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Animal Spirits in Entrepreneurial Innovation: Theory and Evidence

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Abstract

This paper proposes and empirically tests a theory of entrepreneurial innovation to explain its high degree of concentration in space and time. In the model, a successful entrepreneurial project is the result of a search and matching process between entrepreneurs looking for funds and capitalists looking for new ideas to finance. The resulting strategic complementarity between them gives rise to a multiplier effect. Moreover, if complementarity is sufficiently strong, multiple equilibria arise, which can be ranked in terms of entrepreneurial activity. Using data from the European and the US business angels markets for the period 1996-2010, we show that (ii) a complementarity exists between business angels and the entrepreneurial projects submitted to them, and that (ii) the result of multiple equilibria is empirically plausible.

Keywords: Entrepreneurship, financing of innovation, search and matching, strategic complementarities, venture capital, business angels.

JEL Classification: 032, 038, D83, C78, L26.

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1 Introduction

Suppose you think you have a promising idea for a new business venture but you find it hard to finance your project from banks or other conventional sources of capital, because of the high degree of uncertainty associated with your project and because of the lack of good collateral. You might then want to turn to other agents, specialized in screening and evaluating innovative business projects exactly like yours. If they judge your project valuable, these agents decide to provide you with the necessary capital, as well as technical and managerial advice, in exchange for an equity stake in the project. In the standard economics terminology, you are referred to as the entrepreneur, the specialized agents as the capitalists (such as venture capitalists or business angels), and the whole process is usually described as one of *entrepreneurial innovation*.

Two characteristics are salient when we observe the process of entrepreneurial innovation. The first, well-known, characteristic is its high degree of geographic *clusterization*. In the US, for example, roughly half of firms financed by venture capitalists are located in three cities only, San Francisco, Boston, and New York (Chen et Al., 2009). Similar patterns of concentration can be documented for Europe and Asia (think, for instance, of the entrepreneurial clusters in Herzliya (Israel) or in the Guangdong province (China)).

The second, probably less known, feature is the higher *volatility* over time of entrepreneurial investments with respect to all other investments. In Figure 1, we have depicted the volatility of investments in fixed capital and in R&D for the period 1995-2010 in both the US and Europe.[1] The pronounced volatility that we observe in Figure 1 almost *disappears* when we compare it with the one observed on the investments provided by venture capitalists (VCs) and private equity (PE) funds to finance seed and start-ups, which is what we show in Figure 2 for both the US and Europe in the same period.[2,3]

INSERT FIGURES 1 AND 2 HERE

[1] Volatility is measured as percentage deviation of investment components from their Hodrick-Prescott trends with smoothing parameters set at 6.25 (see Ravn and Uhlig, 2002).

[2] Since we are here interested in investments devoted to innovative projects, we limit our attention to the fraction of VC investments on seed and start-ups.

[3] In both figures the aggregate "Europe" refers to EU15 plus Poland, Norway, Switzerland, Russia, Estonia, Czech Republic, Hungary, Slovakia, Slovenia.

A look at Figure 2 clearly suggests a pattern of "boom and bust" of entrepreneurial investments. In particular, both the dotcom bubble (and the subsequent bubble burst), as well as the effect of the economic crisis initiated in 2007 are clearly visible from the figure. The higher volatility of entrepreneurial investments arises even when we restrict the attention to those provided by business angels (BAs), as in Figure 3.[4,5]

INSERT FIGURE 3 HERE

The goal of this paper is twofold. On the one hand, we propose a theory of entrepreneurial innovation which is able to explain both the space clusterization and the high volatility over time that we have documented in the previous figures. On the other hand, we try to validate empirically our main theoretical claims against the available data on entrepreneurial innovation.

The financial market for innovation is imperfect. Both microeconomic theory and empirical evidence have long recognized the potential obstacles hidden in the process of financing innovation, suggesting that innovators may well be financially constrained (for a recent review see for instance Hall and Lerner, 2010). Theoretical arguments, advanced to explain financial market imperfections in this sector, range from transaction costs to agency costs due to informational asymmetries between the innovator (agent) and the financier (principal). While these aspects are common to any financing relationship, a number of additional elements suggest that financing problems can be even more severe for innovative investments: innovations are “unique” events, and the process aimed at producing them is an uncertain and largely unpredictable economic activity.

We capture these market frictions via search theory. In particular, we construct a dynamic, partial-equilibrium model where an entrepreneurial project (or an innovative,

[4]Business angels refer to wealthy individuals that invest *their own funds* in entrepreneurial ventures (differently from VCs, which instead gather funds from insitutional investors, such as pension funds). Because of that, the amount invested in each project by a BA is, on average, considerably lower than the one invested by a VC. BAs and VCs, however, share the following crucial feature: they are expected to contribute to the project not only with financial investments but also with managerial and technical expertise (Gompers and Lerner, 1999).

[5]For lack of data on BAs, we have restricted the time span of the series to the decade 2001-2010. Moreover, and for the same reason, Europe here refers to EU15 plus Norway, Poland, Switzerland and Russia.

start-up firm) is the outcome of a process of *search and matching* between the two main actors of the innovative process: those who come up with new ideas, that we call entrepreneurs (or simply innovators); and those who screen and select the most valuable ideas deserving financing funds, that we call capitalists (or simply financiers). An innovation is the result of a successful matching between an entrepreneur and a capitalist.

In analogy with the labor market literature (Pissarides, 2000), where search theory has been extensively used, the matching process is a modeling tool that is meant to represent all frictions characterizing the process of financing innovation - such as information imperfections, or the entrepreneurs' and capitalists' heterogeneities in skills, location, beliefs etc. (Phelps, 2009). A decentralized market for innovation also allows us to describe the realistic situation in which there may exist, at the same time, promising ideas which are not financed ("unemployed" ideas) and unutilized capital searching for good ideas to finance.

The theoretical model describes a world populated by entrepreneurs (who may come up with new ideas) and capitalists (who select ideas and help launch them in the market). They both must decide whether to participate or not in a *fair of ideas*. This decision depends on the costs and the benefits of this participation. An innovation is the output of an aggregate matching function whose inputs are the total number of entrepreneurs and capitalists attending this fair. The innovation is associated with positive profits, to be shared between them. Hence, the benefit from attending the fair is the chance of a successful matching. The participation cost, representing the cost of developing an innovative project for an entrepreneur and the cost of screening and selecting valuable projects for a capitalist, is heterogeneous for both entrepreneurs and capitalists. Once a successful matching has occurred (that is, once a project has been selected and financed), both entrepreneurs and capitalists go back to the initial stage where they have to decide whether to participate or not. This endless circular process is meant to represent the so called *venture capital cycle* described by Gompers and Lerner (1999). We characterize the number of entrepreneurs and capitalists participating in the fair of ideas along the stationary equilibrium.

In our model, entrepreneurs are willing to spend their time and intellectual resources to discover a new idea only if they have a chance to meet a capitalist. On the other hand, capitalists are willing to spend their time and intellectual resources to evaluate the profitability of ideas only if they have the chance to meet valuable entrepreneurs. More generally, the return to becoming an entrepreneur (capitalist) is higher, the higher the number of capitalists (entrepreneurs) in the market. Hence, and as usual in the class of search and matching models (Diamond, 1982, Kiyotaki and Wright, 1993), a *strategic complementarity* exists between entrepreneurs and capitalists, in that, at equilibrium, the number of entrepreneurs devoting to innovation is an increasing function in the number of capitalists (and viceversa).

The presence of a *thick market externality* across entrepreneurs and capitalists has a number of interesting theoretical implications. The first is the existence of a *multiplier effect* in entrepreneurial innovation, whereby the effect of an exogenous shock on the pace of innovation is magnified by the self-reinforcing nature of the interaction between the two sides of the market for ideas. This multiplier effect may contribute to explain the relatively higher volatility of entrepreneurial investments that we have documented in Figures 2 and 3. In this respect, *animal spirits* matter in the process of entrepreneurial innovation, in the sense that this process may be affected by waves of enthusiasm and/or pessimism.

A second implication is that, if strategic complementarities across entrepreneurs and capitalists are sufficiently strong (which occurs when the matching function exhibits increasing returns to scale), the model may admit a *multiplicity of equilibria*, each characterized by a different pace of entrepreneurial activity. These equilibria can be Pareto-ranked from the lowest to the highest number of innovations (matches) produced by the economy. Welfare is maximized at the equilibrium characterized by the highest number of matches. All other equilibria are sub-optimal and are the result of a coordination failure across the market participants (Diamond, 1982, Cooper and John, 1988). Moreover, a "degenerate" equilibrium always exists in this economy, in which the resources devoted to innovation are null. We call it no-innovation trap.

The possibility of multiple equilibria may contribute to explain the well known phenomenon of geographic *clusterization* of entrepreneurial innovation that we have mentioned above. In principle, even two identical economies in terms of fundamentals may persistently diverge in terms of their innovative performance: some economies may converge towards a "high entrepreneurial activity" equilibrium, others may be trapped into a slow or even stagnant pattern of entrepreneurial activity. Again, the model suggests that *animal spirits* matter in the process of innovation, in the sense that, whether a high or a low activity equilibrium is reached depends on a self-fulfilling mechanism triggered by entrepreneurs' and capitalists' expectations.

In the second part of the paper, we test empirically the main theoretical predictions of the model using data on innovative projects financed by business angels. The first claim is the existence of a strategic complementarity between entrepreneurs and capitalists. To test it, we hand-collect yearly data for the period 1996-2010 across a number of European countries plus the US, on (i) the number of *projects* submitted (potential entrepreneurs), (ii) the number of business *angels* (capitalists), (iii) the number of *deals* (successful matches). The estimated aggregate matching function confirms that, within countries, there exists a statistically and economically significant complementarity between the number of business angels looking for innovative projects and the number of projects submitted to them. We then verify the empirical plausibility of the multiple equilibria by testing the returns to scale of the matching function. In the most reliable estimated model, the scale elasticity of the matching function is slightly above unity, suggesting that multiple equilibria are not unlikely.

