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RISK PREMIA IN LONG-DURATION SOVEREIGN BONDS

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Abstract

In this paper I develop a model of sovereign lending with default and long-duration coupon bonds. Long-duration bonds offer an insurance benefit to the borrower because countries are not required to frequently roll-over outstanding debt. However, investors anticipate that countries might default in the future and ask for returns that compensate for this risk. In this framework, I find that bonds with longer duration offer higher interest rate spreads. Bonds issued by countries that are more likely to receive negative income shocks when investors' consumption is low have significantly higher interest rate spreads because investors anticipate defaults many periods into the future.

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In this paper I develop a model of sovereign lending with endogenous default and long-duration coupon bonds. When a country issue short-period bonds, it is forced to roll-over potentially large negative assets position in times when consumption is low. Long-duration bonds offer an insurance benefit to the borrower because countries are not required to frequently roll-over outstanding debt. However, investors anticipate that countries might default in the future and ask for returns that compensate for this risk. In this framework, I find that bonds with longer duration offer higher interest rate spreads. Bonds issued by countries that are more likely to receive negative income shocks when investors' consumption is low have significantly higher interest rate spreads because investors anticipate defaults many periods into the future.

Emerging countries tend to issue long-term bonds in US dollars. The weighted average maturity of bonds included in the EMBI Global index in 2007 was 15.44 years. Market participants often use duration as an alternative measure of the time to maturity (e.g., the number of years to the maturity date) of a bond. Duration is a measure of how long in years it takes for the price of a bond to be repaid by its internal cash flows. Duration for emerging market bonds is usually significantly lower than the corresponding maturity because most of the bond's price comes from coupon payments made early in the future while the principal is significantly discounted given the commonly high yields. For example, the weighted average duration of bonds included in the EMBI Global index was 7.90 years in 2007.

Regardless of the evidence that emerging countries bonds have long maturity, most existing models of sovereign lending with default assume 1-period bonds. The main difficulty in analyzing multiple periods bonds is that the number of state variables normally increases with the periods to maturity. In this paper, I incorporate long-duration bonds in a dynamic model of sovereign lending with default and risk averse investors. I start from Aguiar and Gopinath (2006). In my model, a small open economy borrows from a large developed country. I consider endowment economies. I introduce two key modifications to Aguiar and Gopinath (2006): I assume that investors are risk averse and countries issue long-duration bonds. A long duration bond pays an infinite stream of coupons until a borrower defaults. The coupon payments decrease geometrically with time. By varying the rate at which coupon payments decay over time, it is possible to vary the duration of a bond. Each period, the price of a bond depends on the probability of a country's default in each of the following periods. In the model, countries default after receiving a series of negative shocks. I show that in this framework, interest rate spreads increase with the duration of the bonds. The spread between a a 5-year and a 1-quarter duration bond is about 700 basis points per annum on average. For a given bond duration, spreads increase with the correlation between the borrower and the lender business cycle with a rate that increases with the duration of the bond.

With 5-year duration bonds, the spread between countries with low and high correlation with the lenders is 350 basis points per annum. With 1-period bonds, a sovereign must re-finance every period any outstanding debt. Given the persistence of the output process, a country must roll-over bonds issued in bad times when the endowment level is low. In these times, outflows of funds are particulary costly and default probabilities very high. As a result, spreads are very low in good times and very high in bad times, for a given amount of debt. In equilibrium, countries borrow only small amount of debt in bad times and spreads are lower than in the data. In my model, the endowment of the borrower is characterized by stochastic shocks to trend growth and countries issue long-duration bonds. In good times for the borrower, the probability of default in the next period is small but spreads may be significantly higher than zero because lenders anticipate a higher default probability many periods ahead. Bonds issued in bad times must not be immediately re-financed. As a result, incentives to default are lower and in equilibrium countries borrow more. Bonds issued by countries that are more likely to receive negative income shocks when investors' consumption is low have significantly higher interest rate spreads because investors anticipate defaults many periods into the future.

