] \quad (12)$$

2.2 Parameterization

Our parameterization relies on standard numerical values in the literature (see Woodford, 2003). The discount factor β is set equal to 0.99, while the slope coefficient of the Phillips curve κ is set equal to 0.21, which is analogous to assuming a Calvo parameter equal to 0.75³. We set the inverse

²See Nisticò (2007) and Lombardo and Vestin (2007), who show that both the Calvo and the Rotemberg pricing models yield an equivalent Phillips curve shape and imply the same relative weights in the welfare criterion.

³Notice that in a model with sticky prices à la Calvo, Phillips curve' slope coefficient κ_c would be equal to

$$\kappa = \frac{(1-\omega)(1-\beta\omega)}{\omega} (\sigma + \psi) \quad (13)$$

Hence we impose $\phi = \frac{(\varepsilon-1)\omega}{(1-\omega)(1-\beta\omega)}$, where $1 - \omega$ stands for the fraction of firms adjusting their prices every period

of labor-supply elasticity equal to 1.5, the coefficient of constant relative risk aversion $\sigma = 1$ and $\varepsilon = 10$. The autoregressive coefficients of potential output, preference and cost-push shocks are all assumed equal to 0.8. The chosen autoregressive coefficient for the inflation target is higher, consistent with the evidence of Ireland's estimates about the statistical properties of the time-varying inflation target for the Federal Reserve. ρ_π is assumed to be equal to 0.99, while the standard deviation of the inflation target is equal to 0.001. Following the literature, the volatility of all other structural shocks is assumed to be considerably higher. In this paper we set it equal to 0.01, ten times higher than that of the inflation target⁴.

3 Information structure and learning

The model is solved under discretion and the private sector is assumed unable to distinguish between shocks to the inflation target and to potential output. Since the private sector observes the interest rate set by the central bank, they can use the policy rule $r_t = f_1\pi_t^* + f_2\bar{y}_t + f_3s_t + f_4a_t$ to obtain a signal composed by the unobservable variables and a random noise.

$$\chi_t = \begin{bmatrix} f_1 & f_2 \end{bmatrix} \begin{bmatrix} \pi_t^* \\ \bar{y}_t \end{bmatrix} + \nu_t = H' \begin{bmatrix} \pi_t^* \\ \bar{y}_t \end{bmatrix} + \nu_t \quad (14)$$

The state equation for π_t^* and \bar{y}_t can be represented in the following way

$$\begin{bmatrix} \pi_{t+1}^* \\ \bar{y}_{t+1} \end{bmatrix} = \begin{bmatrix} \rho_\pi & 0 \\ 0 & \rho_y \end{bmatrix} \begin{bmatrix} \pi_t^* \\ \bar{y}_t \end{bmatrix} + \begin{bmatrix} \eta_{t+1}^\pi \\ \eta_{t+1}^y \end{bmatrix} = R \begin{bmatrix} \pi_t^* \\ \bar{y}_t \end{bmatrix} + \begin{bmatrix} \eta_{t+1}^\pi \\ \eta_{t+1}^y \end{bmatrix} \quad (15)$$

The private sector will compute optimal estimates of the current inflation target and potential output through the Kalman gain matrix K^5 . In the

⁴The results shown in the next sections are robust to using different parameter values as the ones widely used in the Woodford textbook (2003).

⁵The Kalman gain matrix K is found numerically. We let Σ be the variance-covariance matrix of $[\eta_{t+1}^\pi \ \eta_{t+1}^y]'$ and let $P_{t+1|t}$ denote the mean squared error of the forecast of $[\pi_{t+1}^* \ \bar{y}_{t+1}]'$. The Kalman gain matrix and the mean-squared error are found by iterating until convergence on

$$K_t = RP_{t|t-1}H(H'P_{t|t-1}H + \sigma_\nu^2)^{-1}$$

and

$$P_{t+1|t} = (R - K_tH')P_{t|t-1}(R - K_tH')' + \Sigma$$

case of a zero measurement error ($\nu_t = 0$), optimal forecasts of the future inflation target and potential output are computed as

$$\begin{bmatrix} \pi_{t+1|t}^* \\ \bar{y}_{t+1|t} \end{bmatrix} = (R - KH') \begin{bmatrix} \pi_{t|t-1}^* \\ \bar{y}_{t|t-1} \end{bmatrix} + KH' \begin{bmatrix} \pi_t^* \\ \bar{y}_t \end{bmatrix} \quad (16)$$

Repeated substitutions for $\begin{bmatrix} \pi_{t|t-1}^* \\ \bar{y}_{t|t-1} \end{bmatrix}$ lead to the reduced form of the estimation problem in $t + 1$:

$$\begin{bmatrix} \pi_{t+1|t}^* \\ y_{t+1|t} \end{bmatrix} = \sum_{j=0}^{\infty} D^j K \chi_{t-j} \quad (17)$$

where

$$D \equiv [R - KH']$$

More generally, the matrix D is dependent on σ_ν^2 , in spite of R and H being invariant to measurement errors. In presence of higher volatility in the measurement error, the one step ahead forecast error for the policy targets increases, while the elements in the Kalman gain matrix, through which the private sector estimates the inflation target and potential output decrease. These results are documented in table 1, which shows the elements in the Kalman gain matrix K and the weights attached to the signal χ_t for different degrees of volatility of the measurement error ν_t ⁶. Since the private sector knows that potential output is much more volatile than the inflation target, it believes that the signal is more useful to estimate potential output rather than the inflation target. Consequently, the weight attached to the signal χ_t is much higher (in absolute values) when estimating potential output. Figure 1 and 2 show the elements of the Kalman gain matrix $K_{1,1}$ and $K_{2,1}$ respectively for different values of σ_ν : the absolute value of the elements $K_{i,j}$ is decreasing in the measurement error, revealing that the private sector attaches less weight to noisier indicators. As we will see below, this optimal learning process limits the impact that noisy information has on welfare.