This paper is related to the stream of literature, initiated by Arrow (1962), on the market failures associated with the process of innovation, and more particularly, with the process of innovation financing (for a review of the literature see, for instance, Hall and Lerner, 2010, Hall, 2005). The idea of modeling the "venture capital cycle" described by Gompers and Lerner (1999) via a process of search and matching is also present in the entrepreneurial finance literature. A few papers go deeper into the microeconomic foundations of the market frictions in the financing of innovation (Silveira and Wright, 2007, Silveira and Wright, 2010, Silveira and Amit, 2006, Chiu, Meh and Wright, 2011). Others emphisize the contractual content of the relationship between entrepreneurs and capitalists (Boadway et al., 2005, Keuschnigg, 2003, Inderst and Muller, 2004, Michelacci and Suarez, 2004). Neither of them, however, focuses on the complementary nature of this relationship, and on its implications to explain the two stylized facts on entrepreneurial finance highlighted above. Obviously, because of that, neither of them attempts to test empirically these theoretical predictions. Finally, several explanations have been proposed to explain the space clusterization of the entrepreneurial process. All of them are based on the existence of a network externality, such as input sharing, labor market pooling or knowledge spillovers (see, for instance, Jaffe et al., 1993; Audretsch and Feldman, 1996; Chen et al., 2009). We here signal the presence of an additional network externality to explain the same phenomenon: the one between entrepreneurs and capitalists.

The rest of the paper is organized as follows. Section 2 introduces the theoretical framework, characterizes the stationary equilibrium and proves the strategic complementarity between entrepreneurs and capitalists. Section 3 derives the multiplier effect. Section 4 discusses the possibility of coordination failures in the innovation process and develops an example for illustrative purposes. Section 5 carries out the empirical analysis. Section 6 discusses the policy implications of the model and concludes with a few remarks. All proofs and other technical arguments are relegated to an appendix at the end of the manuscript.

2 The Model

2.1 Innovation as a Search and Matching Process

The world is populated by a measure E of entrepreneurs and a measure K of capitalists who can meet at the "fair" of new ideas.[6] For simplicity, but without any loss of generality, both agents have identical linear utility, $u(x) = x \; \forall x \in R_+$. Time is continuous, and new ideas arrive randomly to the entrepreneurs according to a Poisson process with (exogenous) instantaneous probability σ. In order for these raw entrepreneurial ideas to become marketable innovations however, entrepreneurs need the (financial and managerial) support of capitalists.

Once an entrepreneur has come up with a new idea, she has to decide whether to pursue it or abandon it and wait for the next idea. To pursue it, each entrepreneur has to pay a cost c_E (the cost of developing and submitting the project to the financiers). This cost is stochastic and distributed according to a (twice continuously differentiable) cumulative distribution function $F(c_E)$ in the support $[0, \overline{c_E}]$. If the entrepreneur pays c_E, she acquires the right to participate in the fair and hence, as we will see, the *chance* of matching the "right" capitalist and implement her project. Cost c_E can also be interpreted as an (inverse) measure of the *quality* of the entrepreneur's project.

On the other hand, each capitalist sustains an entry cost c_K to participate in the fair of ideas (the cost of screening, evaluating and selecting the entrepreneurial projects). This cost, which can be thought of as an inverse measure of each capitalist's *talent*, is also stochastic and distributed according to a (twice continuously differentiable) cumulative distribution function $G(c_K)$ in the support $[0, \overline{c_K}]$.[7]

To analyze the entry decisions of entrepreneurs and capitalists in the fair of ideas, we now need to specify the potential benefits that accrue to them if they pay the

[6] Probably, the theoretical framework closest to ours is the now classical "coconut model" by Diamond (1982).

[7] One might alternatively interpret c_E and c_K as outside options, that is, as the opportunity costs of devoting to entrepreneurship.

entry fee. Let $L_E \leq E$ and $L_K \leq K$ denote, respectively, the endogenous number of entrepreneurs and capitalists participating in the fair (that is, those that have paid their respective entry cost). An entrepreneurial venture is the result of a process of successful *search and matching* between an entrepreneur and a capitalist both attending the fair. We capture this production process of new ideas via the following aggregate matching function:

$$M = M\left(L_E, L_K\right), \tag{1}$$

with $\partial M/\partial L_i > 0$ and $\partial^2 M/\partial L_i^2 < 0$ for $i = E, K$, implying positive and decreasing marginal returns to both inputs. We also impose $M\left(L_E, 0\right) = M\left(0, L_K\right) = 0$, that is, the absence of entrepreneurs or capitalists implies zero successful matches. No further structure is imposed on the returns to scale of the matching function. The instantaneous probability of matching for, respectively, entrepreneurs and capitalists attending the fair, is then given by

$$\alpha_E = \frac{M}{L_E} \text{ and } \alpha_K = \frac{M}{L_K}. \tag{2}$$

The standard assumptions on the first two derivatives of the matching function imply that $\partial\alpha_i/\partial L_i < 0$ and $\partial\alpha_i/\partial L_{-i} > 0$ for $i = E, K$.[8] That is to say, the matching probability for an entrepreneur decreases with the number of entrepreneurs and increases with the number of capitalists (and the same holds for capitalists).

For an entrepreneur, the value of waiting for a new idea is denoted by V_E^0 and defined by the following asset equation:

$$rV_E^0 = \sigma \int_0^{c_E^*} \left(V_E^1 - V_E^0 - c_E\right) dF\left(c_E\right), \tag{3}$$

where r is the exogenous riskless interest rate, c_E^* is the highest cost for which there is still entry (to be determined at equilibrium), and V_E^1 represents the expected payoff associated with the entrepreneurial venture for an entrepreneur (the lifetime return to an entrepreneur attending the fair). This latter value is defined by[9]

$$rV_E^1 = \alpha_E \left(\theta\pi + V_E^0 - V_E^1\right), \tag{4}$$

[8] This will be explicitly shown in the proof of Lemma 1.

[9] Two implicit assumptions in (4) are worth noticing. First, every match is successful, that is to say, every venture-backed firm raises positive profits. Indeed, observation suggests that only a small fraction of funded projects reaches that stage (anecdotal evidence suggests that this fraction is below 20%). The second assumption is that, once a match has occurred, the entrepreneur goes back to the inventive stage (the so called *serial* entrepreneur) -probably selling her idea (or patent) to a firm

where π represents total instantaneous profits originating from the innovation, and $\theta \in (0,1)$ is the entrepreneurs' fraction of these profits. These asset equations have the usual interpretations. Equation (3) tells us that, for an entrepreneur, the flow of utility from waiting for a new idea is equal to the instantanous probability of a new idea times the corresponding payoff, which is given by the capital gain associated with participating in the fair minus the entry cost. Equation (4) instead says that the flow of utility from venturing into innovation is equal to the probability of a successful matching with a capitalist times the payoff associated with this chance. Values V_E^0 and V_E^1 can be alternatively interpreted as the value to an entrepreneur of being, respectively, *outside* and *inside* the fair of ideas. Note that, in the two expressions above, we have decided to focus directly on the steady state, as we have imposed $\dot{V}_E^j = 0$ for $j = 0, 1$.

Let us now turn to capitalists. The expected payoff associated with being a capitalist outside the fair is denoted by V_K^0 and defined by the following asset equation:

$$rV_K^0 = \int_0^{c_K^*} \left(V_K^1 - V_K^0 - c_K\right) dG\left(c_K\right), \tag{5}$$

where c_K^* is the highest cost for which there is still entry for capitalists, and V_K^1 represents the expected value from participating in the fair of ideas. This value is defined by[10]

$$rV_K^1 = \alpha_K \left[(1-\theta)\,\pi + V_K^0 - V_K^1\right], \tag{6}$$

where $(1-\theta)\,\pi$ is the capitalists' fraction of the profits prevailing in the market (again, along the steady state it is $\dot{V}_K^i = 0$ for $i = 0, 1$). Before analyzing the choice behavior of entrepreneurs and capitalists, let us briefly comment on two issues. First, the allocation of the innovation profits across entrepreneurs and capitalists, as captured by the parameter θ, is here taken as exogenous. This is not because we believe the contractual arrangement between entrepreneurs and capitalists is uninteresting but simply because our focus is different.[11] Secondly, in expression (6) we have implicitly supposed that

which will start production. In a truly *Schumpeterian* perspective, we consider the entrepreneur as the innovator. This formalization is meant to capture the venture capital cycle. Of course, none of these assumptions is necessary for any of our results.

[10]The implicit assumption here is that each capitalist can enter into one and only one project at a time, and that each entrepreneur needs one and only one capitalist. Moreover, the same considerations made in footnote 9 for entrepreneurs hold here for capitalists.

[11]An extensive literature has focused on optimal contracts between capitalists and entrepreneurs

the cost of financing the entrepreneurial project is null (so that the capitalists' contribution to the venture is technical and/or managerial but not financial). This is only to economize on parameters and simplify calculations.

2.2 Complementarities in the Innovation Process

In general, two activities are complementary whenever the return from one activity increases as the intensity of the other activity increases. Bringing this definition to our context, we say that entrepreneurs and capitalists are complementary if the return from attending the fair of ideas for an entrepreneur (capitalist) is increasing in the number of capitalists (entrepreneurs) attending the fair. We now prove that this statement holds true in our model.

Let us focus on entrepreneurs first. Their cost from attending the fair is distributed according to $F(c_E)$ and is independent of the number of capitalists. Their benefit, associated with the chance of a successful matching with a capitalist, is instead measured by the difference $V_E^1 - V_E^0$. Subtracting (3) from (4), and solving the resulting equation for $V_E^1 - V_E^0$, we obtain

$$V_E^1 - V_E^0 = \frac{\alpha_E \theta \pi + \sigma \int_0^{c_E^*} c_E dF(c_E)}{r + \alpha_E + \sigma F(c_E^*)}. \tag{7}$$

Standard differential calculus proves that the expression above is increasing in α_E, and thus in L_K. The intuition for this result is straightforward: the higher the number of capitalists, the higher the matching probability for an entrepreneur, and hence the higher her return from participating in the innovation process.