This paper is a contribution to the theoretical literature on sovereign lending with defaults. The paper closest to mine are Arellano (2008) and Aguiar and Gopinath (2006). They build on the seminal work by Eaton and Gersovitz (1981) and develop a dynamic general equilibrium model of sovereign lending with endogenous default choice. In their models countries issue only 1-period bonds. Hatchondo and Martinez (2009) incorporate long-duration bonds in a similar framework and show that the mean and the volatility of interest spreads increase with the duration[1]. In their model, countries face shocks around a constant trend and investors are risk neutral. In case of default, countries do not lose access to international capital markets but face direct output costs up to 50 percent of their endowment. Borri and Verdelhan (2009) incorporate risk averse investors in the framework of Aguiar and Gopinath (2006) and Arellano (2008) and find that interest rate spreads are higher for countries that are more likely to default in bad times for the lenders. However, the maximum spreads supported by their model are lower than those observed in the data. This paper shows that the introduction of long-duration bonds improves the ability of models of sovereign lending with default to replicate the size and dynamics of emerging market spreads.

The choice of long-duration bonds, as opposed to bonds with multiple maturities, implies that the dimensionality of the model's state space does not increase with respect to the case of 1-period bonds. I identify potential computational problems of this approach. I solve the model by discrete state-space methods and iteration on the value functions and the price functions. I find that the algorithm does not reach convergence for any size of the grids. Chatterjee and Eyigungor (2009)

[1]Leland (1994) and Leland and Toft (1996) consider long-duration bonds in models of optimal corporate capital structure and debt value.

find the same computational problems in a similar framework. This is a source of concern that the literature on models of sovereign lending with long-duration bonds has to address with further research.

1 A General Equilibrium Model of Sovereign Borrowing

In this section, we build a model of sovereign lending and default with long-duration bonds. We start off the seminal models of Eaton and Gersovitz (1981) and Aguiar and Gopinath (2006). But we depart from the previous literature and assume .. This simple departure has key implications on sovereign bond prices ...

1.1 Endowments

In the model, there is one small emerging open economy and one large developed economy. In the small open economy, there is a representative agent who receives a stochastic endowment stream. In what follows, the superscript B (for 'borrower') denotes variables corresponding to the small open economy, the superscript L (for 'lenders') the large developed economy. Upper case variables denote levels, lower case variables denote logs. The borrower's endowment is composed of a transitory component z_t and a trend Γ_t as in Aguiar and Gopinath (2006):

$$Y_t^B = e^{z_t}\Gamma_t. \tag{1.1}$$

The transitory component, z_t follows an AR(1) around a long run mean μ_z:

$$z_t = \mu_z(1-\alpha_z) + \alpha_z z_{t-1} + \epsilon_t^z.$$

The trend is described by:

$$\Gamma_t = G_t\Gamma_{t-1} \tag{1.2}$$

where:

$$g_t = log(G_t) = \mu_g(1-\alpha_g) + \alpha_g g_{t-1} + \epsilon_t^g.$$

We assume that ϵ^g, ϵ^z are $i.i.d$ normal and that shocks to the transitory and trend component are orthogonal ($E(\epsilon^{g'}\epsilon^z) = 0$). In the large developed economy, there is a representative agent that receives every period an exogenous consumption endowment. We assume that idiosyncratic shocks to consumption growth are $i.i.d$. log-normally distributed:

$$\Delta c_t^L = \overline{c}^L + \epsilon_t^L.$$

Shocks to lenders' consumption endowment are potentially correlated to shocks to the borrower's endowment: $E(\epsilon^{g'}\epsilon^L) = \rho^g$ and $E(\epsilon^{z'}\epsilon^L) = \rho^z$.

All variables in the model are real, and we abstract from monetary policies. In the emerging economy, a benevolent government maximizes the welfare of its representative citizen. To do so, the government can borrow resources from the developed country. The government, however, can only trade non contingent long-duration coupon bonds. These debt contracts are perpetuity that pay an infinite stream of coupons and are not enforceable: the government can choose to default on its debt at any point in time. In this set-up, if investors are risk neutral, the price of a sovereign bond depends exclusively on the (expected) endogenous probabilities of default in each of the periods after a bond is issued. These default probabilities vary with the stock of bonds that will be issued and with the expected future endowment levels. As a result, sovereign bond prices only depend on country-specific characteristics and the large common variation in bond prices observed in the data could only be explained by a high degree of correlation across the countries' endowment processes, leaving no role for systematic risk factors related to foreign investors. But if investors are risk-averse, then sovereign bond prices reflect the correlation between the emerging economy' business cycle and the US economy. Bonds issued by countries that are more likely to default when the investors' consumption is low, are riskier. As a result, these bonds have lower prices and offer higher yields.

