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OPTIMAL MONETARY POLICY AND STOCK-PRICE DYNAMICS IN A NON-RICARDIAN DSGE MODEL

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Optimal Monetary Policy and Stock-Price Dynamics in a Non-Ricardian DSGE Model*

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Abstract

In a DSGE model with non-ricardian agents, à la Blanchard-Yaari, stock-price fluctuations affect the dynamics of aggregate consumption through wealth effects. This wealth effects can be characterized as an additional dynamic distortion with respect to the social planner allocation, related to the cross-sectional consumption dispersion that the decentralized allocation implies. By exploiting the specific cross-sectional distribution that the model implies for individual financial wealth, this paper derives the welfare criterion consistent with this economy, and shows that it features an additional target besides output-gap and price stability: financial stability.

The ultimate implication is that price stability is no longer necessarily optimal, even absent cost-push shocks. Given the quadratic form of the welfare criterion, some fluctuations in output and inflation will be optimal as long as they reduce the volatility of financial wealth.

JEL classification: E12, E44, E52
Key words: Monetary Policy, DSGE Models, Stock Prices, Wealth Effects.

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1 Introduction

The new millennium started with two major events in financial markets: the burst of the “dotcom” bubble in 2001 and the recent global financial meltdown. Such events proved, if there was any need, that developments in financial markets are nowadays of great relevance for the business cycle. As a consequence, the economic literature experienced a renewed interest in the links between monetary policy and dynamics in asset markets, and a widespread debate about the desirability that Central Banks be directly concerned with financial stability.

The benchmark theoretical paradigm linking the macroeconomy to financial markets is the “financial accelerator” model of Bernanke, Gertler and Gilchrist (1999), which Bernanke and Gertler (1999, 2001) exploit to analyze the supply-side effects of stock-price fluctuations and assess the implications of an explicit monetary-policy response to stock prices. In their Dynamic New Keynesian (DNK) model, principal-agent problems in the credit market imply real effects on production of fluctuations in the market value of collaterals. The main conclusion is that, since the macroeconomic relevance of stock-price dynamics relies on its links with inflation, a flexible inflation targeting approach is sufficient to achieve both price and financial stability, and that reacting to stock prices induces a perverse outcome in terms of output dynamics. Analogously, Gilchrist and Leahy (2002) find that both standard DNK models and economies featuring financial frictions, best replicate the dynamic properties of the benchmark RBC framework when no dedicated response is granted to stock-price dynamics. On the contrary, Cecchetti et Al. (2000, 2002, 2003), albeit simulating the very same model as in Bernanke and Gertler (1999), reach the opposite conclusion: adding an explicit response to stock prices reduces overall volatility in the economy without implying a perverse outcome for output dynamics as long as the monetary policy rule responds to the output gap as well.

This debate suggests that while this class of quantitative models captures a significant component of aggregate fluctuations, it seems to have implications that are extremely sensitive to specific calibrations of given structural or policy parameters. Moreover, it remains completely silent on the demand-side channel about which the Fed itself expressed some concern after the burst of the “dotcom” bubble.[1]

A different approach was followed by an alternative strand of literature, working with small-scale models from which analytical implications can be drawn. Carlstrom and Fuerst (2001) derive analytically the welfare-maximizing monetary policy in a flexible-price general equilibrium model with financial frictions. They show that, notwithstanding the absence of nominal rigidities – and hence the costs of inflation – a welfare-improving role for reacting to stock prices emerges, insofar as it can counteract the inefficient response of the economy to shocks to the equity market, which propagate through the binding collateral constraints. The same authors (Carlstrom and Fuerst, 2007) re-enter the debate and analyze the issue of equilibrium determinacy in a standard, representative-agent, DNK model in which the Central Bank responds also to stock prices. In their framework, however, the latter are redundant for the equilibrium allocation, unless monetary policy explicitly responds to them. Accordingly, a monetary policy rule including a response to

[1] See, for example, the minutes explaining the interest-rate cuts on March 20th, 2001, referring to “persistent pressures on profit margins [which] are restraining investment spending and, *through declines in equity wealth, consumption*”, and on April 18th and May 15th of the same year: “possible *effects of earlier reductions in equity wealth on consumption*”.

some measure of stock-market dynamics is never optimal (whatever the concept of "optimality" considered). Nisticò (2005) extends the benchmark small-scale DNK model by introducing a small heterogeneity in financial market participants, in the form of a "perpetual youth" demand-side. This extensions activates a demand-side channel through which financial shocks are transmitted to the real economy through wealth effects on aggregate consumption. The main monetary policy implication of this framework is that monetary policy consistent with price stability should respond to stock-price fluctuations when they are originated from demand-side shocks only. Moreover, this channel was found to be empirically relevant by Castelnuovo and Nisticò (2010), who estimate the model by means of structural bayesian techniques.[2]

This paper takes this "perpetual youth"-DNK model a step further, and performs a formal welfare analysis.

The main results are threefold.

First. I show that the wealth effects of stock-price fluctuations on aggregate consumption can be characterized as an additional dynamic distortion with respect to the social planner allocation. Indeed, while the social planner allocation implies cross-sectional equitability besides aggregate efficiency, the decentralized allocation implies cross-sectional consumption dispersion. This result is particularly useful because it provides intuition for the second (and core) result of this paper.

Second. I derive the welfare-relevant monetary policy loss function moving from a second-order approximation (SOA) of social welfare. Methodologically, this requires aggregating the individual utility functions of an infinite number of heterogeneous agents before a SOA is taken. I show that the specific cross-sectional distribution of wealth implied by the model allows for a simple characterization of the aggregate welfare criterion. From a heuristic perspective, the resulting criterion is a quadratic function, featuring not only squared output gap and squared inflation, but also squared financial wealth: financial stabilization is an additional and independent target of a welfare-maximizing Central Bank.

Third. Using the welfare-relevant monetary policy loss function, optimal monetary policy can be studied. In particular, the discretionary equilibrium in this setting implies that an endogenous trade-off emerges – even absent cost-push shocks – between output and price stability on the one hand, and financial stability on the other hand. As a consequence, in this framework strict price stability is no longer an optimal monetary policy regime.

The paper is structured as follows. Section 2 presents the "perpetual-youth"-DNK model of the business cycle, and discuss the frictionless allocation. Section 3 derives the welfare criterion, while Section 4 analyzes the implied stabilization trade-offs and the optimal monetary policy design under discretion. Section 5 concludes.

2 A Structural Model with Stock-Wealth Effects.

The important feature of the model economy is the demand side, which is populated by non-ricardian agents: at the beginning of each period, a constant fraction of consumers, trading in financial markets, is randomly replaced by an equivalent measure of agents with zero-holdings

[2]Di Giorgio and Nisticò (2007, 2010) further extend the framework to an open economy setting, while Airaudo, Nisticò and Zanna (2007) exploit the framework to study the determinacy and learnability properties of monetary policy rules responding to stock prices.

of financial assets. This feature of the demand side generates a non-trivial interaction between "newcomers" owning zero financial wealth (and therefore consuming less) and "old traders" with accumulated wealth (and therefore consuming more). This interaction is crucial in generating real effects of swings in stock prices, as it drives a wedge between the stochastic discount factor pricing all securities and the average marginal rate of intertemporal substitution in consumption – which in the case of infinitely-lived consumers coincide. In equilibrium, this wedge acts as a dynamic distortion to aggregate consumption, triggered by fluctuations in financial wealth.

Specifically, we work with a discrete-time stochastic version of the perpetual youth model introduced by Blanchard (1985) and Yaari (1965): the economy consists of an indefinite number of cohorts, facing a constant probability $\xi \in [0, 1]$ of being replaced as each period begins.