The Kalman gain depends on all elements of R , H , Σ and on the variance of measurement error σ_ν^2 .

⁶In table 1, we label with $K_{i,j}$ the element in row i , column j of K . Moreover, the solution of the signal-extraction problem provides also the covariance matrix of the mean squared error forecast P , whose generic element is labeled $P_{i,j}$. In the last two columns of the table we report the values of the vector DK in equation (17).

The private sector is unable to distinguish which shock is hitting the economy, when there is a shock to the inflation target (potential output) it assigns some probability to the event that also a shock to potential output is occurring (inflation target). For example, after a positive one percent shock to the inflation target, inflation will increase and the central bank will raise the interest rate, as shown in figure 3. The private sector only observes the increased interest rate, but it is unable to distinguish if it was due to a higher inflation target or to a reduced potential output. Under incomplete information, it turns out that the private sector only estimates around 30% of the shock, and perceives a (false) negative potential output shock of 0.015. Therefore, there will be a significant difference in the impulse response function for inflation and output. Figure 4 documents these findings: under incomplete information, inflation is less volatile, while output moves, in contrast with the benchmark case of complete information.

The same line of reasoning can be applied after a positive one percent shock to potential output. The central bank decrease the interest rate, as shown in figure 5. On the other hand, the private sector, observing the lower interest rate, assigns positive probabilities to both a positive shock on potential output and a negative shock to the inflation target. Consequently, inflation goes down, while the reaction of output is milder.

4 Macroeconomic dynamics

In this section we evaluate how noisy signals on potential output and the inflation target affect inflation and output in the discretionary equilibrium. We compare the values of those variables in the presence of imperfect information with the corresponding values under full information. In each period, the central bank possesses direct information about the realizations of the shocks, while the private sector only receives noisy signals about the policy targets. Under complete information, equilibrium inflation is

$$\pi_t = \pi_t^* + \frac{\gamma}{1 - \beta\gamma\rho_s} s_t \quad (18)$$

with $\gamma \equiv \frac{1+\psi}{1+\psi+\kappa^2\phi}$

while output is given by

$$y_t = \bar{y}_t - \frac{\kappa\phi\gamma}{(1+\psi)(1-\beta\gamma\rho_s)} s_t \quad (19)$$

When the private sector does not observe potential output and the inflation target, (18) becomes

$$\pi_{t|t} = \pi_{t|t}^* + \frac{\gamma}{1 - \beta\gamma\rho_s} s_t \quad (20)$$

After substituting the estimation of $\pi_{t|t}^*$, we can rewrite (20) as inflation with complete information plus two additional terms reflecting the deviation of the inflation target and the potential output from their estimated values using the information set available up to period $t - 1$. We then have:

$$\pi_{t|t} = \pi_t - \left(1 - \frac{K_{1,1}f_1}{\rho_\pi}\right) (\pi_t^* - \pi_{t|t-1}^*) + \frac{K_{1,1}f_2}{\rho_\pi} (\bar{y}_t - \bar{y}_{t|t-1}) + \frac{K_{1,1}}{\rho_\pi} \nu_t \quad (21)$$

Analogously, the expression for output in the discretionary equilibrium, under incomplete information, is:

$$y_{t|t} = \bar{y}_{t|t} - \frac{\kappa\phi}{\psi} \left(\frac{\gamma}{1 - \beta\gamma\rho_s}\right) s_t \quad (22)$$

or, after substituting the estimation of $\bar{y}_{t|t}$ into (22):

$$y_{t|t} = y_t + \frac{K_{2,1}f_1}{\rho_y} (\pi_t^* - \pi_{t|t-1}^*) - \left(1 - \frac{K_{2,1}f_2}{\rho_y}\right) (\bar{y}_t - \bar{y}_{t|t-1}) + \frac{K_{2,1}}{\rho_y} \nu_t \quad (23)$$

The macroeconomic effects of imperfect information can therefore be summarized in the following proposition:

Proposition 1. *When the private sector is unable to distinguish between shocks to the inflation target and to potential output, after a positive shock to one of the policy targets, i) the equilibrium levels of inflation and output are lower than under complete information; ii) inflation (output) is positively (negatively) related with the measurement error; iii) contrary to the case of complete information, potential output (inflation target) affects equilibrium inflation (output).*

Proof. We start from the final part of the proposition. Since the private sector is unable to distinguish if the central bank moves the interest rate in response to a shock to the inflation target, or to a shock to potential output, it optimally estimates that a combination of both shocks is hitting the economy. Consequently, inflation will be affected also by potential output and output will be affected also by the inflation target.