1.2 Borrowers

I start with the description of the small open emerging economy. The representative agent in each small open economy maximizes the stream of discounted utilities U^B:

$$U^B = E_t \sum_{t=0}^{\infty} (\beta^B)^t U_t^B = E_t \sum_{t=0}^{\infty} (\beta^B)^t \frac{(C_t^B)^{-\gamma}}{1-\gamma},$$

where β^B denotes the time discount factor, and C_t^B denotes consumption at time t. We let the lenders' and borrowers' discount factors differ because developing countries tend to have higher real risk free rates than emerging countries.[2]

The representative household receives a stochastic stream of the tradable good Y_t^B every period. I assume that y_t^B, the log of the borrower's endowment, follows a Markov process. The representative agent also receives a goods transfer from the government in a lump-sum fashion: i.e, any proceeds from international operations are rebated lump-sum from the government to its

[2]Political economists argue that politicians tend to have shorter time horizons in small developing countries. In Amador (2003) for example, a low value for the discount factor β^B corresponds to the high short-term discount rate of an incumbent party with low probability of remaining in power in a model where different parties alternate.

citizens. The government has access to international capital markets: at the beginning of period t, it can issue a bond with face value F_t and price Q_t, that pays an infinite stream of coupons in the subsequent periods. I assume that the coupon payments decrease geometrically over time at the rate δ. For example, a government that issues a bond with face value F_t, receives $Q_t F_t$ units of the consumption goods in period t, and will repay $\delta^{i-1} F_t$ units of the consumption good in period $t+i$ with $i = 1, ..., \infty$ unless it defaults on its outstanding debt. If the government defaults, all outstanding debt is erased. I define the total coupon payments due in period t on all past bond issuances outstanding as $B_t = \sum_{i=1}^{t} \delta^{i-1} F_{t-i}$. As a result, the law of motion for the stock of coupon payments is:

$$B_{t+1} = \delta B_t + F_t \tag{1.3}$$

A positive (negative) value for B_t^{t+1} represents a negative (positive) net asset position for the borrowing country. The representative household's budget constraint conditional on not defaulting at time t is then:

$$C_t^B = Y_t^B + Q_t F_t - B_t. \tag{1.4}$$

In case of default, all current debt disappears. A sovereign that defaults at date t is excluded from international capital markets for a stochastic number of periods and suffers a direct output loss. In this case, consumption is constrained by the value of output during autarky, which is denoted $Y_t^{B,def}$, and the budget constraint is simply:

$$C_t^B = Y_t^{B,def}. \tag{1.5}$$

Following Aguiar and Gopinath (2006), I assume an fixed-proportion direct output cost of default. In particular, $Y_t^{B,def} = Y_t(1-\theta)$. A second consequence of a country's default is exclusion from international capital markets. In Eaton and Gersovitz (1981) exclusion is permanent, and default is not an equilibrium outcome. I follow Aguiar and Gopinath (2006) and assume that exclusion lasts a stochastic number of periods. Although this assumption implies a degree of coordination by foreign investors that is partially at odds with the assumption that investors behave competitively, it captures the fact that countries in default do not access international capital markets for some time. As Hatchondo, Martinez, and Sapriza (2007) note, in this framework, the equilibrium size of debt is smaller when the exclusion from capital markets is shorter. This is because exclusion works an incentive to repay, thus reassuring lenders, decreasing the risk premium and allowing more borrowing.

1.3 Lenders

I now turn to the description of the lenders. The representative agent receives an exogenous stochastic consumption endowment every period denoted C_t^L. Lenders are risk-averse and behave competitively. I assume that lenders maximize the stream of discounted utilities U^L:

$$U^L = E_t \sum_{t=0}^{\infty} (\beta^L)^t U_t^L = E_t \sum_{t=0}^{\infty} (\beta^L)^t \frac{(C_t^L)^{1-\gamma} - 1}{1-\gamma},$$

where β^L denotes the lenders' discount factor.

Lenders supply any quantity of funds demanded by the small open economy, but they require compensation for the risk they bear. Lenders cannot default. In Arellano (2008), lenders are risk-neutral. In that case, lenders charge the borrower the interest rate that makes them break-even in expected value. In our model, lenders are risk-averse, and require not only a default premium, but also a *default risk premium*. They expect a higher return on average if defaults are more likely in bad times for them.