Households have Cobb-Douglas preferences over consumption and leisure,[3] and demand consumption goods and two types of financial assets: state-contingent bonds and equity shares issued by the monopolistic firms. In equilibrium, this side of the economy implies a state equation for consumption and a pricing equation for the equity shares.

Let $j \in (-\infty, t]$ index the discrete set of cohorts of agents interacting in the financial market in period t, and $i \in [0, 1]$ index the continuum of monopolistic firms in the economy. At time 0, therefore, agents that entered financial markets in period j seek to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t (1-\xi)^t \Big[\delta \log C_t(j) + (1-\delta) \log(1 - N_t(j))\Big]$$

subject to a sequence of budget constraints (expressed in real terms) of the form:

$$C_t(j) + E_t \left\{ \mathcal{F}_{t,t+1} \frac{P_{t+1}}{P_t} B_{t+1}(j) \right\} + \int_0^1 Q_t(i) Z_{t+1}(j,i)\, \mathrm{d}i \leq \frac{1}{1-\xi} \Omega_t(j) + W_t N_t(j) - T_t(j). \quad (1)$$

Consumers belonging to cohort j, therefore, seek to maximize the expected stream of utility flows, discounted to account for impatience (as reflected by the intertemporal discount factor β) and uncertain permanence in financial markets (as reflected by the probability of not being replaced in the next period, $1-\xi$). To that aim, they choose a pattern for individual real consumption $C(j)$, hours worked $N(j)$ and financial-asset holdings. At the end of period t, the latter consist of a set of contingent claims and a portfolio of equity shares. The one-period ahead stochastic nominal payoff of the state-contingent portfolio is $B^*_{t+1}(j)$, for which the relevant stochastic discount factor is $\mathcal{F}_{t,t+1}$. The stock portfolio includes equity shares issued by the monopolistic firms, $Z_{t+1}(j, i)$, whose real price at period t is $Q_t(i)$.[4]

At the beginning of each period, then, the sources of funds consist of the real disposable labor income, $W_t N_t(j) - T_t(j)$, and the real financial wealth carried over from the previous period

$$\Omega_t(j) \equiv \int_0^1 \Big(Q_t(i) + D_t(i)\Big) Z_t(j,i)\, \mathrm{d}i + B_t(j), \quad (2)$$

which includes the nominal pay-off on the equity portfolio and on the contingent claims. Moreover, following Blanchard (1985), financial wealth carried over from the previous period also pays off the

[3]The assumption of log-utility in consumption allows to achieve closed-form solutions for individual and aggregate consumption. See Smets and Wouters (2002) for a non-stochastic framework with CRRA utility.

[4]Throughout the paper, let a superscript asterisk denote nominal values: $X^* \equiv PX$.

gross return ($\frac{1}{1-\xi}$) on the insurance contract that redistributes among agents that have not been replaced (and in proportion to one's current wealth) the financial wealth left over from the ones who have.

The first-order conditions for an optimum consist of the budget constraint (1) holding with equality, the intra-temporal optimality condition with respect to consumption and leisure

$$C_t(j) = \frac{\delta}{1-\delta} W_t(1 - N_t(j)), \tag{3}$$

and the inter-temporal conditions with respect to the two types of financial assets:

$$\mathcal{F}_{t,t+1} = \beta \frac{U_c(C_{t+1}(j))}{U_c(C_t(j))} = \beta \frac{P_t C_t(j)}{P_{t+1} C_{t+1}(j)} \tag{4}$$

$$P_t Q_t(i) = E_t \left\{ \mathcal{F}_{t,t+1} P_{t+1} \left[Q_{t+1}(i) + D_{t+1}(i) \right] \right\}. \tag{5}$$

Equation (4) defines the equilibrium Stochastic Discount Factor (SDF) for one-period ahead nominal payoffs. An important implication of this relation is that – *at the individual level* – the SDF equals the Intertemporal Marginal Rate of Substitution (IMRS) in consumption. This further implies that the *individual* euler equations in this framework and in the Representative Agent (RA) setup are identical and do not depend on ξ. Indeed, rearranging equation (4) and taking conditional expectations yields:

$$\beta P_t C_t(j) = E_t \left\{ \mathcal{F}_{t,t+1} P_{t+1} C_{t+1}(j) \right\}. \tag{6}$$

This important result stems from the existence of the competitive insurance contract à la Blanchard (1985), which insures individuals against the risk of being replaced, and effectively makes them infinitely-lived, when it comes to financial decisions. Although with a finite expected lifetime, therefore, individual agents behave in this framework exactly as they would in the RA setup, smoothing their individual consumption over an infinite horizon.

Equation (5), in turn, defines the stock-price dynamics, by equating the nominal price of an equity share to its nominal expected payoff one period ahead, discounted by the SDF, $\mathcal{F}_{t,t+1}$.

It is useful, at this point, to characterize the agents interacting in the financial markets in period t as belonging to either one of two subsets: the "old traders" are those agents that are in the market since at least one period (i.e. $j \leq t-1$), while the "newcomers" are those that entered the market in the current period (i.e. $j = t$). The latter entered with no holdings of financial assets their sources of funds only consist of their labor income, net of public taxes and/or transfers. The budget constraint of the newcomers is therefore

$$C_t(t) + E_t \left\{ \mathcal{F}_{t,t+1} \frac{P_{t+1}}{P_t} B_{t+1}(t) \right\} + \int_0^1 Q_t(i) Z_{t+1}(t,i)\, \mathrm{d}i \leq W_t N_t(t) - T_t(t). \tag{7}$$

Therefore, in the period in which they enter financial markets, the agents face budget constraint (7); from the next period on, in turn, they face constraint (1).

The heterogeneity in the longevity of the several cohorts of agents in the financial market implies a non-degenerate distribution of individual financial wealth across generations, and a consequent distribution of individual consumption. We assume that the public authority is able to affect such distribution in the steady state, by means of an appropriate redistribution scheme, reflected by $T_t(j)$. Indeed, we assume that public taxes/transfers to individuals consist of two components:

$$T_t(j) = T_t + \phi(j)\mathcal{R}_t. \tag{8}$$

The first component is common across cohorts and is used to collect resources to finance the employment subsidy to monopolistic firms;[5] the second component, instead, is a "participation fee" (subsidy), proportional to the net real return from holding a one-period risk-free bond – inclusive of the extra-return due to the insurance scheme –

$$\mathcal{R}_t \equiv \frac{1}{1-\xi} - E_t \left\{ \mathcal{F}_{t,t+1} \frac{P_{t+1}}{P_t} \right\} \tag{9}$$

where the coefficient of proportionality is generation specific, $\phi(j)$, and defines whether the transfer is a fee ($\phi(j) > 0$) or a subsidy ($\phi(j) < 0$).[6] Such "participation fee" serves the only purpose of redistributing financial wealth across generations, by choosing ϕ so that the steady-state distribution is consistent with an equilibrium allocation around which a quadratic Taylor expansion of expected social welfare is a valid second-order approximation of expected welfare when evaluated using only first-order-approximated equilibrium conditions.[7] As such, this redistribution scheme does not have effects on the aggregate equilibrium dynamics.