The same argument holds for capitalists. Their cost is independent of the number of entrepreneurs, while their benefit is measured by the difference $V_K^1 - V_K^0$. Again, solving the system made up of (5) and (6) for $V_K^1 - V_K^0$, we obtain

$$V_K^1 - V_K^0 = \frac{\alpha_K (1-\theta) \pi + \int_0^{c_K^*} c_K dG(c_K)}{r + \alpha_K + G(c_K^*)}, \tag{8}$$

(for instance in the presence of moral hazard and adverse selection): see, among others, Keuschnigg (2003), Inderst and Muller (2004), Michelacci and Suarez (2004), Silveira and Wright (2007).

which is increasing in α_K, and thus in L_E. The two previous results are summarized in the following

Lemma 1. *Entrepreneurs and capitalists are complementary, in that the return from attending the fair of ideas for an entrepreneur (capitalist) is increasing in the number of capitalists (entrepreneurs) attending the fair.*

The existence of a complementarity between entrepreneurs and capitalists will be verified empirically in Section 5. Before that however, let us turn to characterize a stationary equilibrium for this economy.

2.3 The Stationary Equilibrium

We now consider the optimal entry decisions for both entrepreneurs and capitalists. At each point in time, the choice of the $E - L_E$ entrepreneurs who are outside the fair, as to whether to pursue their project or abandon it, depends on the relative costs and benefits of the project. Again, the cost c_E is distributed according to $F(c_E)$, while the benefit is measured by the difference $V_E^1 - V_E^0$. There exists an *inframarginal* entrepreneur for whom $c_E^* = V_E^1 - V_E^0$. Substituting for the expression given in (7), we obtain

$$c_E^* = \frac{\alpha_E \theta \pi + \sigma \int_0^{c_E^*} c_E dF(c_E)}{r + \alpha_E + \sigma F(c_E^*)}. \tag{9}$$

All entrepreneurs whose entry cost is lower than c_E^* find it profitable to participate in the fair. The expression above links the threshold cost c_E^* to the probability of successful matching for entrepreneurs α_E, and hence to the number of entrepreneurs and capitalists attending the fair, L_E, L_K: as an immediate implication of Lemma 1, a higher L_K and/or a lower L_E leads to an increase in the probability of a successful matching with a capitalist (α_E), which in turn causes an increase in the cutoff value of the entry cost c_E^*.[12]

[12] Define

$$H(c_E^*, \alpha_E) = \frac{\alpha_E \theta \pi + \sigma \int_0^{c_E^*} c_E dF(c_E)}{r + \alpha_E + \sigma F(c_E^*)} - c_E^*$$

as the implicit function of c_E^* with respect to α_E^*. It is immediate to prove, via the implicit function theorem, that $dc_E^*/d\alpha_E > 0$.

In analogy to the previous case, the chance of a successful matching with an entrepreneur is worth $V_K^1 - V_K^0$ to a capitalist. Given that the cost of this chance c_K is distributed according to $G(c_K)$, there exists an inframarginal capitalist for whom $c_K^* = V_K^1 - V_K^0$. Substituting for the expression given in (8), we obtain

$$c_K^* = \frac{\alpha_K (1-\theta)\pi + \int_0^{c_K^*} c_K dG(c_K)}{r + \alpha_K + G(c_K^*)}. \tag{10}$$

All capitalists whose entry cost is lower than c_K^* find it profitable to participate in the fair. This expression captures the positive relationship between α_K and c_K^*.

Finally remind that, for both entrepreneurs and capitalists, the inflows into the "fair of innovation" must be equal to the outflows along the *steady state*, that is

$$\dot{L}_E = \sigma (E - L_E) F(c_E^*) - L_E \cdot \alpha_E = 0, \tag{11}$$

and

$$\dot{L}_K = (K - L_K) G(c_K^*) - L_K \cdot \alpha_K = 0, \tag{12}$$

where $L_E \cdot \alpha_E = L_K \cdot \alpha_K = M$. Equation (11) captures the evolution of entrepreneurs over time. Along the steady state, the number of entrepreneurs deciding to participate in the fair $(\sigma (E - L_E) F(c_E^*))$ must equalize the number of entrepreneurs who have successfully matched with capitalists and have thus returned to the waiting stage $(L_E \cdot \alpha_E)$. An analogous interpretation can be given to (12).

Equation (11) can be interpreted as a positive relationship between L_E and c_E^* (for any given value of L_K). A higher value of c_E^* implies greater entry in the market of innovation. To maintain the steady state, the number of matches must correspondingly increase. Hence, a higher value of L_E (for a constant value of L_K) is required for equation (11) to hold. The same is true, *mutatis mutandis*, for equation (12) capturing L_K as a positive function of c_K^* for given L_E. We are now ready for the following

Definition. *A stationary equilibrium for this economy is any 4-tuple* (L_E, L_K, c_E^*, c_K^*) *that solves the four equations (9), (10), (11) and (12).*

In equilibrium, the number of entrepreneurs venturing in innovative projects depends on the number of capitalists deciding to back these projects, as this affects the chance of a successful matching. On the other hand, the number of capitalists devoting their time and resources to screening and evaluating innovative projects depends on the chances of encountering good potential entrepreneurs. This interdependence across entrepreneurs' and capitalists' behavior along any stationary equilibrium is characterized in the following

Theorem 1 *At equilibrium, the number of entrepreneurs attending the fair of ideas is an increasing function of the number of capitalists attending the fair, and viceversa:* $dL_i/dL_{-i} > 0 \; \forall i = E, K$.

The result stated in Theorem 1 is a consequence of the complementarity between entrepreneurs and capitalists that we have proven in Lemma 1. Intuitively, a higher number of capitalists participating in the fair raises the chance of a successful matching for an entrepreneur, it makes her participation to the fair more profitable, and thus it brings about an increase in the equilibrium number of entrepreneurs (and viceversa).

Notice that this complementarity, that we may call "strategic" -as it is the result of endogenous and interdependent entry choices of the two types of agents- can be linked to the input complementarity of the matching function, that we may instead label as "technological" -as it is related to the assumed functional form of (1). The two inputs in (1) are technological complements when the marginal productivity of one input is increasing in the use of the other input (in which case (1) is said to be *supermodular* in its two inputs). Note that entrepreneurs and capitalists can be strategic complements even when they are not complementary inputs in the matching function.[13] As we will clarify in Appendix B however, technological complementarity positively contributes to detemine the "strength" of the strategic complementarity.

Finally, we close the section with a result "disciplining" the number of equilibria admitted by this economy. We prove the following[14]

Theorem 2 *If the matching function (1) is homogeneous of degree 1, the economy admits one and only one stationary equilibrium.*

[13]The proof of Lemma 1 shows that, in order for the entrepreneurs' return from entering into the fair to be increasing in L_K, it must only be that $\partial M/\partial L_K > 0$ (and the same holds when we consider the capitalists' return). In other words, strategic complementarity only requires function (1) to exhibit positive marginal productivities in both inputs.

[14]A similar result in a different model is provided by Diamond (1984).

In the next two sections we will investigate the theoretical implications of the strategic complementarity between entrepreneurs and capitalists both when the economy admits a multiplicity of stationary equilibria and when the equilibrium is unique. We start with the latter.

3 The Multiplier Effect

Suppose that the matching function is homogenous of degree 1, and hence that the equilibrium is unique. We now prove that the presence of strategic complementarities makes this equilibrium highly sensitive to disturbances. For instance, suppose that a negative shock on innovation profits materializes in this economy, so that $\Delta\pi < 0$ (think, for instance, of the dotcom bubble burst in 2001). This negative shock reduces the payoff associated with the entrepreneurial activity and thus reduces the number of entrepreneurs. The lower number of entrepreneurs weakens the incentive to become capitalist, which in turn further lowers the incentive to entrepreneurship. This process continues *ad infinitum*, describing a vicious circle whereby the aggregate response to the shock is stronger than the initial instantaneous response. The strategic complementarity across the two main actors of the innovation process magnifies the initial effect of the shock and gives rise to what is usually referred to as a *multiplier effect*.

More formally, define $L_i(L_{-i}, \rho)$ as the (positively sloped) reaction function of agents of type i with respect to the agents of type $-i$ (for $i = E, K$), parameterized by $\rho \in R_+$ capturing any feature that affects L_i other than changes in L_{-i}. We are now ready to state the following

Theorem 3 *A multiplier effect characterizes the process of entrepreneurial innovation, in that the total equilibrium response of entrepreneurs and capitalists to an exogenous shock is greater than the instantaneous response:*

$$\frac{dL_i}{d\rho} > \frac{\partial L_i}{\partial \rho} \ \forall i = E, K.$$

The effect of an exogenous shock on the market of ideas is amplified by the strategic complementarity across the two sides of the market for ideas. Hence, any factor that affects the entrepreneurs' or the capitalists' payoff has a big impact on the level of innovative activity. This mutual, self-reinforcing, interaction between entrepreneurs and capitalists may contribute to explain the extremely high volatility of entrepreneurial investments that we have documented in Figures 2 and 3. To use a phrase from Summers (1988), our entrepreneurial equilibrium is *fragile*, in the sense that it is potentially subject to large fluctuations in the level of activity. This suggests that *animal spirits* may play a role in explaining the dynamics of entrepreneurial innovation. It also suggests that even small temporary shocks may have persistent consequences on the innovative process and, hence, long-run implications in terms of overall economic performance.

4 Coordination Failures in Entrepreneurial Activity

The presence of a thick market externality raises the possibility of *coordination failures* across the market participants (see for instance Diamond, 1982, 1984, Cooper and John, 1988). In particular, if the complementarity between entrepreneurs and capitalists is strong enough –that is, if the matching function exhibits increasing returns to scale–, the model may generate multiple equilibria.