Let us consider now the parts i)-ii) of the proposition. The coefficients of the differences between actual and estimated policy targets in (21) and

(23) are negative⁷, hence we need that $(\pi_t^* - \pi_{t|t-1}^*)$ and $(\bar{y}_t - \bar{y}_{t|t-1})$ are positive to prove our claim. When a positive shock to the inflation target occurs, the private sector underestimates the size of the shock because it assigns a positive probability that a negative shock to potential output has occurred. Therefore, the difference between the actual inflation target and the estimated one is positive, while, provided that potential output is not changing, $(\bar{y}_t - \bar{y}_{t|t-1})$ is positive. Even if the measurement error is positively related to inflation in equation (21), quantitatively this effect is small and offset by the differential between actual and estimated policy targets. In the case of a negative shock to the inflation target, the results are symmetrically opposite, because the private sector underestimates the size of the shock to the inflation target and estimates that a positive shock to potential output has occurred.

The same line of reasoning can be applied when a positive shock to potential output hits the economy. The private sector underestimates the size of the shock, because it also estimates that a negative shock to the inflation target has occurred: consequently, the differentials of the actual and estimated policy targets are positive and contribute to have a lower level of output than that with complete information. After a negative shock to potential output, the difference between actual and estimated policy targets are always negative, offset the negative effect of a measurement error, hence output turns out to be higher than in the case of complete information⁸. \square

5 Welfare Analysis

5.1 Macroeconomic Volatility and Uncertain Policy Targets

In this section we compute the value of the loss function previously defined and evaluate how it is affected by uncertainty on policy targets. Under complete information we obtain a loss equal to 0.92, while this value raises to 0.97 when the private sector is unable to distinguish between shocks to

⁷It is sufficient to combine the values of the elements of the Kalman gain matrix with the coefficients of (21) and (23) to prove that $-\left(1 - \frac{K_{1,1}f_1}{\rho_\pi}\right)$, $\frac{K_{1,1}f_2}{\rho_\pi}$, $\frac{K_{2,1}f_1}{\rho_y}$ and $-\left(1 - \frac{K_{2,1}f_2}{\rho_y}\right)$ are negative. To that extent, table 1 presents the values of the Kalman gain matrix for different values of the measurement error, showing that $K_{1,1}$ is positive, while $K_{2,1}$ is negative.

⁸The results of Proposition 1 hold also in case of a shock hitting both the policy targets, not shown here.

potential output and the inflation target, in the case of a zero measurement error.

In order to provide a quantitative evaluation of the macroeconomic effects of the measurement error, we simulate the model for different values of σ_ν . Table 2 shows the value of the loss function, together with the variance of inflation and output for different degrees of volatility in the measurement error ν . Together with figure 7, which presents the loss function graphically for $\sigma_\nu = 0, \dots, 0.25$, we can conclude that the loss function is increasing in the size of the measurement error. As observed in table 1, in fact, when the signal used by the private sector to estimate policy targets becomes less precise, the weights attached to χ are low because the private sector realizes that signals are not very accurate. Consequently, the elements in the Kalman gain matrix K become smaller, and this partly offsets the increase in the noise.

We now study the welfare consequences of having only one unobservable variable for the private sector, either the inflation target or potential output. When the private sector does not observe π_t^* , the transition equation is $\pi_{t+1}^* = \rho_\pi \pi_t^* + \eta_{t+1}^\pi$, while the measurement equation becomes $\chi_t = f_1 \pi_t^* + \nu_t$ ⁹. Following Harvey (1990) we can derive an analytical expression for the Kalman gain matrix κ_1 and the conditional variance of the optimal predictor p_1 :

$$\kappa_1(f_1) = \frac{\rho_\pi f_1 p_1}{f_1^2 p_1 + \sigma_\nu^2} \quad (24)$$

$$p_1 = -\frac{(1 - \rho_\pi^2) \frac{\sigma_\nu^2}{f_1^2} - \sigma_{\eta, \pi}^2}{2} + \frac{\sqrt{\left[(1 - \rho_\pi^2) \frac{\sigma_\nu^2}{f_1^2} - \sigma_{\eta, \pi}^2 \right]^2 + 4 \frac{\sigma_{\eta, \pi}^2 \sigma_\nu^2}{f_1^2}}}{2} \quad (25)$$

The same analytical framework applies for the case in which private agents do not observe \bar{y}_t . In this case the transition equation is $\bar{y}_{t+1} = \rho_y \bar{y}_t + \eta_{t+1}^y$, while the measurement equation is $\chi_t = f_2 \bar{y}_t + \nu_t$ ¹⁰. The Kalman gain matrix κ_2 and the conditional variance of the optimal predictor p_2 are given by :

$$\kappa_2(f_2) = \frac{\rho_y f_2 p_2}{f_2^2 p_2 + \sigma_\nu^2} \quad (26)$$

⁹In this case the measurement error ν_t can be interpreted as a policy shock as in Rudebusch and Williams (2008) and Melecky et al. (2010).