The nominal gross return $(1 + r_t)$ on a safe one-period bond paying off one unit of currency in period $t + 1$ with probability 1 (whose price is therefore $E_t \{\mathcal{F}_{t,t+1}\}$) is defined by the following non-arbitrage condition:

$$(1 + r_t)E_t \{\mathcal{F}_{t,t+1}\} = 1. \tag{10}$$

Using equation (5), the optimal consumption-leisure choice (3), the equilibrium stochastic discount factor (4) and a condition ruling out Ponzi schemes, we can solve equation (1) forward, and derive an equilibrium relation between individual consumption and total wealth:

$$C_t(j) = \sigma H_t + \frac{\sigma}{1-\xi} \Big[\Omega_t(j) - \phi(j) \Big]. \tag{11}$$

Equation (11) defines the equilibrium time–t consumption of an agent transacting in financial markets since period j, as an increasing function of his financial wealth $\Omega_t(j)$ and his human wealth

$$H_t \equiv E_t \left\{ \sum_{k=0}^{\infty} (1-\xi)^k \mathcal{F}_{t,t+k} \frac{P_{t+k}}{P_t} \Big(W_{t+k} - T_{t+k} \Big) \right\}, \tag{12}$$

defined as the expected discounted after-tax value of the life-time labour endowment, and common across cohorts; finally, $\sigma \equiv \delta[1 - \beta(1 - \xi)]$ denotes the propensity to consume out of total (human and financial) wealth.

Analogously, since they enter the market without accumulated financial wealth ($\Omega_t(t) = 0$), in t the newcomers can consume only out of their human wealth, net of the redistributive subsidy that they get as soon as they enter the market:

$$C_t(t) = \sigma \left(H_t + \phi^{nc} \right), \tag{13}$$

where $\phi^{nc} \equiv -\frac{1}{1-\xi}\phi(t) > 0$.

On average, therefore, newcomers consume less than old traders because they hold a smaller amount of total wealth. This partition is important because all the action in this framework is

[5] See Section 2.2.

[6] Hence, for example, for the case of "newcomers" – since they enter with no financial wealth – it will be $\phi(j) < 0$.

[7] Woodford (2003, Ch. 6) derives and discusses the conditions under which such second-order approximation is valid. See also Section 3 for more on this point.

going to originate from the difference between these two subsets of agents, and their interaction in financial markets.

The overlapping-generation structure of households ($\xi > 0$) implies that the propensity to consume out of total wealth is higher than in the RA set up ($\xi = 0$), because a positive ξ reduces the effective rate at which households discount utility (i.e. $\beta(1-\xi)$) and this makes the present even more valuable than the future (σ is increasing in ξ). Moreover, a positive ξ also implies higher returns on current financial wealth through the insurance market.

2.1 Aggregation across Cohorts

The aggregate value of the generic variable X is computed as a weighted average of the corresponding generation-specific counterpart, where each cohort's weight is equal to its own mass:

$$X_t \equiv \sum_{j=-\infty}^{t} n_t(j) X_t(j) = \sum_{j=-\infty}^{t} \xi(1-\xi)^{t-j} X_t(j). \tag{14}$$

Since agents entering the market at time t hold no financial assets, however, all the financial wealth is held by old traders; accordingly, its aggregate value is defined as the average across old traders only:

$$\Omega_t \equiv \sum_{j=-\infty}^{t-1} \xi(1-\xi)^{t-1-j} \Omega_t(j). \tag{15}$$

Thereby, since the aggregator defined in (14) computes the average across *all* agents, it implies

$$\sum_{j=-\infty}^{t} \xi(1-\xi)^{t-j} \Omega_t(j) = (1-\xi)\Omega_t, \tag{16}$$

capturing the fact that all the financial wealth is held by old traders, whose mass is $(1-\xi)$. Moreover, since the fee/subsidy $\phi(j)$ serves redistributive purposes only, it must be such that its aggregate value is zero:

$$\sum_{j=-\infty}^{t} \xi(1-\xi)^{t-j} \phi(j) = 0. \tag{17}$$

We can now characterize the aggregate equilibrium conditions by applying the aggregator (14) to the individual restrictions derived in the previous section. Aggregate consumption is related to total aggregate wealth through the relation

$$C_t = \sigma\Big(\Omega_t + H_t\Big), \tag{18}$$

and the aggregate labor supply schedule reads:

$$N_t = 1 - \frac{1-\delta}{\delta}\frac{C_t}{W_t}. \tag{19}$$

We need now to derive the state equation for aggregate consumption, through aggregation of the Euler equation (6). Notice that, in the RA setup, the aggregation of the individual euler equations is straightforward because all agents are identical. This implies that, in equilibrium, the *individual* and *average* IMRS in consumption are the same, and equal to the stochastic discount factor. Hence, individual consumption smoothing carries over in aggregate terms and the time$-t$ level of average consumption is related only to the discounted value expected for $t+1$. This case is nested in our framework when the turnover rate ξ goes to zero.

On the contrary, when there is turnover in financial markets between agents with accumulated wealth and agents with no financial assets, the aggregation of the individual Euler equations is no longer straightforward, because agents in the financial market change from one period to the next. In order to see this, first notice that the aggregate time$-t+1$ consumption can be expressed as the weighted average of two components:

$$C_{t+1} = \xi C_{t+1}^{nc} + (1-\xi) C_{t+1}^{ot}, \tag{20}$$

where the first term is the per-capita consumption of newcomers

$$C_{t+1}^{nc} \equiv C_{t+1}(t+1),$$

the second is the per-capita consumption of old traders

$$C_{t+1}^{ot} \equiv \sum_{j=-\infty}^{t} \xi(1-\xi)^{t-j} C_{t+1}(j),$$

and the weights are given by the mass of agents in each subset. Second, applying the aggregator (14) to the individual euler equation (6) yields on the left-hand side the aggregate time$-t$ consumption, and on the right-hand side the average consumption of *old traders only*:

$$\beta P_t C_t = E_t \left\{ \mathcal{F}_{t,t+1} P_{t+1} C_{t+1}^{ot} \right\}. \tag{21}$$

In order to derive a state equation for aggregate consumption moving from the above relation, therefore, we need to account for the wedge between the average level of consumption across old traders only and the average across *all* agents that are in the market at $t+1$. Equations (13), (18) and (20) show that this wedge is proportional to the difference in per-capita consumption between the two subsets, and to the aggregate stock of financial wealth held at $t+1$:

$$C_{t+1}^{ot} - C_{t+1} = \xi \left(C_{t+1}^{ot} - C_{t+1}^{nc} \right) = \frac{\xi\sigma}{1-\xi} \left(\Omega_{t+1} - \phi^{nc} \right), \tag{22}$$

An increase in financial wealth (even temporary) enlarges this wedge because it makes the difference between the average consumption of old traders and that of newcomers larger, and thus distorts the dynamics of aggregate consumption.

Equation (22) shows that the magnitude of such distortion depends upon two factors. First, higher rates of replacement (ξ), for given swings in stock prices, imply a larger fraction of people entering the market tomorrow and being unaffected by variations in financial wealth. Second, higher levels of aggregate stock-market wealth (Ω), for a given rate of replacement, imply larger effects on current consumption, and therefore a higher difference with the expected future level.

We can use now this formulation of the wedge to substitute C_{t+1}^{ot} out of equation (21), rearrange and get

$$C_t = \frac{\xi\sigma}{\beta(1-\xi)} E_t \left\{ \mathcal{F}_{t,t+1} (1+\pi_{t+1}) \left(\Omega_{t+1} - \phi^{nc} \right) \right\} + \frac{1}{\beta} E_t \left\{ \mathcal{F}_{t,t+1} (1+\pi_{t+1}) C_{t+1} \right\}. \tag{23}$$

The first term in the equation above captures the stock-wealth effects, and is increasing in the turnover rate ξ. Therefore, while *individual* equilibrium conditions are identical to the RA case, the equilibrium dynamic path for *aggregate* consumption is generally different. Indeed, in a framework with non-ricardian agents the growth rate of aggregate consumption is distorted by fluctuations in financial wealth, whereas in the RA setup the latter are neutral, as shown by equation (23) when the turnover rate ξ is zero.