How do we interpret equilibrium multiplicity? For the sake of illustration, suppose that the economy admits two (non-degenerate) equilibria, respectively denoted by superscripts O,P, with $(L_i)^O > (L_i)^P$ for $i = E, K$ (an example of this kind is developed later in this section). It is easy to interpret these two equilibria as *self-fulfilling* equilibria triggered, respectively, by optimistic or pessimistic expectations. Whenever entrepreneurs expect a high number of capitalists to be matched with ($L_K^e = (L_K)^O$ where the superscript e stands for "expected"), their number will be high as well, $(L_E)^O$. Similarly, whenever capitalists expect a high number of entrepreneurs ($L_E^e = (L_E)^O$), their number will also be high, $(L_K)^O$. Equilibrium O can be labelled as the optimistic (or *thick*) equilibrium. Via a totally symmetric argument, expecting few entrepreneurs and capitalists entering the market makes the agents converge towards the low-entry equilibrium P, which can be referred to as the pessimistic (or *thin*) equilibrium.

Given that in our model only profitable innovations are pursued, whenever multiple equilibria exist, they can be *Pareto-ordered* from the lowest to the highest number of innovations (matches) produced by the economy. Welfare is thus maximized at the equilibrium characterized by the highest number of matches: all other equilibria are sub-optimal and are the result of a coordination failure between entrepreneurs and capitalists.

Finally note that, in our theoretical framework, the economy is also potentially subject to a most disruptive coordination failure. If entrepreneurs expect *no* capitalist participating in the fair ($L_K^e = 0$), the number of expected matches and thus the probability of matching a capitalist for an entrepreneur are both null ($M(L_E, 0) = 0$, $\alpha_E = 0$). As a result, the expected value from participating in the fair is zero ($V_E^1 = 0$), implying no entrepreneur entering into the market of innovation at equilibrium ($(L_E)^T = 0$). Symmetrically, if capitalists expect no entrepreneur at the fair of ideas ($L_E^e = 0$), none of them will participate either ($(L_K)^T = 0$). The result of this extreme form of miscoordination is a (degenerate) equilibrium in which $(L_E)^T = (L_K)^T = 0$. We call this equilibrium a *no-innovation trap*, as no innovation ever takes place in this economy.

Example. Consider the model developed in Section 2 and further suppose that (i) the matching function (1) is Cobb-Douglas with increasing returns to scale: $M = \delta L_E^{\beta_E} L_K^{\beta_K}$ with $\delta \in R_+$, $\beta_E, \beta_K < 1$ and $\beta_E + \beta_K > 1$, and that (ii) entry costs are the same for every entrepreneur and every capitalist, $\underline{c_E}$ and $\underline{c_K}$.[15] This economy admits three stationary equilibria. The first (thin) equilibrium is given by the pair $\left((L_E)^P, (L_K)^P\right)$ that solves the following system:[16]

$$\begin{cases} \underline{c_E} = \frac{\frac{M}{L_E}\theta\pi + \sigma \underline{c_E}}{r + \frac{M}{L_E} + \sigma} \\ \underline{c_K} = \frac{\frac{M}{L_K}(1-\theta)\pi + \underline{c_K}}{r + \frac{M}{L_K} + 1}. \end{cases}$$

The second (thick) equilibrium is instead given by the pair $\left((L_E)^O, (L_K)^O\right)$ that solves the system given by the two steady-state conditions:

$$\begin{cases} \sigma(E - L_E) = \delta L_E^{\beta_E} L_K^{\beta_K} \\ K - L_K = \delta L_E^{\beta_E} L_K^{\beta_K}. \end{cases}$$

Finally, the third equilibrium is the (degenerate) no-innovation trap, $\left((L_E)^T, (L_K)^T\right) = (0, 0)$.

For illustrative purposes, let us carry out a numerical simulation of this economy. First fix these numerical values for the following parameters (which are drawn from a

[15] In some respects, this example resembles the one developed by Diamond (1982) in Section IX.

[16] Under constant returns to scale, this system is impossible, and hence this equilibrium disappears.

non-linear estimation of a Cobb-Douglas matching function for Europe only[17]): $\beta_E = 0.726$, $\beta_K = 0.438$, $\delta = 0.046$. Further assume $r = 0.05$, $\pi = 100$, $\theta = 0.5$, $\sigma = 1$, $\underline{c_E} = 30$, $\underline{c_K} = 35$, $K = 400$, $E = 180$, (the last two values are the average number of business angels and yearly submitted projects for Europe).

The first system defining the thin equilibrium becomes

$$\begin{cases} 30 = \frac{0.046(L_E)^{-0.274}(L_K)^{0.438}\cdot 50+30}{0.05+0.046(L_E)^{-0.274}(L_K)^{0.438}+1} \\ 35 = \frac{0.046(L_E)^{0.726}(L_K)^{-0.562}\cdot 50+35}{0.05+0.046(L_E)^{0.726}(L_K)^{-0.562}+1} \end{cases}$$

whose solution is given by $\left((L_E)^P, (L_K)^P\right) \simeq (64, 41)$. The second system is instead given by

$$\begin{cases} 180 - L_E = 0.046\,(L_E)^{0.726}\,(L_K)^{0.438} \\ 400 - L_K = 0.046\,(L_E)^{0.726}\,(L_K)^{0.438}, \end{cases}$$

whose solution is given by $\left((L_E)^O, (L_K)^O\right) \simeq (156, 376)$, which is the thick equilibrium. Hence, this economy admits two non-degenerate equilibria plus the no-innovation trap.

The possibility of multiple equilibria provides an alternative explanation to the strong space clusterization that we observe in entrepreneurial innovation. This is not to deny the importance of fundamentals in explaining the different patterns of entrepreneurial behavior that we observe across different regions: for economic, institutional, or even cultural reasons, some regions may simply provide more powerful incentives to entrepreneurship. What we claim here is that, all other things equal, *animal spirits* matter in entrepreneurial innovation, in the sense that, at least to a certain extent, a favourable entrepreneurial climate (or the lack of it) may turn out to be self-fulfilling. In this respect, the role of the policy maker might be crucial in providing a coordination device towards a path of bouncing entrepreneurial activity. Evidence on public intervention across the developed world confirms this claim (Lerner, 2010). We come back to this issue in the concluding section.

5 Empirical Evidence

This section is devoted to the empirical validation of the main theoretical claims of Sections 2, 3 and 4. We first test the complementarity between entrepreneurs and capitalists by directly estimating the aggregate matching function given in (1). We

[17] The estimates table is available upon request from the authors.

then verify the empirical plausibility of the result of multiple equilibria by analyzing the returns to scale of the estimated matching function.

5.1 Data

The matching function expresses the output -the number of innovations- as a positive function of two inputs -the number of potential entrepreneurs and the number of capitalists. A key challenges of our analysis is the search of suitable data to estimate this function. One of the two inputs, the "number of potential entrepreneurs", is particularly difficult to measure. Usually, we observe the number of actual entrepreneurs, which is a proper subset of the group of those who are willing to become entrepreneurs but may or may have not been financed yet. The European Association for Business Angels (EBAN) and the US Center for Venture Research (CVR) at the University of New Hampshire, however, have recently started to collect more detailed yearly data, at macro level, across angel investors. In particular, they both record the total number of entrepreneurial projects *submitted* to each business angel. We then use this number of projects as a proxy for the number of potential entrepreneurs. Moreover, EBAN and CVR collect yearly data on two other dimensions, which may well capture our remaining two variables of interest: the number of business angels (as a proxy for the input "number of capitalists"), and the number of deals (as a proxy for the output "number of undertaken entrepreneurial projects"). We hand-collect data over the three mentioned dimensions of the business angels activity across EU-15 countries, plus Norway, Poland, Switzerland, Russia and the US over the period 1996-2010.[18] A summary description of these data is provided in Table 1.

INSERT TABLE 1 HERE

5.2 Estimation Models and Results

Using the data illustrated above, we carry out a pooled regression estimation of different specifications for the matching function in order to estimate the existence and the

[18] Data for European countries and the US are recorded in the annual reports compiled by, respectively, EBAN and CVR (in particular, EBAN Annual Reports from 2005 to 2010, and CVR Angel Market Activity Reports from 2003 to 2010). Note that these data cover most but not all the business angels activity across the countries considered. The reason is that angel networks are not obliged to release any data.

degree of complementarity between capitalists and entrepreneurs.

5.2.1 Strategic and Technological Complementarities

Theorem 1 has proven that entrepreneurs and capitalists are strategic complements. This strategic complementarity arises because, as shown in Lemma 1, the return from attending the fair of ideas for an entrepreneur (capitalist) is increasing in the number of capitalists (entrepreneurs) attending the fair, whenever the matching function exhibits positive marginal productivities in both inputs. Moreover, as shown in Appendix B, the degree of technological complementarity between two inputs - captured by the cross-partial derivative of the matching function - strengthens the strategic complementarity (in that it weakens the negative effect that an increase in the number of entrepreneurs (capitalists) has on their own return from participating in the fair).

We now estimate a CES-type matching function and verify whether the marginal productivities of both inputs and the cross-partial derivative are strictly positive. Consider the following matching function:

$$M_{it} = A(\beta_E (L_E)_{it}^{\theta} + \beta_K (L_K)_{it}^{\theta})^{\frac{v}{\theta}} \exp(\boldsymbol{\beta_c} \mathbf{c}_{it} + \varepsilon_{it}) \tag{13}$$

where M_{it} is the number of deals in country i at time t; $(L_E)_{it}$ and $(L_K)_{it}$ are the number of projects submitted and of business angels in country i at time t, respectively; $\mathbf{c}_{it}$ is a vector of controls; v is the return-to-scale parameter; β_E and β_K are share parameters, A is a scale technology parameter. For this function, the (constant) Hicks elasticity of substitution between the two input factors is given by $\sigma = 1/(1-\theta)$. The CES collapses to a Cobb-Douglas function when $\sigma \to 1$ (or, $\theta \to 0$).