¹⁰The measurement error is assumed to be identical to the case in which the private sector does not observe the inflation target with the aim of evaluating the cost of not observing one of the two inflation targets independently of the measurement error.

$$p_2 = -\frac{(1 - \rho_y^2) \frac{\sigma_\nu^2}{f_2^2} - \sigma_{\eta,y}^2}{2} + \frac{\sqrt{\left[(1 - \rho_y^2) \frac{\sigma_\nu^2}{f_2^2} - \sigma_{\eta,y}^2 \right]^2 + 4 \frac{\sigma_{\eta,y}^2 \sigma_\nu^2}{f_2^2}}}{2} \quad (27)$$

It is now possible to derive the variance of equilibrium inflation under the assumption that the private sector does not observe the inflation target. This will be equal to the variance of inflation with complete information plus a term proportional to the conditional variance of the optimal predictor for the unknown policy target:

$$\text{Var}(\pi_{t|t}) = \text{Var}(\pi_t) + \left(\frac{f_1 \kappa_1}{\rho_\pi} - 1 \right)^2 p_1 + \left(\frac{\kappa_1}{\rho_\pi} \right)^2 \sigma_\nu^2 \quad (28)$$

Similarly, the variance of equilibrium output, when potential output is unobservable, is given by the variance of output under complete information plus a term proportional to the conditional variance of the optimal predictor for potential output:

$$\text{Var}(y_{t|t}) = \text{Var}(y_t) + \left(\frac{f_2 \kappa_2}{\rho_y} - 1 \right)^2 p_2 + \left(\frac{\kappa_2}{\rho_y} \right)^2 \sigma_\nu^2 \quad (29)$$

The value of the loss function in the two alternative scenarios may be written respectively as

$$L_\pi = W_1 (\text{Var}(\pi_{t|t} - \pi_t^*)) + W_2 (\text{Var}(y_t - \bar{y}_t)) \quad W_1 \equiv \frac{\phi}{2} \quad W_2 \equiv \frac{1 + \psi}{2} \quad (30)$$

$$L_y = W_1 (\text{Var}(\pi_t - \pi_t^*)) + W_2 (\text{Var}(y_{t|t} - \bar{y}_t)) \quad (31)$$

We can then prove the following:

Proposition 2: *The loss function from not observing the inflation target is higher with respect to that of not observing potential output, that is $L_\pi > L_y$.*

Proof: See Appendix.

The policy message which arises from this result is that welfare benefits from political transparency about inflation target are higher. The economic intuition may be found in the fact that while potential output is only determined by the economic model, through the production function, the inflation target is a purely exogenous policy goal. In figure 8 we present the relative loss with respect to the baseline case of complete information when the private sector does not observe π_t^* or \bar{y}_t . Clearly this relative loss is increasing in the size of σ_ν , with L_π above L_y , both expressed as percent deviation from the complete information case.

5.2 Robustness Analysis: Alternative Measurement Error

We have assumed that the signal received by the private sector is a linear combination of the true policy targets, with or without an explicit measurement error. It is interesting to evaluate the consequences of observing each policy target but with different measurement error. In this case we can model the signal perceived by the private sector in the following way:

$$\begin{bmatrix} \pi_t^{*o} \\ \bar{y}_t^o \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \pi_t^* \\ \bar{y}_t \end{bmatrix} + \begin{bmatrix} \nu_t^\pi \\ \nu_t^y \end{bmatrix} \quad (32)$$

The signal perceived by the private sector is now independent on the policy followed by the central bank and it boils down to the real policy targets plus a measurement error. Notice that the observation equation (32) entails a significant signal-extraction problem only in presence of non-zero measurement errors. In the previous section, on the other hand, a signal-extraction problem arose also with a zero measurement error, because the signal was modeled as a linear combination of the true policy targets.

Simulations conducted by varying the variance of the perceived inflation target and the variance of potential output show that it is more costly to have a noisier signal about the inflation target, as documented in table 3. Moreover, we simulate the model by varying alternatively σ_ν^π (for a given value of $\sigma_\nu^y = 0.1$) and σ_ν^y (for a given value of $\sigma_\nu^\pi = 0.1$). Then we compute the percent change of the loss under the latter scenario and plot it in figure 9. We have another evidence that transparency about the inflation target is more welfare enhancing than transparency about potential output. The relative loss has a peak for a value of $\sigma_\nu = 0.09$, while afterwards it decreases, without, however, arriving to modest values. After a threshold value of $\sigma_\nu = 0.09$, the weight attached to the noisy indicator of the inflation target becomes considerably lower so that it offsets the increase in the variance of the measurement error, and diminishing the relative loss with respect to the case of $\sigma_\nu = 0.09$. These finding confirms our previous conclusion about the desirability of political transparency on the inflation target.

6 Concluding remarks

Central bank's policy targets are not always public knowledge: in the last two decades we have observed a sharp increase in the degree of central bank

transparency. In this paper we show that the absence of transparency about policy targets entails welfare losses. We quantify such losses in a DSGE model featuring uncertainty on potential output and imperfect knowledge of the central bank's inflation target by the private sector. Welfare loss is increasing in the measurement error, hence this paper suggests that central banks must be transparent about policy targets, especially the inflation target. Political transparency speeds the agents' learning process and is a tool to manage their expectations.

The results of this paper are derived in a workhorse DSGE model with Rotemberg pricing, where financial markets are complete and there is no heterogeneity among agents. We believe that only by enriching the financial side of the model we can derive a trade-off between the beneficial effect of transparency, in terms of managing the private sector's expectations, and the overreaction of the financial markets to public signals. We leave this issue for future research.