This mechanism, therefore, implies a direct channel by which the dynamics of stock prices can feed back into the real part of the model. Indeed, a rise in stock prices at time t reflects an increase in the expected financial wealth for period $t+1$, as shown by equation (5). All individuals in the financial market at t will then increase their current consumption expenditures to optimally smooth their intertemporal profile. At $t+1$, however, a fraction of these individuals will be replaced by newcomers who hold no equity shares, and whose consumption will therefore not be affected by the shock to the stock wealth. Consequently, the increase in stock prices affects current aggregate consumption more than the aggregate level expected for tomorrow.

Such distortion in the dynamics of aggregate consumption, with respect to the RA case, vanishes in either one of two cases: *i)* there is no heterogeneity in financial market participants ($\xi = 0$); *ii)* the aggregate stock of financial wealth is stable at the value $\Omega_{t+1} = \phi^{nc}$. In both cases, indeed, the wedge between the average consumption of old traders and that of newcomers is closed, and the optimal degree of individual-consumption smoothing goes through to aggregate consumption as well.

2.2 Firms and Price-Setting.

The supply-side of the economy is standard as in most New-Keynesian literature. A competitive retail sector produces the final consumption good Y_t packing the continuum of intermediate differentiated goods by means of a CRS technology,

$$Y_t = \left[\int_0^1 Y_t(i)^{1/(1+\mu)}\, di \right]^{(1+\mu)},$$

in which $\mu > 0$ captures the degree of market power in the market for inputs $Y_t(i)$. Equilibrium in this sector yields the input demand function and the aggregate price-index:

$$Y_t(i) = \left[\frac{P_t(i)}{P_t} \right]^{-(1+\mu)/\mu} Y_t \qquad\qquad P_t = \left[\int_0^1 P_t(i)^{-1/\mu}\, di \right]^{-\mu}. \tag{24}$$

A monopolistic wholesale sector produces a continuum of differentiated perishable goods, indexed $i \in [0,1]$, out of labor services, according to the following CRS production function

$$Y_t(i) = A_t N_t(i), \tag{25}$$

in which $A_t \equiv A \exp(a_t)$ captures aggregate, log-stationary, productivity shocks. Aggregating across firms and using the demand for intermediate goods (24) yields

$$\mathcal{Z}_t Y_t = A_t N_t, \tag{26}$$

in which $N_t \equiv \int_0^1 N_t(i)\,\mathrm{d}i$ is the aggregate level of hours worked and

$$\mathcal{Z}_t \equiv \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{1+\mu}{\mu}} \mathrm{d}i$$

is an index of price dispersion over the continuum of intermediate goods-producing firms, such that $\zeta_t \equiv \log(\mathcal{Z}_t)$ is second-order. We assume that the government subsidizes employment at the constant rate τ^e, to offset – on average – the monopolistic distortion. Given this assumption, and the production function (25), the equilibrium real marginal costs are constant across firms and given by:

$$MC_t = (1-\tau^e)\frac{W_t}{A_t}. \tag{27}$$

The price-setting mechanism follows Calvo's (1983) staggering assumption, with $1-\theta$ denoting the probability for a firm of having the chance to re-optimize in a given period. When able to set its price optimally, each firm seeks to maximize the expected discounted stream of future dividends, otherwise they keep the price unchanged.

Equilibrium in this side of the economy implies the familiar New-Keynesian Phillips Curve, relating the current inflation rate to the discounted expected one and real marginal costs.

2.3 General Equilibrium.

The fiscal authority runs a balanced budget in every period, and finances the employment subsidies through lump-sum taxes to households. In equilibrium, all output is absorbed by private consumption ($Y_t = C_t$), the net supply of state-contingent bonds is nil ($B_t = 0$), and the aggregate stock of outstanding equity for each wholesale firm equals the corresponding total amount of issued shares, normalized to 1 ($Z_t(i) = 1$ for all $i \in [0,1]$). As a consequence, the dynamics of stock prices are defined by the following pricing equation, micro-founded on the consumers' optimal behavior and deriving from the aggregation across firms of equation (5):

$$Q_t = E_t\left\{\mathcal{F}_{t,t+1}(1+\pi_{t+1})\left(Q_{t+1}+D_{t+1}\right)\right\}. \tag{28}$$

In the equations above, D_t and Q_t define total real dividend payments and the aggregate real stock-price index, respectively:[8]

$$D_t \equiv \int_0^1 D_t(i)\,\mathrm{d}i \qquad\qquad Q_t \equiv \int_0^1 Q_t(i)\,\mathrm{d}i. \tag{29}$$

The conditions above finally imply that the stock of aggregate wealth is simply

$$\Omega_t = Q_t + D_t. \tag{30}$$

[8] For the sake of accuracy, we should point out that the second term in (29) actually defines the aggregate real market capitalization. However, given the assumptions regarding the mass of wholesalers and the total outstanding shares, both normalized to 1, the aggregate market capitalization is also equivalent to the aggregate stock-price index.

2.4 The Efficient Allocation.

This section derives the efficient allocation which will serve as welfare-relevant benchmark to compute optimal policies.

We derive the efficient allocation as the solution to the problem faced by a social planner who allocates consumption and hours among households present in the market. Specifically, at time $t = 0$, the social planner maximizes the expected discounted stream of weighted averages of households' utility flows, where each cohort is given a weight corresponding to its mass,

$$E_0 \sum_{t=0}^{\infty} \beta^t \sum_{j=-\infty}^{t} \xi(1-\xi)^{t-j} \Big[\delta \log C_t(j) + (1-\delta)\log(1-N_t(j))\Big]$$

subject to the resource constraint

$$Y_t = C_t \equiv \sum_{j=-\infty}^{t} \xi(1-\xi)^{t-j} C_t(j)$$

and the aggregate production function

$$Y_t/A_t = N_t \equiv \sum_{j=-\infty}^{t} \xi(1-\xi)^{t-j} N_t(j),$$

in both of which we related aggregate variables to cohort-specific levels, using the aggregator (14).

Given the absence of endogenous state variables, this problem can be formulated as a sequence of static, unconstrained maximizations:

$$\max_{\{C_t(j),N_t(j)\}_{j=-\infty}^{t}} \sum_{j=-\infty}^{t} \xi(1-\xi)^{t-j} \Big[\delta \log C_t(j) + (1-\delta)\log(1-N_t(j)) + \Lambda_t\big(A_t N_t(j) - C_t(j)\big)\Big],$$

for each $t = 0, 1, ..., \infty$. Since both the resource and technological constraints are defined in terms of aggregate variables, the lagrange multiplier Λ_t is independent of j. The optimality conditions of this problem are:

$$\delta = \Lambda_t C_t(j) \tag{31}$$

$$1-\delta = \Lambda_t A_t (1 - N_t(j)) \tag{32}$$

for all $j \in \{j \leq t : j \in \mathbb{Z}\}$ and all t. By aggregating across generations, we get

$$\delta = \Lambda_t C_t \tag{33}$$

$$1-\delta = \Lambda_t A_t (1 - N_t), \tag{34}$$

which, together with the resource and technological constraints, imply

$$C_t^e(j) = C_t^e = Y_t^e = \delta A_t \tag{35}$$

$$N_t^e(j) = N_t^e = \delta \tag{36}$$

for all t. Therefore, the solution to a social planner problem in this case implies an equilibrium allocation that is both *efficient and equitable* (hence the superscript ^{e}): all generations demand the same amount of consumption and supply the same amount of hours worked.

In our decentralized economy, instead, the equilibrium is not equitable, because fluctuations in the stock of financial wealth affects the wedge between the average consumption of old traders and the average consumption of newcomers, as implied by equation (22).