1.4 Recursive equilibrium

In order to describe the economy at time t, I need to keep track of the borrower's endowment stream and his outstanding debt. Let y^B denote the history of events up to t: $y^B = (y_0^B, ..., y_t^B)'$. Given that the stochastic endowment process is Markovian, I denote $f(y^{B'}, y^B)$ the conditional density of $y^{B'}$, e.g. the value of y^B at time $t+1$ given the initial value of y^B at time t. In what follows, the value of a variable in period $t+1$ is denoted with a *prime* superscript and the value of a variable in period $t+1$ with a *double prime* superscript.

Given the initial state of the economy, the value of the default option is:

$$v^o(B, y^B) = max\{v^c(B, y^B), v^d(y^B)\},$$

where $v^c(B, y^B)$ denotes the contract continuation value and v^d the value of defaulting. If the government chooses to repay the coupon payments coming to maturity, it can purchase new debt with face value F. I use the law of motion for the coupon payments 1.3, to express the bond face value as $F = B' - \delta B$. With this simple variable substitution, I reduce the number of state variables because I need only to keep track of the outstanding stock of coupon payments, and not of the face value of each outstanding bond. In period t, if the government does not default on outstanding coupon payments B, it choose the new optimal debt level by picking a value for B'. If $B' > \delta B$, the government issues new debt and F is positive. If $B' < \delta B$, the government is saving. As a result, the value of staying in the contract is a function of the exogenous states y^B, the quantity of coupon coming to maturity B and future coupon payments B'. In case of default, all outstanding debt is erased, and the small economy is forced into autarky for a stochastic number of periods. Hence, the only state variables that influence the value v^d of defaulting is y^B. I now define more precisely v^c and v^d.

The value of default depends on the probability of re-accessing financial markets in the future and on the current output loss:

$$v^d(y^B) = u_B(y^{def}) + \beta \int_{y^{B'}} [\lambda v^o(0, y^{B'}) + (1-\lambda) v^d(y^{B'})] f(y^{B'}, y^B) dy^{B'},$$

where λ is the exogenous probability of re-entering international capital markets after a default. As I have seen, when a borrower defaults, consumption is equal to the autarky value of output. In the following period, the borrower regains access to international capital markets with no outstanding debt with probability λ, or remains in autarky with probability $1-\lambda$.

The value of staying in the contract and repaying debt coming to maturity is:

$$v^c(B, y^B) = Max_{B'} \{u(c) + \beta \int_{y^{B'}} v^o(B', y^{B'}) f(y^{B'}, y^B) dy^{B'}\}, \tag{1.6}$$

subject to the budget constraint (1.4). Note, I have used (1.3) to replace F with B' in (1.6). The borrower chooses B' to maximize utility and anticipates that the equilibrium bond price depends on the exogenous state variables and on the new debt B'.

Let Υ denotes the set of possible values for the exogenous states y^B. For each value of B, the small open economy default policy is the set $D(B)$ of exogenous states such that the value of default is larger than the value of staying in the contract:

$$D(B) = \{y^B \in \Upsilon : v^d(y^B) > v^c(B, y^B)\}.$$

Similarly, $R(B)$ is the set of exogenous states such that the value of default is smaller than the value of staying in the contract. The repayment set $R(B)$ is the complement to $D(B)$:

$$R(B) = \{y^B \in \Upsilon : v^d(y^B) \leq v^c(B, y^B)\}.$$

The default probability dp is endogenous and depends on the amount of outstanding debt and on the endowment realization. In particular, the default probability is related to the default set through:

$$dp(B', y^B) = \int_{D(B')} f(y^{B'}, y^B) dy^{B'},$$

where $dp(B', y^B)$ denotes the expectation at time t of a default at time $t+1$ for a given level B' of outstanding debt due at time $t+1$.

1.5 Bond prices

Bonds issued by the small open economy are perpetuities that promise to repay an infinite stream of income which decays at a geometrical rate. If the borrower defaults, all outstanding debt is erased δ. Bond prices are a function of the current state vector y^B and the face value of the desired level of borrowing F. I use the law of motion for the stock of coupons (1.3) to substitute F with $B' - \delta B$. If the government does not default at date $t+i$, lenders receive coupon payments equal to a decaying fraction (δ^{i-1}) of the bond face value, which is normalized to 1. In case of default, payoffs are zero from that date onward. Starting from the investor's Euler equation, the bond price function is:

$$Q = E[M' 1_{1-dp(B', y^B)}] + \delta E[M' M' 1_{1-dp(B'', y^{B'})}] + ... \tag{1.7}$$

where M' is the investors' stochastic discount factor and is equal to:

$$M' = \beta^L \frac{U_{c^L}(C')}{U_{c^L}(C)} = \beta^L e^{-\gamma \overline{c}^L + \frac{\gamma^2}{2}\sigma^2_{\epsilon^L}}$$