References

- [1] Cornand, Camille and Frank Heinemann, (2006). "Optimal Degree of Public Information Dissemination", *mimeo*.
- [2] Cukierman, Alex and Francesco Lippi, (2005). "Endogenous Monetary Policy with Unobserved Potential Output", *Journal of Economic Dynamics and Control*, 29(11), 1951-1983.
- [3] Dennis Richard, (2004). "Inferring Policy Objectives from Economic Outcomes", *Oxford Bulletin of Economics and Statistics*, 66, 735-764.
- [4] Dennis, Richard and Federico Ravenna, (2008). "Learning and optimal monetary policy", *Journal of Economic Dynamics and Control*, 32, 1964-1994.
- [5] Di Giorgio, Giorgio and Guido Traficante, (2010). "Uncertainty and Transparency of Monetary Policy", in George Kaufman and Robert Bliss (Eds.) *Financial Institutions and Markets*, Palgrave MacMillan, volume 3 (2010) .

- [6] Ehrmann, Michael and Frank Smets, (2003). "Uncertain potential output: implications for monetary policy", *Journal of Economic Dynamics and Control*, 27, 611-638.
- [7] Faust, Jon and Lars Svensson, (2001). "Transparency and Credibility: Monetary Policy with Unobservable Goals", *International Economic Review*, 42, 369-397.
- [8] Faust, Jon and Lars Svensson, (2002). "The Equilibrium Degree of Transparency and Control in Monetary Policy", *Journal of Money, Credit and Banking*, 34, 520-539.
- [9] Geraats, Petra M., (2002). "Review: Central Bank Transparency", *Economic Journal*, 112, 532-565.
- [10] Harvey, Andrew C., (1990). "Forecasting, Structural Time Series Models and the Kalman Filter", *Cambridge University Press*.
- [11] Ireland, Peter N., (2007). "Changes in the Federal Reserve's Inflation Target: Causes and Consequences", *Journal of Money, Credit and Banking*, 39(8), 1851-1882.
- [12] Lombardo, Giovanni and David Vestin, (2007). "Welfare Implications of Calvo vs. Rotemberg Pricing Assumptions", *ECB Working Papers* 770.
- [13] Melecky, Martin, Diego Rodríguez Palenzuela and Ulf Söderström (2010). "Inflation target transparency and the macroeconomy", *Series on Central Banking, Analysis, and Economic Policies, Volume XIII, Central Bank of Chile*, 371-411.
- [14] Nisticò Salvatore, (2007). "The welfare loss from unstable inflation", *Economic Letters*, 96, 51-57.
- [15] Orphanides, Athanasios, (2001). "Monetary Policy Rules Based on Real-Time Data", *American Economic Review*, 91(4), 964-985.
- [16] Orphanides, Athanasios, (2003). "Monetary Policy with noisy information", *Journal of Monetary Economics*, 50, 605-631.

- [17] Orphanides, Athanasios, (2003). "The quest for prosperity without inflation", *Journal of Monetary Economics*, 50, 633-663.
- [18] Rudebusch, Glenn and John Williams (2008). "Revealing the Secrets of the Temple: The Value of Publishing Central Bank Interest Rate Projections", *University of Chicago Press*, 247-284.
- [19] Svensson, Lars E.O. and Michael Woodford, (2003). "Indicator variables for optimal policy", *Journal of Monetary Economics*, 50, 691-720.
- [20] Svensson, Lars E.O. and Michael Woodford, (2004). "Indicator variables for optimal policy under asymmetric information", *Journal of Economic Dynamics and Control*, 28, 661-690.
- [21] Walsh, Carl E., (2003). "Accountability, Transparency and Inflation Targeting", *Journal of Money, Credit and Banking*, 35, 829-849.
- [22] Walsh, Carl E., (2006). "Transparency, Flexibility and Inflation Targeting", in Frederic Mishkin and Klaus Schmidt-Hebbel (editors) *Monetary Policy under Inflation Targeting*, Santiago, Chile: Banco Central de Chile.
- [23] Walsh, Carl E., (2007). "Optimal Economic Transparency", *The International Journal of Central Banking*, 3(1), 5-36.
- [24] Woodford, Michael, (2003). "Interest and Prices: Foundations of a Theory of Monetary Policy", *Princeton University Press*.

A Proof of Proposition 2

Proposition 2: *The loss function from not observing the inflation target is higher with respect to that of not observing potential output, that is $L_\pi > L_y$.*

The difference in term of welfare loss between (30) and (31) is given by

$$L_\pi - L_y = W_1 [Var(\pi_{t|t} - \pi_t^*) - Var(\pi_t - \pi_t^*)] + W_2 [Var(y_t - \bar{y}_t) - Var(y_{t|t} - \bar{y}_t)] \quad (33)$$

The difference in the losses is therefore function of two terms: the first one, weighted by the coefficient W_1 reflects the cost of not observing inflation target, while the second one, weighted by the coefficient W_2 expresses how it is detrimental in welfare terms not observing potential output. From (28) and (29) and the analysis in the main text we know that the variance of inflation and output is higher when there is imperfect information. Therefore, after some computation, (33) can be rearranged in the following way:

$$L_\pi - L_y = W_1 \left(\frac{f_1 \kappa_1}{\rho_\pi} - 1 \right)^2 p_1 - W_2 \left(\frac{f_2 \kappa_2}{\rho_y} - 1 \right)^2 p_2 + \left[W_1 \left(\frac{\kappa_1}{\rho_\pi} \right)^2 - W_2 \left(\frac{\kappa_2}{\rho_y} \right)^2 \right] \sigma_\nu^2 \quad (34)$$

All the terms in (34) are positive, hence the net result is determined by the relative weights W_1 , W_2 , by the variances p_1 , p_2 and by how the private sector estimates the unobservable target (κ_1 and κ_2)¹¹. With the parameterization used in the main text it can be shown that, even if $p_2 > p_1$ and the absolute value of $\kappa_2 > \kappa_1$, the influence of having a central bank attaching a bigger weight on inflation stabilization rather than output stabilization makes the welfare loss with unobservable inflation target higher than in the case of unobservable potential output¹².