2.4.1 The Steady-State Equilibrium.

The solution to the Social Planner problem outlined above implies the following efficient and equitable steady state:

$$C^e(j) = C^e = Y^e = \delta A \tag{37}$$
$$N^e(j) = N^e = \delta. \tag{38}$$

In turn, the decentralized economy converges to the following non-stochastic, zero-inflation steady state:

$$Y = \frac{\delta}{1-\delta} W(1-N) \tag{39}$$
$$[\beta(1+r)-1]Y = \frac{\xi\sigma}{1-\xi}\left(\Omega - \phi^{nc}\right) \tag{40}$$
$$(1+r)Q = \Omega \tag{41}$$
$$\Omega = Q + D \tag{42}$$
$$W = \frac{A}{(1+\mu)(1-\tau^e)} \tag{43}$$
$$Y = AN \tag{44}$$
$$D = Y - (1-\tau^e)WN. \tag{45}$$

By using (39), (43) and (44), we can derive the steady state level of output in the decentralized economy:

$$Y = \frac{\delta A}{(1+\mu)(1-\tau^e)(1-\delta)+\delta}. \tag{46}$$

Such market economy can replicate both aggregate efficiency and intergenerational equitability, with appropriate redistributive schemes. Indeed, *aggregate efficiency* can be replicated by appropriately setting the employment subsidy, such that

$$(1+\mu)(1-\tau^e) = 1, \qquad \text{i.e.} \qquad \tau^e = \frac{\mu}{1+\mu}. \tag{47}$$

Moreover, *intergenerational equitability* can be replicated by appropriately setting the financial fee/subsidy, so that it transfers resources from wealthy agents to poor ones. Specifically, the redistributive scheme that implements intergenerational equitability requires

$$\phi(j) = \Omega(j) - (1-\xi)\Omega, \tag{48}$$

so that individual steady-state consumption is

$$C(j) = C = \sigma\left(\Omega + H\right),$$

for all j. This further implies that all agents whose steady-state level of individual financial wealth is below the average receive a subsidy ($\phi < 0$) while those with a higher stock of individual wealth pay a fee. The optimal redistributive scheme, moreover, by implying $\phi^{nc} = \Omega$, also implies $\beta(1+r) = 1$, through equation (40).

The equilibrium values of the remaining variables in the efficient and equitable steady state are:

$$N = \delta \tag{49}$$

$$C = \delta A \tag{50}$$

$$W = A \tag{51}$$

$$D = \frac{\mu}{1+\mu}\delta A \tag{52}$$

$$Q = \frac{\mu}{1+\mu}\frac{\beta}{1-\beta}\delta A \tag{53}$$

$$\Omega = \frac{\mu}{1+\mu}\frac{1}{1-\beta}\delta A. \tag{54}$$

2.5 The Linear Model.

First-order approximation around the efficient and equitable steady state derived in the previous section yields the following log-linear system for the demand-side of our model economy:[9]

$$w_t = y_t + \varphi n_t \tag{55}$$

$$y_t = E_t y_{t+1} + \psi E_t \omega_{t+1} - (r_t - E_t \pi_{t+1} - \rho) \tag{56}$$

$$\omega_t = \beta E_t \omega_{t+1} + (1-\beta) d_t - \beta(r_t - E_t \pi_{t+1} - \rho) \tag{57}$$

$$\omega_t = \beta q_t + (1-\beta) d_t \tag{58}$$

$$d_t = y_t - \frac{1}{\mu} mc_t, \tag{59}$$

in which $\varphi \equiv \frac{N}{1-N} = \frac{\delta}{1-\delta}$ is the inverse of the (steady-state) Frisch elasticity of labor supply and $\psi \equiv \xi \frac{1-\beta(1-\xi)}{1-\xi} \frac{\delta}{1-\beta} \frac{\mu}{1+\mu}$.

Equation (56) is the linear state equation for consumption and output. Notice that a positive turnover rate ($\xi, \psi > 0$) distorts the inter-temporal path of aggregate consumption, and make the dynamics of financial wealth matter for the equilibrium dynamics of real output. Indeed, as in the RA framework, current real output depends upon the real interest rate and expectations about the future. While in the RA setup, however, the expectations about the future are only related to output, here also expectations about future financial wealth matter explicitly, the more the higher the turnover rate ξ. This makes the dynamics of aggregate financial wealth relevant for current aggregate consumption and the transmission of real and monetary shocks.

As to the supply-side, moving from the definition of the general price level in (24) and considering that a fraction $(1-\theta)$ of all firms revise their price at t at the common level P_t^o, and that a fraction θ keeps the price constant at last period's general price level, we can describe inflation dynamics with a familiar New Keynesian Phillips curve:

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1-\theta)(1-\theta\beta)}{\theta} mc_t. \tag{60}$$

Linearization of equation (27) yields

$$mc_t \equiv \log((1+\mu)MC_t) = w_t - a_t = (1+\varphi)(y_t - a_t), \tag{61}$$

[9]In what follows lower-case letters denote log-deviations from the steady state: $x_t \equiv \log(X_t/X)$. Note that, $(1+r_t)$ being the gross interest rate, r_t is (to first order) the actual net interest rate. The log-deviation of the gross interest rate from its steady state is therefore $r_t - \rho$, where we set $\rho \equiv \log(1+r) = -\log\beta$.

where the last equality is obtained using the labor supply (55), the resource constraint ($y_t = c_t$) and the production function.

Imposing on equation (61) the condition that marginal costs be constant at their steady state level, finally, implies the equation for the natural level of output

$$\overline{y}_t = a_t, \tag{62}$$

and therefore link short-run real marginal costs to the output gap $x_t \equiv y_t - \overline{y}_t$:

$$mc_t = (1 + \varphi)x_t. \tag{63}$$

For future reference, it is useful to use the above equations (61) and (63) to obtain the following formulation for aggregate dividends:

$$d_t = \overline{y}_t - \frac{1 + \varphi - \mu}{\mu} x_t, \tag{64}$$

which highlights that the elasticity of real dividends with respect to the output gap is decreasing in the steady state markup (μ) and the Frisch elasticity of labor supply ($1/\varphi$).

Using the above to substitute for real dividends on equation (58), we can describe the private sector of our model economy by means of the following linear system:

$$y_t = E_t y_{t+1} + \psi E_t \omega_{t+1} - (r_t - E_t \pi_{t+1} - \rho) \tag{65}$$

$$\omega_t = \beta E_t \omega_{t+1} - (1 - \beta) \frac{1 + \varphi - \mu}{\mu} x_t - \beta(r_t - E_t \pi_{t+1} - \rho) + (1 - \beta)\overline{y}_t \tag{66}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t. \tag{67}$$

In such economy, therefore, three sources of distortion affect the equilibrium allocation in a given period: two of them are common to the baseline New Keynesian model, while the third is specific to our framework with stock–wealth effects. First, a static distortion due to monopolistic competition implies a lower level of aggregate supplied output, both in the short and in the long run. This distortion can be corrected by an appropriate (constant) employment subsidy. Second, a dynamic distortion due to nominal rigidities implies a positively-sloped supply schedule, price dispersion across different production sectors and ultimately a lower level of short-run equilibrium output (this distortion fades away in the long-run). Third, a dynamic distortion due to the interaction in financial markets of heterogeneous agents with different levels of financial wealth implies consumption dispersion across households and, coupled with nominal rigidities making aggregate demand relevant for equilibrium output, affects the short-run rate of growth in consumption and output. This distortion can be corrected in the steady-state by an appropriate redistribution scheme.