In specification (13), the marginal return to L_i is given by $\partial M/\partial L_i = Av\beta_i L_i^{\theta-1}(\beta_i L_i^{\theta} + \beta_{-i} L_{-i}^{\theta})^{\frac{v}{\theta}-1} \exp(\boldsymbol{\beta_c} \mathbf{c}_{it} + \varepsilon_{it})$ (for $i = E, K$). Strategic complementarity requires $\beta_E, \beta_K, v, A > 0$. The cross-partial derivative is instead given by

$$\frac{\partial M}{\partial L_i \partial L_{-i}} = Av(v-\theta)\beta_i \beta_{-i} (L_i L_{-i})^{\theta-1} (\beta_i L_i^{\theta} + \beta_{-i} L_{-i}^{\theta})^{\frac{v}{\theta}-2} \exp(\boldsymbol{\beta_c} \mathbf{c}_{it} + \varepsilon_{it}),$$

which is strictly positive if $(v-\theta) > 0$. Hence, a positive difference between v and θ signals the existence of a technological complementarity between the two input levels.

The nonlinear estimation results of the log-CES matching function are shown in Table 2.[19] Estimates are in line with our theoretical predictions. Both A and v, and

[19] Estimates are computed using nonlinear least squares, and the residuals have an approximately normal distribution.

the share parameters are significantly positive. The same is true for the estimated difference between v and θ. Hence, the higher the number of business angels, the greater (more positive) the effect of the number of entrepreneurial projects submitted on the innovation process, and viceversa. Or equivalently, the impact of one additional project submitted on the number of deals is positive and increasing in the number of business angels, and viceversa.

INSERT TABLE 2 HERE

Our estimates also suggest a unitary elasticity of substitution between the two inputs, because the θ parameter is positive but not significantly different from zero. This brings us to consider a Cobb-Douglas (CD) matching function of the following form:

$$M_{it} = A\left(L_E\right)_{it}^{\beta_E}\left(L_K\right)_{it}^{\beta_K}\exp(\boldsymbol{\beta}_{\mathbf{c}}\mathbf{c}_{it} + \varepsilon_{it})$$

Two inputs in a CD function are always complementary to the extent that the input shares (β_E, β_K) are strictly positive.[20] We estimate the following log-transformation of the CD matching function:

$$m_{it} = \beta_0 + \beta_E\left(l_E\right)_{it} + \beta_K\left(l_K\right)_{it} + \boldsymbol{\beta}_{\mathbf{c}}\mathbf{c}_{it} + \varepsilon_{it} \tag{14}$$

where m_{it} is the log of the number of deals in country i at time t; $\left(l_E\right)_{it}$ and $\left(l_K\right)_{it}$ are the logs of the number of projects submitted and of the business angels in country i at time t, respectively; $\mathbf{c}_{it}$ is a vector of controls.

Model (14) is estimated via a *robust* regression to deal with the presence, in the dataset, of outliers that can distort the ordinary least squares estimator (OLS). By considering squared residuals, OLS tend to give an excessive importance to observations with very large residuals and, consequently, distort the parameters' estimation in presence of outliers. Adopting the graphical tool proposed by Rousseeuw and Van Zomeren (1990), Figure 4 shows that several outliers are present, suggesting that there is a serious risk that the OLS estimator be strongly attracted by outliers (Rousseeuw

[20]The cross-partial derivative of a CD function writes as $dM/\left(dL_i dL_{-i}\right) = A\beta_i\beta_{-i}M/\left(L_i L_{-i}\right)\exp(\boldsymbol{\beta}_{\mathbf{c}}\mathbf{c}_{it} + \varepsilon_{it})$, which is higher than zero as long as $\beta_i > 0$ for $i = E, K$.

and Leroy, 1987).[21] To tackle this issue, and following the recent literature (Verardi and Croux, 2009), we adopt the MM-estimators method which has been found to be the most suitable to combine a high resistance to outliers and high efficiency.[22]

INSERT FIGURE 4 HERE

Estimation results of the log-linear CD matching function are shown in the first column of Table 3. The share parameters are both significantly positive. In particular, a 1% increase in the number of submitted projects (business angels) leads to a 0.53% (0.55%) increase in the number of deals.

INSERT TABLE 3 HERE

5.2.2 Technological Complementarity in Elasticities

We have so far verified the existence of a (strategic and technological) complementarity between the *levels* of the two inputs: a 1 unit increase in L_i leads to an increase in the output which is increasing in the level of the other input. By construction however, the log-linear Cobb-Douglas specification in (14) implicitly assumes a constant elasticity of the output with respect to each input, that is to say: a 1% increase in L_i leads to a constant increase in the output. In this section, we consider two generalizations of model (14) to verify the existence of a complementarity in *elasticities*, that is, to explicitly test whether a 1% increase in L_i leads to an increase in the output which depends positively on the other input.

[21] In particular, two observations for Belgium and Norway are *bad leverage points*, meaning that their explanatory variables are slightly different from those of the rest of data and their outcomes are higher than they should be according to the fitted model. The collected data for US are large *good leverage points*, suggesting that the characteristics of the US business angels market are rather different from the other countries but that the number of deals is consistent with what the model predicts. Finally, few other observations (i.e., for Italy, Portugal, Denmark, Netherlands and Poland) are *vertical outliers*, being standard in their characteristics but more or less successful in terms of number of deals than the model would suggest.

[22] The intuition behind the method is simple. In the classical OLS estimation, the objective is to minimize the variance of the residuals. Given that the variance is sensitive to outliers, this may result in distorted OLS estimates. The class of robust S- and MM-estimators instead minimize a measure of dispersion of the residuals that is less sensitive than the variance to extreme values.

The first generalization is a log-linear Cobb-Douglas with a log-*interaction term* between the demand and the supply of financial funds:

$$m_{it} = \beta_0 + \beta_E (l_E)_{it} + \beta_K (l_K)_{it} + \beta_{EK} [(l_E)_{it} \cdot (l_K)_{it}] + \boldsymbol{\beta}_{\mathbf{c}} \mathbf{c}_{it} + \varepsilon_{it}, \tag{15}$$

The coefficient β_{EK} is meant to capture the degree of complementarity in elasticities. We expect it to be strictly higher than zero.[23]

The second model we estimate is a trascendental logarithmic (translog) function, which generalizes the log-Cobb-Douglas form by allowing the output elasticity to vary with the size of the two input shares:[24]

$$\begin{aligned} m_{it} &= \beta_0 + \beta_E (l_E)_{it} + \beta_K (l_K)_{it} + \beta_{EK} [(l_E)_{it} \cdot (l_K)_{it}] + \\ &\quad + \beta_{EE} [(l_E)_{it}]^2 + \beta_{KK} [(l_K)_{it}]^2 + \boldsymbol{\beta}_{\mathbf{c}} \mathbf{c}_{it} + \varepsilon_{it} \end{aligned} \tag{16}$$

The translog considers the squares of the two log-inputs. Our assumption on decreasing marginal returns for both inputs in (1), induces us to expect $\beta_{EE}, \beta_{KK} < 0$.

The four models that we have estimated are all closely related to each other. The log-linear CD matching function (14) is nested into the interaction-augmented log-linear CD matching function (15), which is nested into the translog specification (16). In particular, (14) and (15) are directly obtained from (16) by applying the following restrictions, respectively: $\beta_{EK} = \beta_{EE} = \beta_{KK} = 0$ and $\beta_{EE} = \beta_{KK} = 0$. Finally, it can be demonstrated that the translog specification can be obtained from a second-order Taylor approximation of the logarithmic transformation of the CES specification (13).[25]

As with model (14), also (15) and (16) are estimated via *robust* regressions. Results are shown in columns (2) and (3) of Table 3. The estimated elasticities of the matching function all have the expected signs and are highly statistically significant.

In column 2, the interaction term between the logarithms of the two explanatory variables is positive and highly statistically significant, which suggests the existence

[23] The cross-partial derivative is given by $dm/dl_i dl_{-i} = \beta_{EK}$, which is strictly positive if and only if $\beta_{EK} > 0$.

[24] The scale elasticity of a translog is defined by $\epsilon = \epsilon_E + \epsilon_K$, where $\epsilon_E = \beta_E + \beta_{EK} (l_K)_{it} + 2\beta_{EE} (l_E)_{it}$ is the elasticity of new deals with respect to the number of submitted projects, and $\epsilon_K = \beta_K + \beta_{EK} (l_E)_{it} + 2\beta_{KK} (l_K)_{it}$ is the elasticity of new deals with respect to the number of BAs.

[25] When the elasticity of substitution is in the neighborhood of unity, a two-input CES function may be approximated by a Taylor expansion which has the form of (16) under the following restrictions: $\beta_{EK} = -2\beta_{EE} = -2\beta_{KK}$ (Kmenta, 1967).

of an input complementarity, not only between the levels of BAs and entrepreneurial projects, but also between their elasticities. In other words, the impact of a 1% increase in the number of business angels on the number of deals is positive and *increasing* with the number of entrepreneurial projects, and viceversa.

Given that the log-linear CD specification is nested in the interaction-augmented log-linear CD specification and that the latter's parameter estimates are all significant, we conclude that model (15) is to be preferred to model (14). Notice also that, moving from the first to the second specification leads to a significant reduction in the estimated first-order elasticity of the number of deals to the number of entrepreneurs (which drops to 0.27%). This means that, in the simplest specification, the estimated elasticity of m_{it} to l_E erroneously captures the positive role of the omitted interaction term.

The best fit is obtained under model (16), which estimates the more general translog matching function. Moving from model (15) to model (16), the role of the first-order terms on the outcome of interest remains substantially unchanged, while the impact of the interaction term significantly improves, going from 0.02% to 0.31%. This change is accompanied by a significant and negative impact of the squared values of the two inputs, thus confirming our assumption of diminishing marginal returns to each input.