¹¹The measurement is assumed to be the same to isolate its effect on the final result.

¹²Computational details are available upon request. This result can be reversed if we assume a very low autoregressive coefficient on the inflation target, which seems at odd with current practices adopted by the major central banks.

Table 1: The Kalman gain matrix and the weight attached to the signal in the estimation of the inflation target and potential output respectively. The following values are computed for different degrees of volatility in the measurement error.

σ_ν	$K_{1,1}$	$K_{2,1}$	Weight (inflation target)	Weight (potential output)
0	0.41	-2.37	0.41	-2.38
0.01	0.06	-0.24	0.06	-0.24
0.05	0.0098	-0.016	0.0098	-0.0161
0.1	0.0036	-0.0043	0.0036	-0.0043

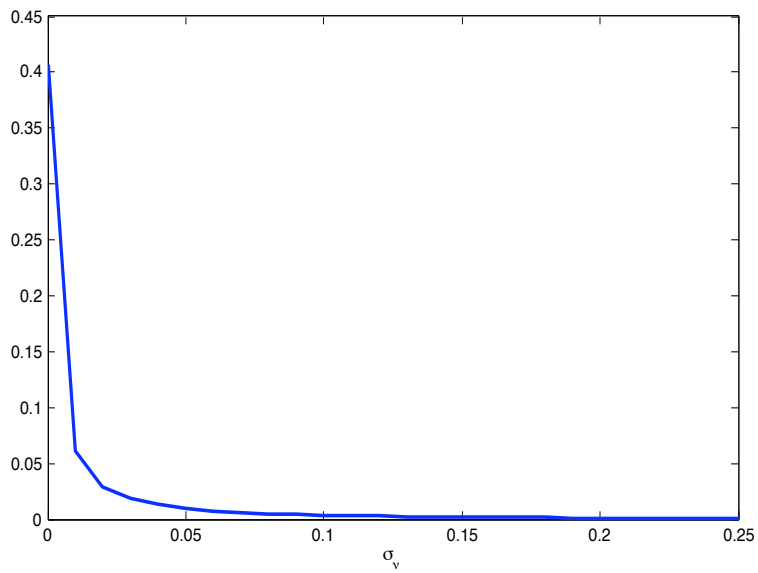


Figure 1: The element $K_{1,1}$ in the Kalman gain matrix