2.5.1 The Flexible-Price Allocation

The Flexible-Price Equilibrium (FPE) arises in this model when the probability of having to charge last period's price goes to zero ($\theta = 0$). In this case, denoting variables in the FPE with an upper bar, real marginal costs are constant: $\overline{MC}_t = (1 + \mu)^{-1}$. Moreover, given the optimal employment subsidy, equation (27) implies the efficiency condition $\overline{W}_t = A_t$. Accordingly, labor market clearing implies the equilibrium level of aggregate potential output and hours worked:

$$\overline{Y}_t = \delta A_t \tag{68}$$

$$\overline{N}_t = \delta. \tag{69}$$

In a first-order approximation, the equilibrium allocation is derived as the solution of the system:

$$\overline{y}_t = a_t \tag{70}$$

$$\overline{\pi}_t = 0 \tag{71}$$

$$\overline{rr}_t = E_t \Delta a_{t+1} + \psi E_t \overline{\omega}_{t+1} \tag{72}$$

$$\overline{\omega}_t = \beta E_t \overline{\omega}_{t+1} - \beta \overline{rr}_t + (1-\beta) a_t \tag{73}$$

which implies that the level of stock-wealth follows aggregate productivity

$$\overline{\omega}_t = \frac{1-\beta\rho_a}{1-\beta\rho_a(1-\psi)} a_t,$$

and the real interest rate dynamics supporting such allocation is

$$\overline{rr}_t = E_t \Delta a_{t+1} + \frac{1-\beta\rho_a}{1-\beta\rho_a(1-\psi)} \psi \rho_a a_t. \tag{74}$$

The FPE aggregate allocation is *efficient* because the static distortion related to monopolistic competition is corrected by the employment subsidy (47), but it is not *equitable*, because fluctuations in the stock of financial wealth imply fluctuations in the wedge between the average consumption of newcomers and that of old traders. However, since in the FPE the Phillips Curve is vertical, the equilibrium level of real activity is determined on the supply side of the economy only, and fluctuations in financial wealth are neutral. While the natural level of output is the same as that arising in the RA version of the model ($\xi = 0$), though, the natural levels of real interest rate and financial wealth are not. In the RA setup, indeed, the natural real rate of interest would only have to accommodate the pressures on output coming from productivity shocks ($\overline{rr}_t = E_t \Delta a_{t+1}$), and stock market wealth would follow productivity one-for-one ($\overline{\omega}_t = a_t$). Equation (74), instead, shows that when there are stock-wealth effects, the natural interest rate partially sterilizes productivity shocks, to absorb the fluctuations in financial wealth that would otherwise distort the dynamics of aggregate consumption and output.

2.5.2 The Efficient and Equitable Allocation

As shown in Section 2.4, the efficient and equitable allocation implies the same level of consumption across all agents. The Efficient and Equitable Equilibrium (EEE), therefore, can be defined as an equilibrium in which not only the output gap and inflation rate are zero, but also the stock of financial wealth is constant at its steady state value: $\omega_t^e = 0$. Indeed, as implied by equation (22), and given the optimal redistribution scheme (48), when the stock of financial wealth equals its steady state value, the wedge in individual consumption shrinks to zero, and equitability of the equilibrium allocation is restored. Therefore, the equilibrium allocation ought to be:

$$y_t^e = \overline{y}_t = a_t \tag{75}$$

$$\pi_t^e = 0 \tag{76}$$

$$\omega_t^e = 0 \tag{77}$$

However, there is no real interest-rate path that support this equilibrium allocation.

Result 1. *In a decentralized economy with non-Ricardian agents and an active stock market, there is no real interest rate dynamics supporting the Efficient and Equitable Equilibrium allocation.*

Proof. Impose $y_t = \overline{y}_t = a_t$ and $x_t = \pi_t = \omega_t = 0$ on system (65)–(67). Then equation (65) implies $rr_t^e = E_t \Delta a_{t+1}$, while equation (73) implies $rr_t^e = \frac{1-\beta}{\beta} a_t$, which are the same only as long as $a_t = 0$ for all t.

So one question that arises at this point is: given the proposition above and the fact that, in turn, in the FPE aggregate consumption is at its efficient level and fluctuations in financial wealth do not affect its dynamics, can we consider the FPE allocation as a proper target for monetary policy? Should we instead rely on the EEE allocation? In order to understand this, we now turn to the computation of a monetary policy loss function derived from the households' preferences.

3 The Welfare-Based Loss Function.

The natural Welfare Criterion in our framework is the weighted average of the agents' utility flows, where each cohort is given a weight corresponding to the mass of agents belonging to it:

$$\mathcal{U}_t \equiv \sum_{j=-\infty}^{t} \xi(1-\xi)^{t-j} \Big[u(C_t(j)) - v(N_t(j)) \Big], \tag{78}$$

in which

$$u(C_t(j)) \equiv \delta \log C_t(j) \qquad v(N_t(j)) \equiv -(1-\delta)\log(1 - N_t(j)).$$

Following Woodford (2003), we derive the Welfare-Based Loss Function by taking a second-order approximation of $\mathcal{U}_t$ around the efficient and equitable steady state detailed in the previous Section.

Let lower-case variables denote the log-deviation from the steady state: $z_t \equiv \log(Z_t/Z)$. Accordingly, the utility from consumption at the cohort level can be written as

$$u(C \exp(c_t(j))) = \delta \Big(c_t(j) \Big) + t.i.p. + \mathcal{O}(\|\chi\|^3), \tag{79}$$

where $\|\chi\|$ denotes the bound on the magnitude of the vector of disturbances and $t.i.p.$ collects all the terms that are independent of policy. The cohort-level disutility from labor, instead, can be expressed as

$$v(N \exp(n_t(j))) = (1-\delta)\varphi \Big(n_t(j) + \frac{1+\varphi}{2} n_t(j)^2 \Big) + t.i.p. + \mathcal{O}(\|\chi\|^3). \tag{80}$$

Given the equitability of the steady state, the aggregator defined in (14) works also for log-deviations, and defines the cross-sectional mean:

$$z_t = \sum_{j=-\infty}^{t} \xi(1-\xi)^{t-j} z_t(j) = E_j z_t(j).$$

Moreover, the optimal employment subsidy implies the efficiency condition $\varphi(1-\delta)=\delta$.

Given the above, we can write the *aggregate* utility from consumption as

$$U_t \equiv \sum_{j=-\infty}^{t} \xi(1-\xi)^{t-j} u\Big(C\exp(c_t(j))\Big) = \delta\Big(c_t\Big) + t.i.p. + \mathcal{O}(\|\chi\|^3), \tag{81}$$

which does not display quadratic terms in consumption, given the assumption of log utility. The *aggregate* disutility from labor, in turn, can be expressed as

$$\begin{aligned} V_t \equiv \sum_{j=-\infty}^{t} \xi(1-\xi)^{t-j} v(N\exp(n_t(j))) \\ = \delta\Big(n_t + \frac{1+\varphi}{2}E_j\left(n_t(j)^2\right)\Big) + t.i.p. + \mathcal{O}(\|\chi\|^3) \\ = \delta\Big(n_t + \frac{1+\varphi}{2}n_t^2 + \frac{1+\varphi}{2}var_j n_t(j)\Big) + t.i.p. + \mathcal{O}(\|\chi\|^3), \end{aligned} \tag{82}$$

in which the last equality uses $E(z^2) = E(z)^2 + var(z)$. The equation above shows that the aggregate disutility from labor displays an additional second-order term, with respect to the RA setup, which is related to the cross-sectional dispersion in hours: $var_j n_t(j)$.

Using the resource constraint and a second-order approximation of the aggregate production function, we obtain

$$U_t = \delta\Big(y_t\Big) + t.i.p. + \mathcal{O}(\|\chi\|^3), \tag{83}$$

and

$$V_t = \delta\Big(y_t + (1+\varphi)a_t y_t + \frac{1+\varphi}{2}y_t^2 + \zeta_t + \frac{1+\varphi}{2}var_j n_t(j)\Big) + t.i.p. + \mathcal{O}(\|\chi\|^3), \tag{84}$$

in which ζ_t denotes the log-price dispersion across monopolistic firms.