In 1.7 I use the fact that $E(M'') = E(M')$, because lenders' endowment growth is $i.i.d.$. The bond price in (1.7) depends on the current level of the endowment and on the entire future stock of coupon payments, because the outstanding debt levels in each future periods influence the future probabilities of default. If the probability of default is zero in each of the future periods, the bond price is equal to $Q^{rf} = 1/(R^{f,1} - \delta)$, where $R^f = E[M']^{-1}$ is the (gross) risk free rate on a 1-period zero-coupon bond that investors can purchase. The bond price function (1.7) implies that the price of a bond depends on the expected probabilities of coupon repayments in each future periods up to a default, weighted by the investors' stochastic discount factors. If future defaults tend to occur in bad times for the investors (e.g., when their marginal utility of consumption is high), bond prices are low and yields are high. In 1.7, the price of a bond depends on an infinite series of future debt levels. It is possible to express it in recursive form as follows:

$$\begin{aligned} Q(B', B, y^B) &= E[M'(1_{1-dp(B', y^B)} + Q'(B'', B', y^{B'}))] \\ &= E[M'](E[1_{1-dp(B', y^B)}] + E[Q'(B'', B', y^{B'})]) + \\ &+ cov[M', (1_{1-dp(B', y^B)} + Q'(B'', B', y^{B'}))], \end{aligned} \tag{1.8}$$

where $Q'(B'', B', y^{B'})$ is the bond price at date $t+1$, when the borrower choose the optimal quantity of debt B'' given the initial debt level B' and the endowment level $y^{B'}$. Here I assume that investors know the borrower's debt policy function in order to form expectations on $Q'(B'', B', y^{B'})$. If, for a given state y^B, $B' < \delta B$, then the government is saving. I assume that investors in the large developed economy have no commitment problem and repay any debt with probability one. As a result, in this special case the bond price is equal to the price of a risk free long-duration bond with coupon decay factor δ. The price of this risk-free bond is equal to:

$$Q^{rf} = \frac{1}{R^{f,1} - \delta}.$$

I assume for simplicity that the small open economy cannot issue or purchase one-period bonds. As a result, $R^{f,1}$ is the risk-free rate on a one-period zero-coupon bond issued and purchased by investors in the large developed economy. Figure 1 plots the price function of a long-duration bond with face value equal one for the case of no default risk. When δ is close to 0, the bond price converge to the price of a 1-period risk free bond. The higher is δ, the higher the bond price. With long-duration bonds, the bond price can be significantly higher than 1. A country issuing a bond with high δ in period t, will receive a multiple of the bond face value at time t and will repay a decaying fraction of the bond face value thereafter.

1.6 Bond duration

The duration of a bond is a measurement of how long it takes for the price of a bond to be repaid by its internal cash flows. It is expressed as number of periods (e.g., years, quarters, etc). The maturity of a bond is the number of years before the bond face value is repaid. For example, the duration of a zero-coupon bond is equal to its maturity. For coupon bonds with long maturity the return on principal has very low value. As a result, duration is a better measure of the sensitivity of a bond price to changes in the interest rate. The duration of a bond that pays an infinite stream of coupons that decay geometrically at rate δ is equal to[3]:

$$D_t = \frac{1}{Q_t} \sum_{n=1}^{\infty} n \frac{\delta^{n-1}}{R^n}, \tag{1.9}$$

where R is the yield to maturity of the long-duration bond. I use the standard definition for yield to maturity: the constant rate that equates the present value of the stream of coupon payments to the bond price:

$$Q_t = \sum_{n=1}^{\infty} \frac{\delta^{n-1}}{R^n}.$$

[3]This definition of bond duration is an approximation of the Macaulay (1938) duration. The true Macaulay definition weights each coupon payment time by the yield to maturity of a zero-coupon with maturity in that period. In 1.9 the weight on each coupon payment time is the constant yield to maturity of the long-duration bond.