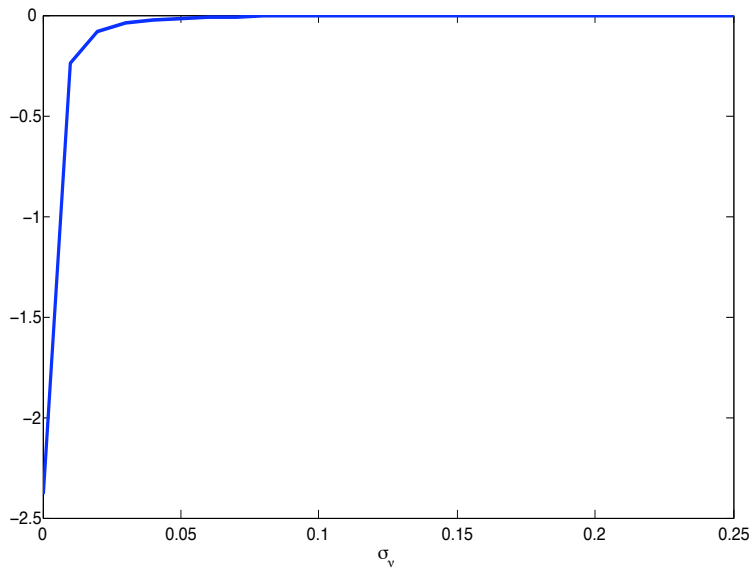


Figure 2: The element $K_{2,1}$ in the Kalman gain matrix

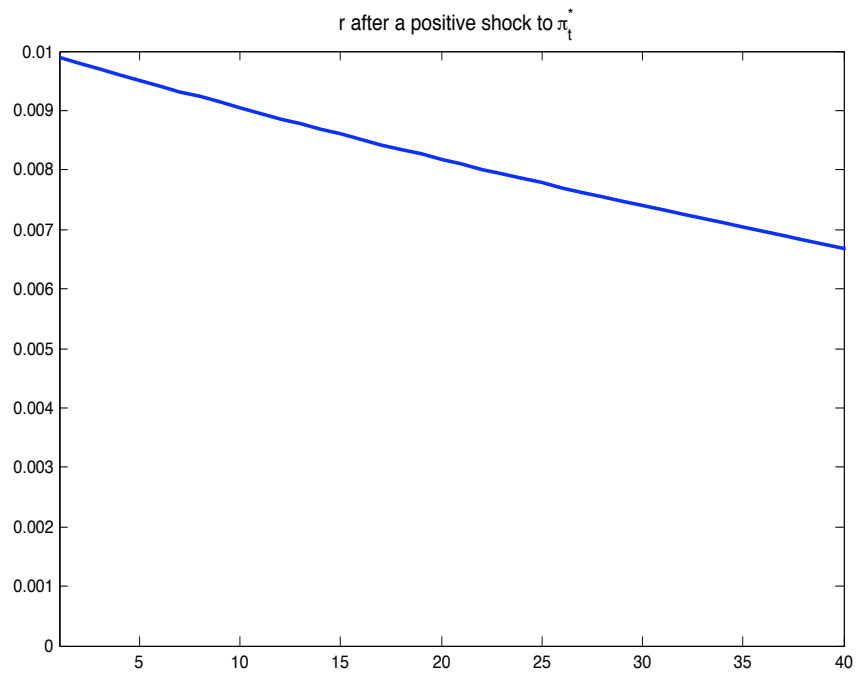


Figure 3: The reaction of interest rate after a positive 1% shock to π_t^*

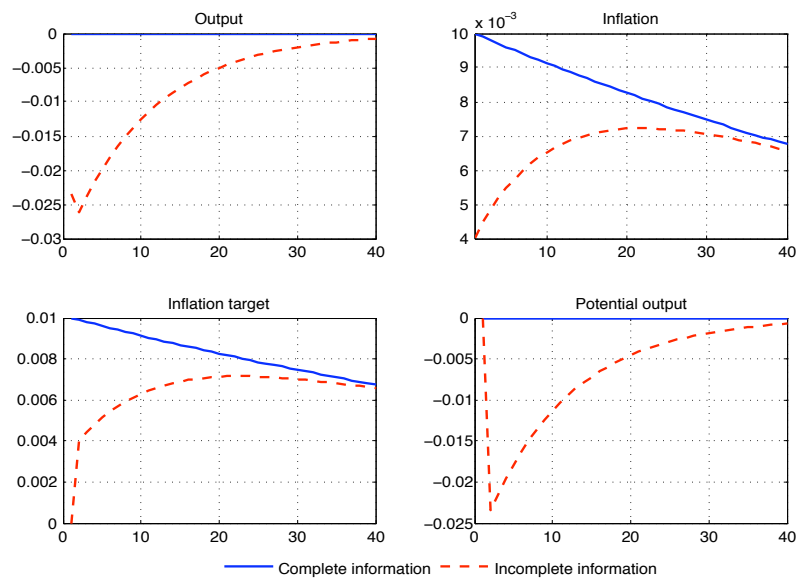


Figure 4: Impulse response function after a positive 1% shock to π_t^*

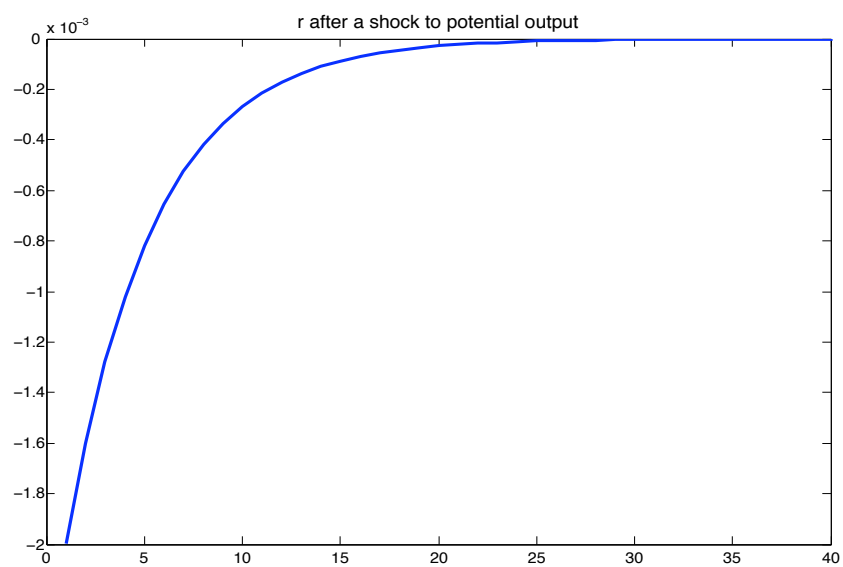


Figure 5: The reaction of interest rate after a positive 1% shock to \bar{y}_t

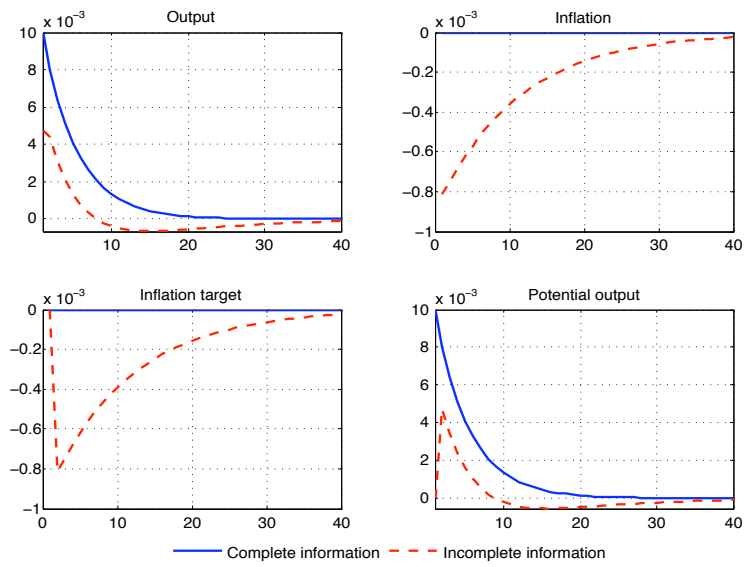


Figure 6: Impulse response function after a positive 1% shock to \bar{y}_t

Table 2: Welfare loss and volatility of key variables

Regime	Var(π_t)	Var(y_t)	Loss
Complete information	0.011	0.514	0.9232
$\sigma_\nu = 0$	0.0117	0.5017	0.9734
$\sigma_\nu = 0.01$	0.0119	0.5374	1.005
$\sigma_\nu = 0.05$	0.0135	0.5413	1.094
$\sigma_\nu = 0.1$	0.0147	0.542	1.1516