By taking the difference of the two terms above, we can write the Welfare Criterion (78) – expressed as a percentage of steady state consumption – in terms of squared output gap, price dispersion across firms and hours dispersion across households:

$$\begin{aligned} \mathcal{U}_t = U_t - V_t = -\delta\Big((1+\varphi)a_t y_t + \frac{1+\varphi}{2}y_t^2 + \zeta_t + \frac{1+\varphi}{2}var_j n_t(j)\Big) + t.i.p. + \mathcal{O}(\|\chi\|^3) \\ = -\frac{\delta(1+\varphi)}{2}\Big(x_t^2 + \frac{2}{1+\varphi}\zeta_t + var_j n_t(j)\Big) + t.i.p. + \mathcal{O}(\|\chi\|^3), \end{aligned} \tag{85}$$

in which the last equality uses the definition of potential output, $\overline{y}_t = a_t$, and of output gap, $x_t \equiv y_t - \overline{y}_t$.

We can now use a first-order approximation of the individual labor supply (3), to relate the cross-sectional variance of hours worked to that of individual consumption: $\varphi^2 var_j n_t(j) = var_j c_t(j)$. Therefore, we can express the Welfare Criterion, up to second-order, as

$$\mathcal{U}_t = -\frac{\delta(1+\varphi)}{2}\Big(x_t^2 + \frac{2}{1+\varphi}\zeta_t + \frac{1}{\varphi^2}var_j c_t(j)\Big) + t.i.p. \tag{86}$$

The above criterion differs from the one we would get in a RA setup only for the last term in parentheses, which measures the consumption dispersion across households. Indeed, if all agents were to consume *in every period* t the same amount of goods, the cross-sectional variance would be zero, and the benchmark case with a Representative Agent would arise.

To understand the implications of this additional term, it is useful to recall the partition of the set of agents into the two subsets of "newcomers" and "old traders". In particular, in a first-order approximation, individual consumption is

$$c_t(j) = \begin{cases} \tilde{h}_t & \text{for newcomers } (j = t)\text{, whose mass is } \xi \\ \frac{\psi}{\xi}\frac{\Omega(j)}{\Omega}\omega_t(j) + \tilde{h}_t & \text{for old traders } (j \leq t-1)\text{, whose mass is } (1-\xi), \end{cases} \tag{87}$$

in which $\tilde{h}_t$ collects all the terms that are not cohort-specific (related to the average human wealth). Let $\Delta_t \equiv var_j(c_t(j)|j \leq t)$ denote the cross-sectional dispersion in individual consumption across *all* individuals. Using the law of total variance on such partition, we can express Δ_t as:

$$\Delta_t = E\Big[var_j\big(c_t(j)\big|j=t\big),\ var_j\big(c_t(j)\big|j \leq t-1\big)\Big] + var\Big[E_j\big(c_t(j)\big|j=t\big),\ E_j\big(c_t(j)\big|j \leq t-1\big)\Big]. \tag{88}$$

The first term is the mass-weighted average of two cross-sectional variances of individual consumption: the one among old traders and the one among newcomers. Since the newcomers are all identical, they all consume the same amount of goods: this term is, therefore, proportional to the consumption dispersion among old traders only:

$$E\Big[var_j\big(c_t(j)\big|j=t\big),\ var_j\big(c_t(j)\big|j \leq t-1\big)\Big] = (1-\xi)var_j(c_t(j)|j \leq t-1). \tag{89}$$

In turn, the dispersion of consumption among old traders in period t reflects the composition between newcomers and old traders, characterizing the financial market in $t-1$. Indeed, using a first-order approximation of equation (4), we notice that:

$$var_j(c_t(j)|j \leq t-1) = var_j(c_{t-1}(j) - f_{t-1,t} - \pi_t|j \leq t-1) = var_j(c_{t-1}(j)|j \leq t-1) = \Delta_{t-1} \tag{90}$$

The second term in (88), instead, captures the dispersion in the *average* consumption, between old traders and newcomers. Since financial wealth is held by old traders only, the average consumption of the latter is proportional to *average per-capita* financial wealth,[10] while that of newcomers is independent of it. As a consequence, the variance of average consumption between the two subsets is proportional to the squared average financial wealth:

$$\begin{aligned} var\Big[E_j\big(c_t(j)\big|j=t\big),\ E_j\big(c_t(j)\big|j \leq t-1\big)\Big] &= var\left[\tilde{h}_t,\ \frac{\psi}{\xi}\omega_t + \tilde{h}_t\right] \\ &= E\left[\tilde{h}_t^2,\ \left(\frac{\psi}{\xi}\omega_t + \tilde{h}_t\right)^2\right] - \left[E\left(\tilde{h}_t,\ \frac{\psi}{\xi}\omega_t + \tilde{h}_t\right)\right]^2 = \psi\frac{\sigma}{1-\beta}\frac{\mu}{1+\mu}\omega_t^2. \end{aligned} \tag{91}$$

This component, as argued above, captures the dispersion in average consumption between newcomers and old traders, and is therefore the relevant term driving the wedge that distorts the growth rate of aggregate consumption, as shown in Section 2.

[10]Notice that, since the stock of accumulated wealth (gross of the fee/subsidy) is not the same across households, the average per-capita financial wealth, up to first-order, is properly computed using:

$$\omega_t = \sum_{j=-\infty}^{t-1} \xi(1-\xi)^{t-1-j}\frac{\Omega(j)}{\Omega}\omega_t(j).$$

Therefore, the individual-consumption dispersion across all agents in the economy consists of two components: the financial-wealth dispersion *within* the subset of old traders and the squared *aggregate* financial wealth. Accordingly, we can characterize the evolution over time of the consumption dispersion, as:

$$\Delta_t = (1-\xi)\Delta_{t-1} + \psi \frac{\sigma}{1-\beta}\frac{\mu}{1+\mu}\omega_t^2. \tag{92}$$

Moving from an arbitrary initial level of consumption dispersion Δ_{-1}, which is independent of policies implemented from $t=0$ onward, we can therefore write the level at time t as

$$\Delta_t = (1-\xi)^{t+1}\Delta_{-1} + \psi \frac{\sigma}{1-\beta}\frac{\mu}{1+\mu}\sum_{s=0}^{t}(1-\xi)^{t-s}\omega_s^2, \tag{93}$$

and the discounted value over all periods $t > 0$ as

$$\sum_{t=0}^{\infty}\beta^t\Delta_t = \frac{\psi\delta\mu}{(1-\beta)(1+\mu)}\sum_{t=0}^{\infty}\beta^t\omega_t^2 + t.i.p. \tag{94}$$

Finally, the Welfare-Based Loss Function $\mathcal{L}_0$ is defined as the expected discounted stream of utility losses, expressed as a share of steady state consumption. Ignoring the terms independent of policy and those of third or higher order, we can therefore write it as

$$\mathcal{L}_0 \equiv -E_0\sum_{t=0}^{\infty}\beta^t\mathcal{U}_t = \frac{\delta(1+\varphi)}{2}E_0\sum_{t=0}^{\infty}\beta^t\left(x_t^2 + \alpha_\pi\pi_t^2 + \alpha_\omega\omega_t^2\right), \tag{95}$$

in which we defined $\alpha_\pi \equiv \frac{1+\mu}{\mu\kappa}$ and $\alpha_\omega \equiv \frac{\psi\delta\mu}{\varphi^2(1-\beta)(1+\mu)}$, and we made use of the following two lemmata:

Lemma 1. $\zeta_t = \frac{1}{2}\frac{1+\mu}{\mu}var_i p_t(i) + \mathcal{O}(\|\chi\|^3)$

Proof. Galì and Monacelli (2005),

Lemma 2. $\sum_{t=0}^{\infty}\beta^t var_i p_t(i) = \frac{\theta}{(1-\theta)(1-\theta\beta)}\sum_{t=0}^{\infty}\beta^t\pi_t^2$

Proof. Woodford (2003, Ch. 6).