Finally, we test the robustness of our previous empirical findings by introducing a few control variables in the $\mathbf{c}_{it}$ vector of controls. In particular, we consider the following World Bank Indicators: i) the value of the market capitalization of listed companies in percentage of GDP (MC); (ii) the amount of the domestic credit provided by the banking sector in percentage of GDP (DC). Stock market capitalization should have a positive effect on BA activity (because a well-developed stock market facilitates the exit of the business angels through IPOs). On the other hand, a mostly bank-based financial sector is usually seen as detrimental to entrepreneurial innovation. Results, shown in Table 4, are substantially similar to our previous findings. The coefficients of the controls all have the expected signs, and the stock market capitalization seems to significantly affect the (log-)number of deals.

INSERT TABLE 4 HERE

5.3 The Returns to Scale of the Matching Function

As proven in Theorem 2, constant returns to scale of the matching function imply that the equilibrium is unique. We now test the returns to scale (RTS) of this function in order to verify whether one or more than one equilibrium is to be expected in the BA market.

Table 2 presents the estimated returns to scale for the CES matching function and the results of an F-test for the null hypothesis of constant returns to scale ($\upsilon = 1$). Following Yashiv (2000) and Warren (1996), the last rows of both tables 3 and 4 present the estimated RTS at the sample mean of the explanatory variables and the results of an F-tests for the null hypothesis of constant returns to scale (CRS) of the respective log-linear matching function specification (for the translog model, the null hypothesis implies the following three linear restrictions on the parameters: $\beta_E + \beta_K = 1$, $\beta_{EE} + \beta_{EK} = 0$, $\beta_{KK} + \beta_{EK} = 0$).

Overall, our results suggest that returns to scale are either constant or slightly increasing. The nonlinear estimation of the log-CES matching function (Table 2) gives us an estimated scale elasticity significantly positive and in the neighborhood of 1. The null hypothesis of constant returns to scale cannot be rejected. Moving to the log-linear matching function specifications without further control variables (Table 3), our evidence supports the presence of increasing returns to scale of the estimated matching function. In particular, in the translog model the output elasticity evaluated at the sample mean of explanatory variables is around 1.06, and the null hypothesis of CRS is rejected at 99%. When the selected World Bank Indicators are added among the control variables (Table 4), the translog model continues to exhibit mildly increasing returns to scale of the order of 1.01, but the null hypothesis of CRS cannot be rejected.

6 Conclusions and Policy Implications

This paper has built a model of the market for innovation that focuses on the relationship between innovators and financiers. An innovation is the outcome of a search and matching process between an innovator with a new project and a financier backing that project. The model has investigated the choice of innovators and financiers as to whether or not to participate in a "fair of innovation" and has determined the equilibrium number of innovators and financiers contributing to the innovation process along the steady state. The main purpose of the modeling strategy that we have followed has been the one of representing the "venture capital cycle" described in the literature on entrepreneurial finance (Gompers and Lerner, 1999).

We have shown that a strategic complementarity exists between innovators and financiers, in that an increase in participation of the former induces an increase in participation of the latter (and viceversa). Two main implications are drawn on this basis. First, the innovation process is subject to a multiplier effect which magnifies the effects of any shock on the innovative performance of the system. Secondly, coordination failures between innovators and financiers may occur, which are driven by pessimistic beliefs about the attendance of the fair of innovation. These two results may contribute to explain the concentration of the entrepreneurial activity in both space and time that we observe in the real world.

Using data on the business angel market for the period 1996-2010 across a group of European countries plus the US, our empirical analysis has confirmed that the number of angel investors looking for promising entrepreneurial projects to finance and the number of projects submitted to them are complementary. We have then verified the empirical plausibility of the multiple equilibria by testing the returns to scale of the matching function. In the most reliable model (the translog specification), the estimated scale elasticity of the matching function is slightly above unity, suggesting that multiple equilibria are not unlikely.

Three main policy implications may be drawn from our analysis. First, government intervention may be useful in the form of a "stabilization policy", that is, in order to attenuate the pronounced cyclicality of entrepreneurial innovation that we have documented above and that, according to our theory, originates from the presence of a multiplier effect in this process. Secondly, given that our model of innovation admits the possibility of coordination failures -that is, of equilibria characterized by sub-optimally low paces of innovation-, an effective government intervention might be able to initiate a *virtuous* cycle, that is, to favor the coordination of economic agents towards a path of faster innovation. For instance, public policy could in principle help drive the economy out of "bad equilibria" (such as the "no-innovation trap"). This task may not be as easy as it appears from a theoretical model: the policy maker might be incompetent or captured by special interests. Yet, empirical evidence confirms that, behind every successful story of entrepreneurial innovation (from Silicon Valley to the Singapore VC industry), the role of public policy has always been crucial at the very early stages of development. In the words of Lerner (2010, p.42), "every hub of cutting-edge entrepreneurial activity in the world today had its origins in proactive government intervention. Similarly, the venture capital industry in many nations has been profoundly shaped by government intervention".

This, however, does not necessarily imply that a simple "big push" strategy is the best innovation policy that a public authority can implement, which brings us to the third policy implication. Our model allows us to assess the role of the financier not only as someone who provides innovators with the necessary funds, but also as someone who actively participates in the innovation process by evaluating and selecting potentially profitable ideas. Given that the government is likely to be less skillful than professional financiers in this function, its most valuable task is probably not the one of financing directly entrepreneurs' ideas, but rather the one of fostering the emergence of a class of active capitalists. In other words, this paper suggests that the government preferably subsidize financiers rather than entrepreneurs, so as to exploit their expertise in terms of selection of the most promising innovative ventures.

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A Proofs

Proof of Lemma 1. Differentiating (7) with respect to L_K, we obtain

$$\frac{d\left(V_E^1-V_E^0\right)}{dL_K}=\frac{\partial\left(V_E^1-V_E^0\right)}{\partial\alpha_E}\cdot\frac{\partial\alpha_E}{\partial L_K}=\frac{\theta\pi r+\sigma\left(\theta\pi F\left(c_E^*\right)-\int\limits_0^{c_E^*}c_E dF\left(c_E\right)\right)}{\left[r+\alpha_E+\sigma F\left(c_E^*\right)\right]^2}\cdot\frac{1}{L_E}\frac{\partial M}{\partial L_K},$$

which is always strictly positive, given that rational entrepreneurs pursue profitable projects ($\theta\pi > c_E^*$), and that the marginal productivity of capitalists is strictly positive ($\partial M/\partial L_K > 0$). This completes the proof of the first part of the theorem.

To prove the second part of the theorem, an entirely analogous argument can be developed starting from the differentiation of (8) with respect to L_E. We omit it for brevity.

Proof of Theorem 1. We prove this statement via a simple *reductio ad absurdum* argument: an initial increase in L_i cannot be followed by a (weak) decrease in L_{-i} ($\forall i = E, K$) if we want the 4 expressions (9), (10), (11), (12) defining the stationary equilibrium to hold all at the same time.

Suppose instead that, following an increase in L_K, L_E has diminished (a totally symmetric argument can be developed for the opposite case). From equation (9) ($c_E^* = c_E^*\left(\overset{+}{\alpha_E}\right) = c_E^*\left(\overset{+}{L_K},\overset{-}{L_E}\right)$) and from equation (10) ($c_K^* = c_K^*\left(\overset{+}{\alpha_K}\right) = c_K^*\left(\overset{-}{L_K},\overset{+}{L_E}\right)$), we then know for sure that, respectively, c_E^* has increased and c_K^* has decreased. If we now equalize the left-hand sides (LHS) of equations (11) and (12) (which we can do, given that the right-hand side (RHS) of these equations coincide), we obtain

$$\sigma\left(1-L_E\right)F\left(c_E^*\right)=\left(K-L_K\right)G\left(c_K^*\right).$$

The increase in L_K and the decrease in c_K^* both imply that the RHS of the equation above (and hence the number of matches) has decreased. On the other hand, the decrease in L_E and the increase in c_E^* both imply that the LHS of the equation above (and hence the number of matches) has increased. These two statements exclude each other. A situation of a decrease in L_E following an increase in L_K is then in contradiction with the definition of stationary equilibrium for this economy.

Proof of Theorem 2. We here prove that, if the matching function exhibits constant returns to scale (CRS), the stationary equilibrium is unique. First pose $\Omega \equiv$

L_K/L_E. Given that (1) has CRS, we can write $\alpha_E \equiv M/L_E = m(\Omega)$, and $\alpha_K \equiv M/L_K = (1/\Omega)\, m(\Omega)$. The entry conditions, (9) and (10), are then both functions of Ω only, the former increasing, the latter decreasing, that is, $c_E^*\left(\overset{+}{\Omega}\right)$ and $c_K^*\left(\overset{-}{\Omega}\right)$. By substituting these functions respectively into (11) and (12), we obtain

$$\sigma(1-L_E)\, F(c_E^*(\Omega)) - L_E m(\Omega) = 0 \tag{17}$$

and

$$(K-L_K)\, G(c_K^*(\Omega)) - L_K(1/\Omega)\, m(\Omega) = 0. \tag{18}$$

Standard differential calculus proves that $L_E(\Omega)$ defined in (17) is monotone increasing in Ω, while $L_K(\Omega)$ defined in (18) is monotone decreasing in Ω. Hence, the function defined as the ratio between them, L_K/L_E is unambiguously decreasing in Ω. Given that it is $\Omega \equiv L_K/L_E$, a stationary equilibrium is a fixed point of function $L_K/L_E(\Omega)$. We now prove that this function admits one and only one fixed point.

Define $g(\Omega) \equiv L_K/L_E(\Omega) - \Omega$. There exist sufficiently low values of Ω such that $g(\Omega) > 0$, as well as sufficiently high values of Ω such that $g(\Omega) < 0$.[26] Given that $g(\Omega)$ is a continuous and monotone decreasing function in Ω, the intermediate value theorem guarantees the existence of one and only one Ω^* such that $g(\Omega^*) = 0$, that is, such that $L_K/L_E(\Omega^*) = \Omega^*$. Finally, it might still be the case that multiple equilibria exist, even though they are all characterized by a unique ratio Ω^*. This instance, however, can be excluded once we realize that $L_E(\Omega)$ and $L_K(\Omega)$, defined in (17) and (18), are monotone functions of Ω.