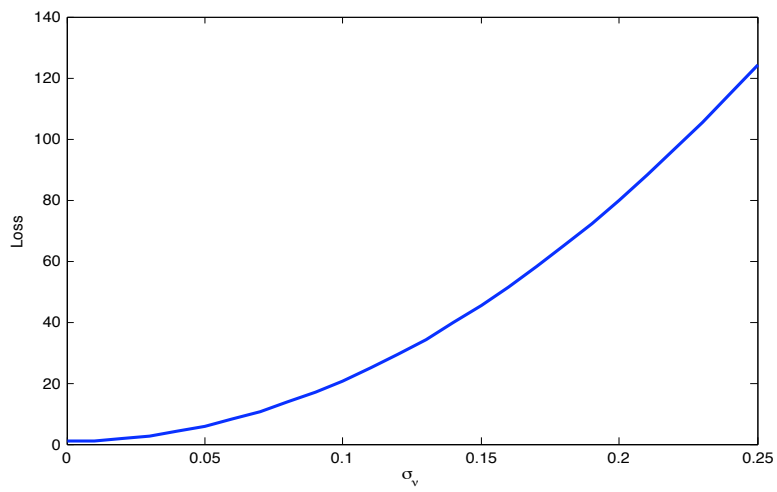


Figure 7: The value of the loss function varying the measurement error.

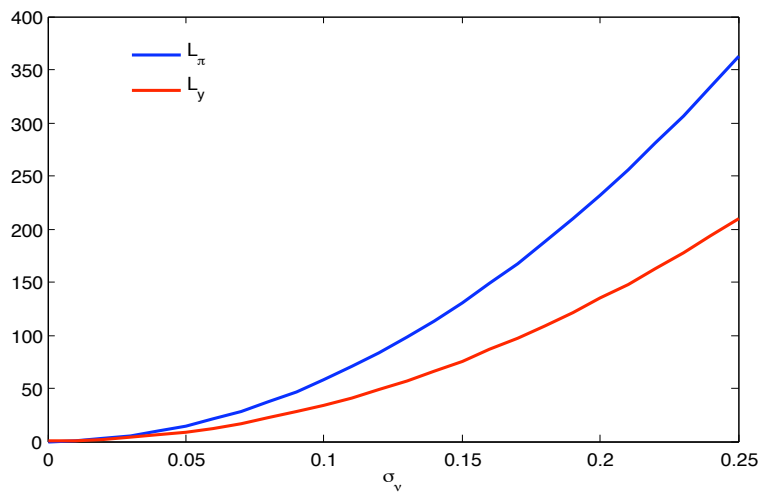


Figure 8: The value of the loss function (relative to the complete information case) when only one policy target is unobservable varying the measurement error.

Table 3: Welfare loss and volatility of key variables with observation equation (32) and different measurement error for the two policy targets.

σ_ν^π	σ_ν^y	$\mathbf{Var}(\pi_t)$	$\mathbf{Var}(y_t)$	Loss
0.1	0.1	0.0146	0.5389	1.1430
0.1	0.05	0.0146	0.5341	1.1370
0.1	0.03	0.0146	0.5291	1.1307
0.1	0.01	0.0146	0.5196	1.1189
0.05	0.1	0.0135	0.5389	1.0853
0.03	0.1	0.0127	0.5389	1.0433
0.01	0.1	0.0116	0.5114	0.9908

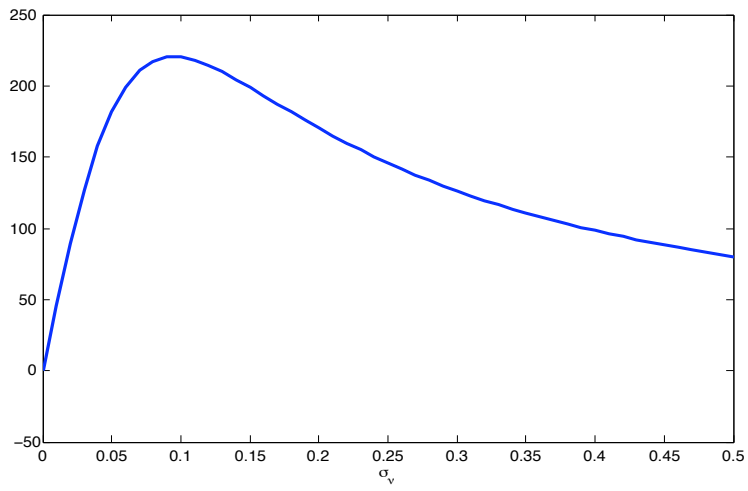


Figure 9: The percent change in the loss function varying alternatively the variance of either the inflation target or potential output.