As a result, we can express the yield to maturity as:

$$R = \frac{1}{Q_t} + \delta. \tag{1.10}$$

In what follows I refer to yield spreads as the difference between the yield to maturity of the long-duration bond and the risk-free rate on the 1-period bond:

$$R^e = R - R^{f,1}$$

By substituting the definition of yield to maturity 1.10 in 1.9, I can express the duration of a long-duration bond as:

$$D_t = \frac{R}{R - \delta}.$$

As a result, the bond duration is a function of both the coupon decay factor δ and the bond yield to maturity. If δ is equal to 0, the duration is equal to 1. This is the case of a 1-period zero-coupon bond. The higher δ, the larger the fraction of the bond price that is repaid in future periods and the longer the duration. The higher the yield to maturity, the smaller is the present value of future coupon payments. Hence, bond duration decreases with the yield to maturity. Figure 2 plots the duration of a bond that pays an infinite stream of coupons as a function of the yield to maturity (R) and the coupon decay parameter δ. Duration is high when R is low and δ is closer to 1. The difference between the duration of a 1-period bond and that of a bond that pays an infinite series of coupons is small for high vales of R and when δ is close to 0.

2 Simulation

I simulate the model at quarterly frequency. I start by reviewing its parameters.

2.1 Calibration

I calibrate the borrower's endowment process described in 1.1 using the parameters in Aguiar and Gopinath (2006). These parameters describe Argentina. In order to focus on permanent shocks to trend growth, I shut down the transitory components of the endowment by setting $\sigma_{\epsilon^z} = 0$. I calibrate lenders' consumption growth using post-war U.S. economy as a reference. Table 1 reports all the parameters used in the simulation.

The direct output cost of default θ is equal to 2 percent per period. The probability of re-entering capital markets after a default λ is equal to 10 percent, implying an average exclusion of 10 quarters. The risk aversion parameter γ^B in the borrower's utility function is set equal to 2.

I use a higher value for the risk aversion parameter $\gamma^L = 10$ in order to match the large spreads between bonds issued by country with different correlation with the US business cycle. Lenders discount future at the annualized rate $\beta^L = 0.81$, while the borrower has a lower time discount factor $\beta^B = 0.40$. The value of γ^L and β^L are calibrated in order to match the average US real log risk-free rate of 0.94 percent per annum. The coupon decaying rate δ is equal to 0.95 and implies a duration for a risk free bond of approximately 5 years.

2.2 Results

Table 2 reports results of the model's simulation. In the first panel I report the first moments. In the second panel I report standard deviations. In the first column, I report results in Aguiar and Gopinath (2006). They consider a small open economy issuing 1-period zero-coupon bonds and risk neutral investors. My model incorporates long duration bonds and risk averse investors. I assume the same endowment for the small open economy as in Aguiar and Gopinath (2006). In particular, in order to focus on shocks to trend growth I shut down transitory shocks z. Columns 2,3 and 4 in table 2 correspond to simulations of three different economies that differ only with respect to their endowment correlation (ρ) with respect to lenders' consumption growth. Reported moments are from simulations of 100,000 quarters, and are computed on the last 90,000 quarters.

The model with risk averse investors and long-duration bond generates interest rate spreads that are significantly higher than in Aguiar and Gopinath (2006). The maximum annual spread in their simulation is 151 basis points per annum. The maximum spreads in the model with long-duration bonds are on average 1000 basis points per annum. Interest rate spreads are higher because the default probabilities are higher. In the model with long duration bonds, default probabilities are about 6.5 percent per annum. As a result, countries default 6 times more frequently than in the model of Aguiar and Gopinath (2006). The high spreads do not depend on the fact that γ^L is higher than γ^B. A model with γ^L equal 2 generates average spreads of approximately 700 basis points per annum (these results are not reported in the paper, but are available upon request). However, a high value for γ^L is important to generate a cross-section of interest rate spreads as a function of the correlation between the business cycles in the small open economies and in the large developed economy.

If the model with long duration bonds improves dramatically simulated average bond spreads, it fails completely to account for debt quantities. The average debt levels in column 2,3 and 4 are approximately 1.5 percent of a country GDP. This contrast with the debt to GDP ratio in Aguiar and Gopinath (2006) which is 19 percent. Hatchondo and Martinez (2009) develop a model that incorporates long duration bonds. Their model generates debt to GDP ratios of up to 50 percent. However, they assume very large costs of default of up to 50 percent of GDP. In fact, in their setting there is a strong correlation between cost of default and average debt level. In my model, I assume a direct cost of default of 2 percent and exclusion from capital markets for a stochastic number of periods.

In panel II of table 2, I report standard deviations of hp-trended quarterly series. Standard deviations are annualized and in percentage points. In the model with long duration bonds, consumption is more volatile than output. A consistent feature of emerging countries business cycle shared with the model of Aguiar and Gopinath (2006). The standard deviation of the interest rate spread is low despite the fact that the model generates a large variation in the trade balance.