Proof of Theorem 3. Define $L_E(L_K, \rho)$ as the implicit function of L_E, and where ρ parameterizes this function. By convention, suppose that $\partial L_i/\partial\rho > 0$ for $i = E, K$. Then it is

$$\frac{dL_E}{d\rho} = \frac{\partial L_E}{\partial\rho} + \frac{dL_E}{dL_K}\frac{dL_K}{d\rho}.$$

[26]The standard assumptions on the matching function imply that

$$\lim_{\Omega\to 0}\frac{L_K}{L_E}(\Omega) = +\infty$$

and

$$\lim_{\Omega\to+\infty}\frac{L_K}{L_E}(\Omega) = 0.$$

Even though they are not necessary, these two results ensure the existence of the two regions where $g(\Omega) > 0$ and $g(\Omega) < 0$.

On the other hand,

$$\frac{dL_K}{d\rho} = \frac{\partial L_K}{\partial \rho} + \frac{dL_K}{dL_E}\frac{dL_E}{d\rho}.$$

Substituting the second expression into the first, we obtain

$$\frac{dL_E}{d\rho} = \frac{1}{1 - \frac{dL_E}{dL_K}\frac{dL_K}{dL_E}}\left(\frac{\partial L_E}{\partial \rho} + \frac{dL_E}{dL_K}\frac{\partial L_K}{\partial \rho}\right) > \frac{\partial L_E}{\partial \rho},$$

given that $\partial L_K/\partial \rho > 0$ and that -as ensured in Theorem 1- $dL_i/dL_{-i} > 0$ for $i = E, K$.

B Strategic and Technological Complementarity

As we said in the main text, entrepreneurs and capitalists can be strategic complements even though they are not complementary inputs in the matching function. Technological complementarity, which is measured by the cross-partial derivative of (1), is however closely related to the strategic complementarity in a way that is clarified in this section.

Theorem 1 has proven that entrepreneurs and capitalists are strategic complements. As shown in Lemma 1, this strategic complementarity arises whenever the return from attending the fair of ideas for an entrepreneur (capitalist) is increasing in the number of capitalists (entrepreneurs) attending the fair, that is, whenever

$$\frac{d\left(V_E^1 - V_E^0\right)}{dL_K} = \frac{\partial\left(V_E^1 - V_E^0\right)}{\partial \alpha_E} \cdot \frac{1}{L_E}\frac{\partial M}{\partial L_K} > 0$$

(and the same is true, *mutatis mutandis*, for capitalists). Let us focus on the second multiplicative term, representing $\partial \alpha_E/\partial L_K$ (this is the only term affected by technological complementarity). This term is a decreasing function in L_E, the intuition being that, when the number of entrepreneurs grows, the probability of matching diminishes and thus strategic complementarity becomes weaker. This weakening effect, however, is *attenuated* by the degree of technological complementarity. To show this, let us calculate the cross-partial derivative of this matching probability:

$$\frac{\partial \alpha_E}{\partial L_K \partial L_E} = -\frac{1}{L_E^2}\frac{\partial M}{\partial L_K} + \frac{1}{L_E}\frac{\partial M}{\partial L_K \partial L_E}.$$

The expression above captures how the positive marginal effect of an increase in the number of capitalists on the entrepreneurs' matching probability varies in the number of entrepreneurs. Technological complementarity implies a positive second addend (as $\partial M/\partial L_K \partial L_E > 0$), and the more so the higher the complementarity. This means that a higher L_E has a less negative (or more positive) effect on the entrepreneurs' matching probability, implying *coeteris paribus* a more *persistent* degree of strategic complementarity as the market of innovation becomes thicker.

	No. Deals		No. Angels		No. Projects		Obs.
	mean	sd	mean	sd	mean	sd	
Austria	5	3	79	30	63	21	10
Belgium	29	17	142	136	197	105	9
Switzerland	6	2	206	162	160	108	4
Germany	18	10	145	59	147	109	7
Denmark	17	29	76	86	38	30	4
Catalonia (Spain)	16	10	251	173	227	109	8
Finland	8	5	185	127	35	13	10
France	206	69	2563	877	833	259	4
Greece	1	1	11	3	8	4	5
Italy	8	9	252	96	153	139	7
Luxembourg	1	0	8	0	30	0	1
Netherlands	48	26	367	554	174	82	10
Norway	3	1	101	72	32	13	4
Poland	4	2	56	28	100	83	4
Portugal	3	4	117	151	137	258	6
Russia	3	1	88	46	35	7	2
Sweden	43	28	284	157	358	306	4
United Kingdom	78	73	1444	1663	369	214	13
USA	50914	8157	238820	22876	370787	115773	9
All countries	3815	13571	18140	63099	27746	102121	121

Table 1: Summary statistics.

	log-CES
$ln(A)$	0.9358
	(0.3136)
β_K	0.0967**
	(0.0057)
β_E	0.9033***
	(0.0057)
v	0.9200**
	(0.0258)
θ	0.5161
	(0.1799)
Year controls	YES
Country controls	YES
N	116
R-squared	0.95
$(v - \theta) > 0$ p-value	0.0147
$\theta \to 0$ p-value	0.2135
CRS p-value	0.1987

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 2: Results of the nonlinear estimation of the log-CES matching function.

	(1)	(2)	(3)
	log-CD	log-CD with interaction	translog
	Elasticities	Elasticities	Elasticities
β_K	0.5517**	0.5672***	0.6087***
	(0.2214)	(0.0380)	(0.0896)
β_E	0.5362***	0.2689***	0.3264***
	(0.1843)	(0.0535)	(0.0609)
β_{EK}		0.0154***	0.3074***
		(0.0030)	(0.0425)
β_{KK}			-0.1356***
			(0.0246)
β_{EE}			-0.1612***
			(0.0204)
Year controls	YES	YES	YES
Country controls	YES	YES	YES
N	116	116	116
R-squared	0.92	0.92	0.90
RTS	1.09	1.00	1.06
CRS p-value	0.0372	0.8925	0.0017

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

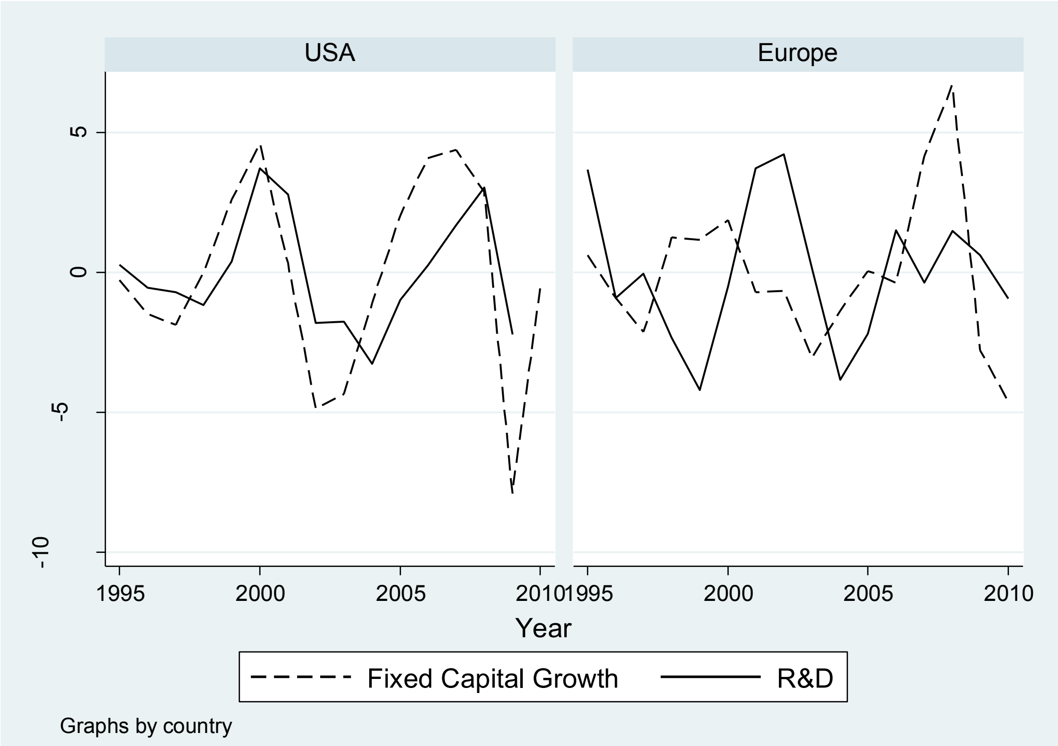
Table 3: Results of the estimations of the log-log matching function specifications.