Notice that $\alpha_\pi \equiv \frac{1+\mu}{\mu\kappa}$ is the familiar weight to inflation volatility, i.e. the ratio between the price-elasticity of demand $\frac{1+\mu}{\mu}$ and the slope of the Phillips Curve κ. On the other hand, $\alpha_\omega \equiv \frac{\psi\delta\mu}{\varphi^2(1-\beta)(1+\mu)}$ denotes the – additional – weight on aggregate financial-wealth volatility, which arises in our framework with non-ricardian agents: since variations in the aggregate financial wealth deepens the wedge between old traders and newcomers, they constitute a *dynamic* distortion to aggregate consumption growth and thereby imply a welfare loss with respect to the efficient and equitable allocation.

Indeed, an immediate implication of (95) is the answer to the question we posed above. Consumption dispersion across agents is a source of welfare loss. Accordingly, fluctuations of aggregate financial wealth around its steady state value imply a lower level of aggregate welfare. As a consequence, the FPE allocation, by implying an equilibrium level of financial wealth evolving in response to productivity shocks, is not a proper target for a welfare-maximizing Central Bank, which should instead target the Efficient and Equitable Allocation.

Result 2. *In a decentralized economy with non-Ricardian agents and an active stock market, a welfare maximizing Central Bank should target the Efficient and Equitable allocation. Financial stabilization ($\omega_t = 0$) becomes an explicit target of welfare-maximizing monetary policy, additional to output and inflation stabilization ($x_t = \pi_t = 0$). The relative weight on financial stabilization (α_ω) is the higher the stronger the stock-wealth effect (ψ) and the higher the market power of monopolistic firms (μ).*

4 Stabilization Trade-offs and Optimal Monetary Policy

An interesting implication of the two Results derived earlier is that a stabilization tradeoff endogenously arises, even in the absence of cost-push shocks. Indeed, since there is no interest rate dynamics which is able to support the EEE allocation, minimization of the loss function (95) implies that some fluctuations in inflation and the output gap will be optimal in order to reduce the fluctuations in financial wealth.

We can see this formally, by deriving the Optimal Monetary Policy (OMP) in this framework.

The complete system in terms of the relevant gaps can be written as follows:

$$x_t = E_t x_{t+1} + \psi E_t \omega_{t+1} - (r_t - E_t \pi_{t+1} - \overline{rr}_t) \tag{96}$$

$$\omega_t = \beta E_t \omega_{t+1} - (1-\beta)\frac{1+\varphi-\mu}{\mu} x_t - \beta(r_t - E_t \pi_{t+1} - \overline{rr}_t) + (1-\beta\rho_a)a_t \tag{97}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t, \tag{98}$$

in which ρ_a denotes the persistence parameter of the productivity shock.

In order to derive the OMP, however, we will use the following reduced form:

$$\omega_t = \beta(1-\psi)E_t \omega_{t+1} + \eta x_t - \beta E_t x_{t+1} + (1-\beta\rho_a)a_t \tag{99}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t, \tag{100}$$

where we set $\eta \equiv \frac{\mu-(1-\beta)(1+\varphi)}{\mu}$ (which is positive for all reasonable calibrations).

4.1 Optimal Monetary Policy under Discretion

The OMP under Discretion is the solution to the following problem

$$\min_{\{x_t,\pi_t,\omega_t\}} x_t^2 + \alpha_\pi \pi_t^2 + \alpha_\omega \omega_t^2$$

such that

$$\omega_t = \eta x_t + K_{\omega,t}$$

and

$$\pi_t = \kappa x_t + K_{\pi,t},$$

where $K_{\pi,t}$ and $K_{\omega,t}$ collect the terms that the Central Bank cannot affect.

The solution to this problem requires the following optimality condition

$$x_t + \alpha_\pi \kappa \pi_t + \alpha_\omega \eta \omega_t = 0.$$

Using the above relation in the reduced system (99)–(100), we can solve for the optimal inflation rate under discretion

$$\pi_t^d = -\Phi_\pi^d a_t, \tag{101}$$

where

$$\Phi_\pi^d \equiv \frac{\alpha_\omega \kappa \eta (1-\beta\rho_a)}{\mathbf{A} + \alpha_\omega \mathbf{B} + \psi \mathbf{C}}$$

and

$$\mathbf{A} \equiv (1-\beta\rho_a)(1+\alpha_\pi\kappa^2-\beta\rho_a) \qquad \mathbf{B} \equiv (1-\beta\rho_a)(\eta-\beta\rho_a)\eta \qquad \mathbf{C} \equiv \beta\rho_a(1+\alpha_\pi\kappa^2-\beta\rho_a)$$

the optimal level of the output gap

$$x_t^d = -\Phi_x^d a_t, \tag{102}$$

where

$$\Phi_x^d \equiv \frac{\alpha_\omega \eta (1-\beta\rho_a)^2}{\mathbf{A} + \alpha_\omega \mathbf{B} + \psi \mathbf{C}}$$

and the optimal level of the stock of financial wealth

$$\omega_t^d = \Phi_\omega^d a_t, \tag{103}$$

where

$$\Phi_\omega^d \equiv \frac{\mathbf{A}}{\mathbf{A} + \alpha_\omega \mathbf{B} + \psi \mathbf{C}}.$$

In the benchmark case with a Representative Agent ($\psi = \alpha_\omega = 0$), in which stock prices are redundant for the equilibrium allocation, Optimal Monetary Policy under discretion implies price stability ($\pi_t^d = 0$), output gap stability ($x_t^d = 0$) and all volatility is borne by the stock of financial wealth ($\omega_t^d = a_t$). This is the familiar case of the optimality of price stability in an economy with no cost-push shock, when the FPE allocation is efficient.

The picture changes in our framework with non-ricardian agents and real effects of stock-price fluctuations.

Result 3. *In a decentralized economy with non-Ricardian agents and an active stock market, Optimal Monetary Policy requires the volatility implied by productivity shocks to be shared among the three targets, given the convexity of the loss function. Accordingly, following a positive productivity shock, financial wealth will rise less than one-for-one, and both inflation and the output gap will decline. Price stability is no longer optimal.*

5 Conclusion

This paper derives a welfare criterion for a DSGE model with non-ricardian agents, à la Blanchard-Yaari, in which stock-price fluctuations have real effects on consumption through wealth effects. Such welfare criterion is then used to characterize the interplay between Optimal Monetary Policy and stock-price dynamics.

The first contribution of the paper is the derivation of the welfare criterion *per se*: since the demand side of our economy features an indefinite number of heterogenous agents, it implies non-trivial issues of utility aggregation, when deriving a second-order approximation of social welfare.

Analogous procedures can be used to derive welfare criteria for stochastic perpetual youth models, used in recent literature for the analysis of non-balanced-budget fiscal policy.

The second contribution is heuristic. I show that the non-ricardianness of agents implies an additional dynamic distortion in the model, which results in an additional term in the welfare criterion. Indeed, the welfare-based loss function features as an explicit additional term also fluctuations in financial wealth. When fluctuations in stock prices affect the demand side of the economy through a wealth effect, therefore, a welfare maximizing Central Bank should target financial stability in addition to stability of the output gap and inflation. The optimal allocation, which requires not only aggregate efficiency but also equitability across agents, implies an endogenous trade-off between output–inflation stabilization and financial stabilization, even in the absence of cost-push or financial shocks. The ultimate implication is that price stability is no longer necessarily optimal. Given the quadratic form of the welfare criterion, some fluctuations in output and inflation will be optimal as long as they reduce the volatility of financial wealth.

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