3 Conclusion

In this paper, I show that incorporating long-duration bonds in a model of sovereign lending with default and risk averse investors helps increases predicted spreads. Long-duration bonds offer an insurance benefit to the borrowers because countries are not required to frequently roll-over outstanding debt. However, investors anticipate that countries might default in the future and ask for returns that compensate for this risk. As the possibility to issue long-duration bonds improves the ability of models of sovereign lending with default to match the observed spreads in the market for emerging market bonds, it does not help to account for the debt quantities. Further research is needed to develop models that jointly account for debt prices and quantities in the market for sovereign debt.

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Table 1: Parameters Choices

Parameter	Variable	Value
Mean lenders' consumption growth (%)	g^L	1.89
Standard deviation of lenders' consumption growth (%)	σ_{ϵ^L}	1.50
Persistence of borrowers' endowment	α_g	.17
Standard deviation of borrowers' endowments (%)	σ_{ϵ^g}	3
Mean trend growth rate (%)	μ_g	2.5
Direct default cost (%)	θ	2
Probability of re-entry(%)	λ	10
Risk-aversion parameter small open economy	γ^B	2
Risk-aversion parameter large developed economy	γ^L	10
Lenders' time discount	β^L	.81
Borrowers' time discount	β^B	.40
Coupon decay parameter	δ	0.95

The table reports benchmark values for the parameters used in the simulation. These parameters imply an annualized risk-free rate r^f in the large developed country equal to .94 percent per annum. The coupon decay parameter is calibrated to match a duration for the case of a risk-free bond of 5 years. The values for the direct output cost and the probability of re-entering financial markets after a default are per quarter. All the other parameters are annualized, e.g. they are reported as $4g^L$, $2\sigma_{\epsilon^L}$, $2\sigma_{\epsilon^g}$,α_g^4, β^{L^4}, β^{B^4} and $4r^f$ since the model is simulated at quarterly frequency. Values describing lenders' consumption growth are from Campbell and Cochrane (1999) and correspond to post-war US consumption data. Values describing the borrowers' endowments are from Aguiar and Gopinath (2006).

Table 2: Results

	Aguiar and Gopinath (2006)	LD Bond ($\rho = -.5$)	LD Bond ($\rho = 0$)	Long Duration ($\rho = -.5$)
		Panel I: First Moments		
Maximum Spread	151	800	938	1150
Mean Spread	100	665	758	875
Def Prob (%)	0.92	6.90	6.95	6.90
Debt/Y (%)	19	1.5	1.5	1.5
		Panel II: Second Moments		
$\sigma(y^B)(\%)$	8.9	8.85	11.80	8.85
$\sigma(c^B)(\%)$	9.42	12.09	8.85	11.70
$\sigma(R)$ (%)	0.32	0.23	0.27	0.32
$\sigma(TB/Y)(\%)$	1.9	6.97	6.75	6.44

This table presents results from simulation of the model at quarterly frequency. The first column reports results in Aguiar and Gopinath (2006). They use a model with risk neutral investors, 1-period zero coupon bonds and shocks to trend growth. The remaining columns report results of simulations for the model presented in this paper with long-duration bonds, risk averse investors and shocks to trend growth. Column 2, 3 and 4 report results of simulations for different values of the correlation coefficient ρ between investors' endowment growth and borrower's trend shocks. Spreads are annualized and in basis points. Standard deviations are for hp-trended quarterly series and are annualized and in percentage points.