	(1)	(2)	(3)
	log-CD	log-CD with interaction	translog
	Elasticities	Elasticities	Elasticities
β_K	0.1690*	0.4242***	0.5304***
	(0.0909)	(0.1488)	(0.1748)
β_E	0.8198***	0.4403***	0.3326**
	(0.0761)	(0.1340)	(0.1360)
β_{EK}		0.0101***	0.2627***
		(0.0031)	(0.0460)
β_{KK}			-0.1156***
			(0.0375)
β_{EE}			-0.1347***
			(0.0208)
β_{DC}	-0.0006	0.0025	0.0021
	(0.0014)	(0.0017)	(0.0021)
β_{MC}	0.0065***	0.0042***	0.0034***
	(0.0021)	(0.0014)	(0.0007)
Year controls	YES	YES	YES
Country controls	YES	YES	YES
N	114	114	114
R-squared	0.92	0.93	0.92
RTS	0.99	0.97	1.01
CRS p-value	0.6971	0.4483	0.8907

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

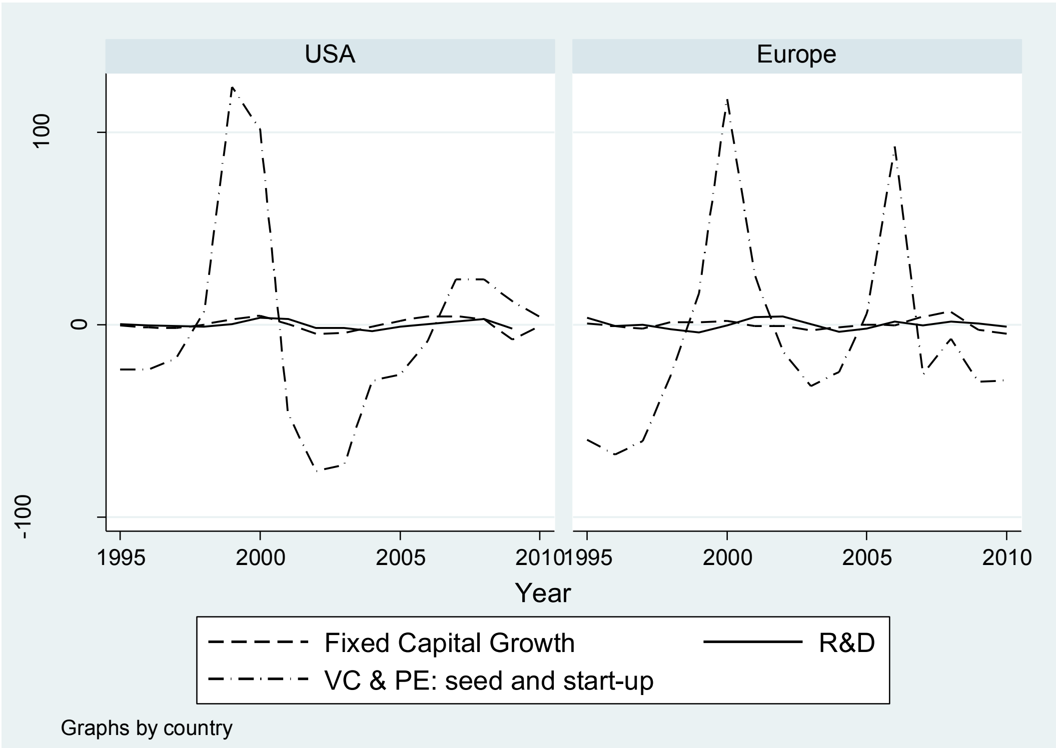
Table 4: Results of the estimations of the log-log matching function specifications (continued).

Figure 1. Volatility of investments in fixed capital and in R&D, years 1995-2010.



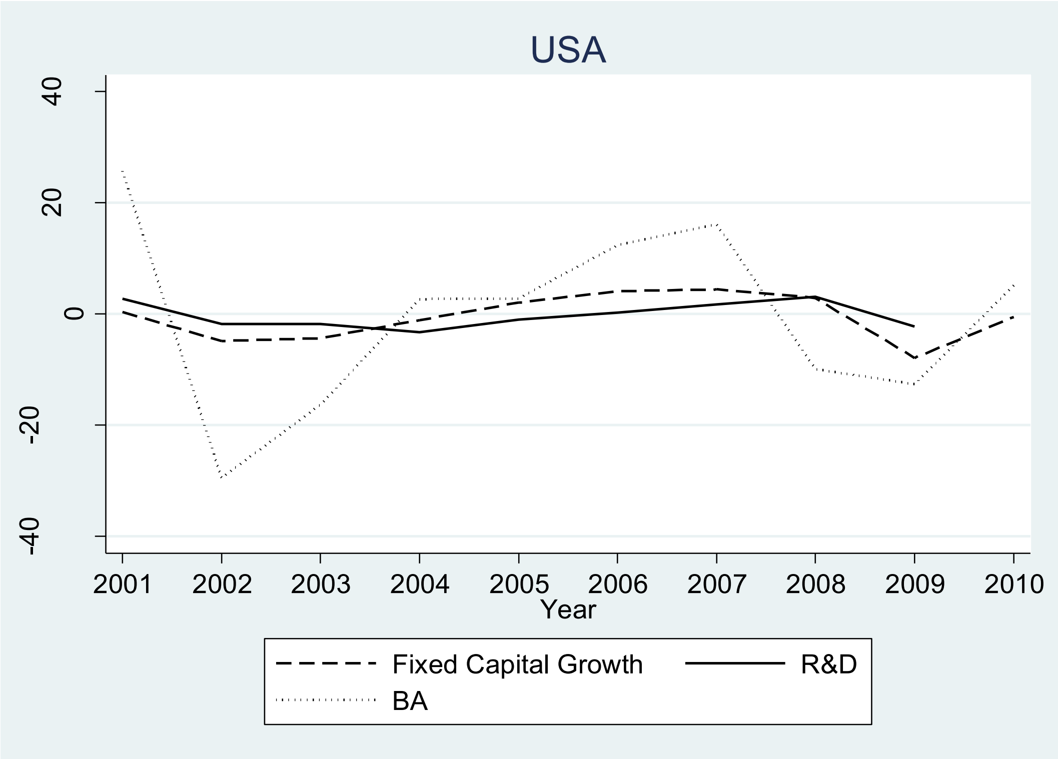
Note: The value of US investments in R&D for the year 2010 is still missing from the OECD database.
Source: Own elaborations from OECD.Stat.

Figure 2. Volatility of investments in fixed capital and in R&D, and volatility of investments in seed-and start-ups provided by Venture Capitalists (VC) and Private Equity (PE) funds, years 1995-2010.



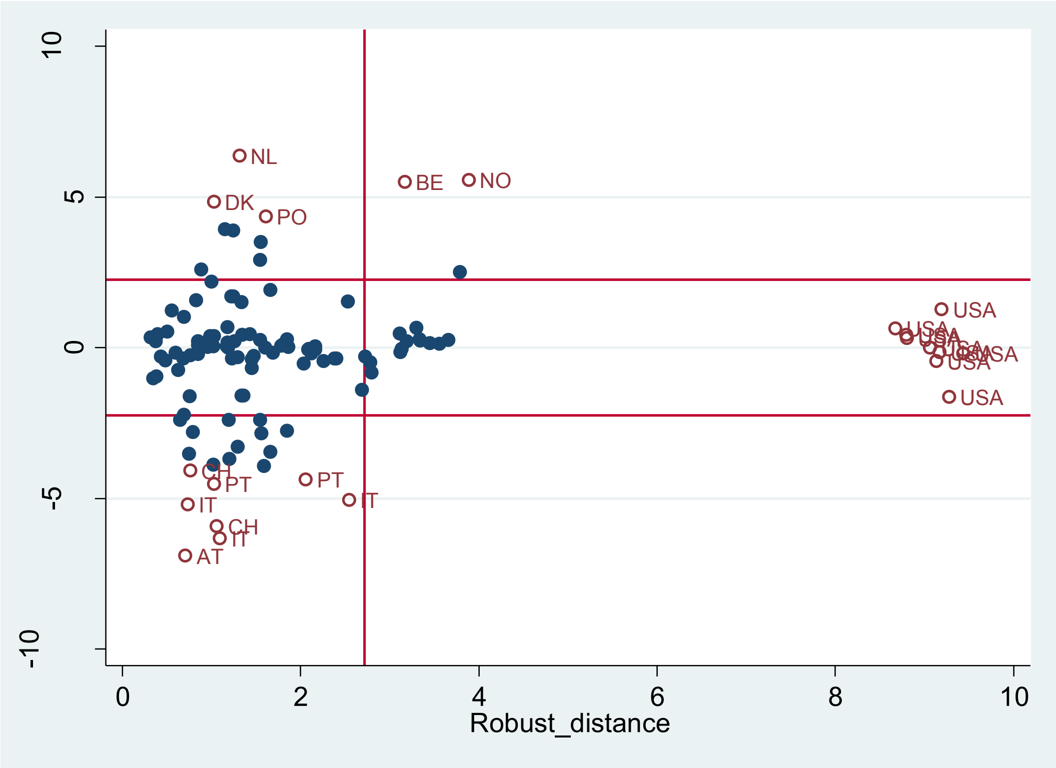
Note: The value of US investments in R&D for the year 2010 is still missing from the OECD database.
Source: Own elaborations from OECD.Stat, MoneyTree Report, EBAN Annual Reports.

Figure 3. Volatility of investments in fixed capital and in R&D, and volatility of investment funds provided by business angels (BA), years 2001-2010.



Note: The value of US investments in R&D for the year 2010 is still missing from the OECD database.
Source: Own elaborations from OECD.Stat, MoneyTree Report, CVR Angel Market Activity Reports.

Figure 4. Diagnostic plot of standardized robust residuals versus robust Mahalanobis distance of the vector of covariates from the vector of their means.



Note. The Mahalanobis distance of a multivariate vector $\mathbf{x}$ of $1\times p$ dimension with mean vector $\boldsymbol{\mu}$ and covariance matrix Σ is defined as: $D(x)=\sqrt{(\mathrm{x}-\boldsymbol{\mu})^{T}\boldsymbol{\Sigma}^{-1}(\mathrm{x}-\boldsymbol{\mu})}$, which follows a chi-squared distribution with p degree of freedom under normality. Observations lying at the right hand side of the vertical limit (set at $\sqrt{X^2_{p,0.975}}$) are defined as *good leverage points*. Their presence does not affect the OLS-estimation but it affects the statistical inference since they do deflate the estimated standard errors. Observations lying above or below the area delimited by the two horizontal limits (set at -2.25 and +2.25, respectively) are defined as *vertical outliers* and affect the estimated intercept of an OLS-estimation. Observations lying both at the right hand side of the vertical limit and outside the 95% confidence interval of the Standard Normal are considered *bad leverage points*. Their presence significantly affects the OLS-estimates of both the intercept and the slope.