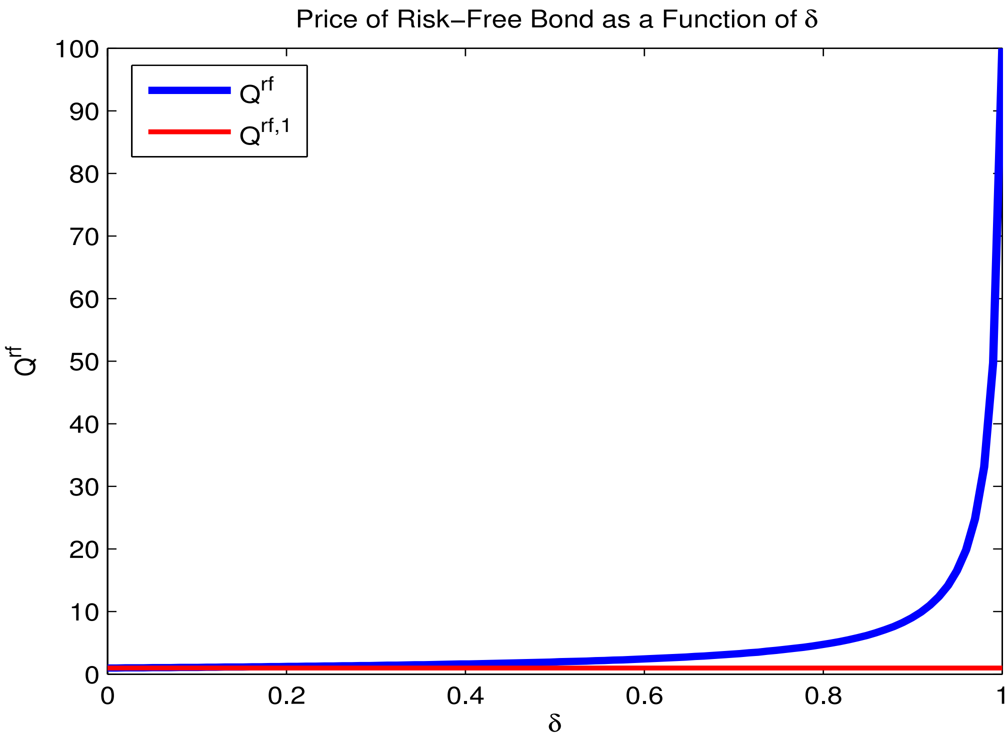

Figure 1: Price of a Long Duration Bond

This figure plots the price of a bond that pays an infinite series of coupons that decay at a geometric rate δ as function of δ. This figure assumes that the probabilities of default are zero in every period. The red horizontal line represents the price of a 1-period risk-free zero coupon bond.

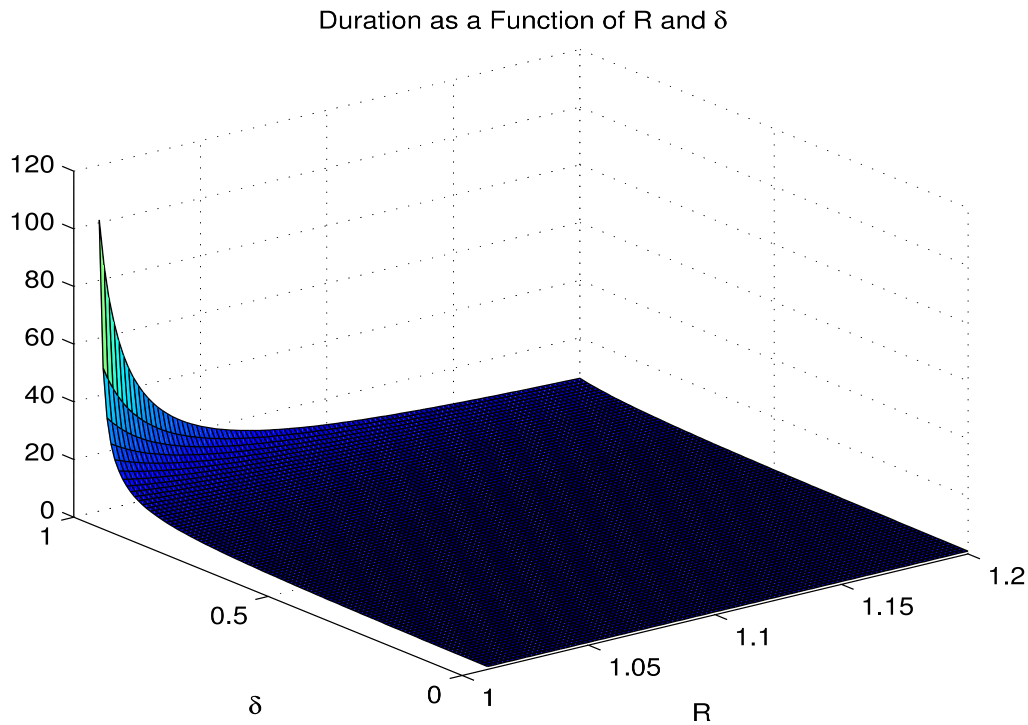

Figure 2: Bond duration

The figure plots the duration of a bond that pays an infinite series of coupons that decay at a geometric rate as a function of the yield to maturity (R) and the rate of decay of the coupons (δ).