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MONETARY POLICY, LIQUIDITY, AND CONSUMPTION INEQUALITY: A COMMENT ON WILLIAMSON (2012)

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Monetary policy, liquidity, and consumption inequality.*

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Abstract

The article shows that expansionary open-market operations are non-neutral, even with price flexibility, when the purchased assets are partially liquid. A permanent injection of money generates a real-balance effect that helps monetary transactions, reduces consumption inequality and, by this way, increases social welfare. The mechanism is effective when the economy is trapped in a stationary equilibrium with an abundant supply of high-yield assets, which also implies extreme consumption inequality – as at the sunrise of the Great Recession. Improving on this equilibrium requires the asset purchases by the central bank or, equivalently, a fall in the supply of partially-liquid assets, which reduce consumption inequality – as observed during the Great Recession. To establish the result, I borrow the Williamson (2012)'s model and return it back with a financial contract that efficiently insures individuals against their uncertain demands for liquidity. Compared to the original contract, the new arrangement enlarges the Pareto frontier to allocations with higher social welfare.

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1 Introduction

Until the burst of the financial crisis in 2008, the amount of interest-bearing assets in the financial system was huge and accompanied by a very attractive interest rate, if compared with the actual one. Up to the same moment, the rise of consumption inequality in US since the 1980s is a robust empirical evidence[1].

In the aftermath of the financial turmoil, and still now in a number of cases, central banks have used the conventional instrument of open-market operation (OMO) to buy unconventional assets for an unconventional period of time, namely the permanent purchase of sovereign bonds (QE) and private bonds (achieved under different programs). One of the result in US was the reduction of consumption inequality[2].

Any expansionary OMO wields its positive effects by lowering the targeted interest rate – whatever channel the reader may have in mind – and by changing the relative supply of outside money and interest-bearing assets in the economy. As long as the assets do not provide liquidity services to the public, their amount and return are not going to affect consumption directly. The direct impact on consumption is supposed to come from a real-balance effect of money holdings in case of sticky prices, or does not come at all with flexible prices[3].

The neutrality of OMO does no longer holds if the interest-bearing assets are only partially illiquid, i.e. they are medium of exchange to some extent. In this case a reduction of their supply and of the interest rate may have direct consequences over consumption profiles[4]. Indeed, Williamson (2012) shows that in this case any OMO that injects money is at best neutral, if not contractionary.

Building on the contribution of Williamson (2012), I reverse his conclusion by showing that a permanent purchase of partially-liquid interest-bearing assets by the central bank has expansionary effect, i.e. it increases social welfare, when the economy is trapped in a stationary equilibrium characterized

[1] Attanasio and Pistaferri (2016) report the evolution of consumption inequality over time as measured by different papers, and the trend is positive until 2009 in all of them.

[2] In Attanasio and Pistaferri (2014), "there is evidence of a substantial slowing down of consumption inequality during the Great Recession".

[3] The neutrality proposition, or the Modigliani-Miller theorem, for the OMOs has been initially proposed by Wallace (1981) and has been recently questioned by Benigno and Nisticò (2015). They restrict the feasible set of contracts between the public, the treasury, and the central bank, in order to catch the current institutional arrangements. However, they find those restrictions sub-optimal.

[4] Away from the stationary equilibrium and with frictional prices, an exogenous drop in the liquidity of interest-bearing assets is the focus of Benigno and Nisticò (2013).

by an abundant supply of high-yield assets – as it was at the sunrise of the Great Recession. More generally, away from the Friedman rule, I show that the society is better off when the return on assets is lower than the rate of time preference.

The equilibrium with abundant high-yield assets features extreme consumption inequality between agents that can pay with assets and agents that can only use money. When the authority increases the relative supply of money and lowers the interest rate (up to some point), consumption inequality shrinks – as it was observed during the Great Recession – and social welfare benefits from it. Interestingly, agents using money consume more because the model produces a real balance effect, even though prices are flexible.

As mentioned, this result emerges in the framework of Williamson (2012), an economy where assets facilitate trade and banks provide liquidity risk-sharing to their depositors. The spectrum of liquidity is represented by interest-free money, like cash or some forms of deposits, and interest-bearing assets, generically modeled as bonds. A private shock in the form of trade friction prevents a fraction of agents to use bonds as medium of exchange.

When agents discover their payment technology, i.e. only money or money and bonds, an atomistic banking sector provides an essential mechanism to reallocate liquidity across depositors. Banks in the original contribution offer a very intuitive contract: independently of the initial deposit, all the money goes to depositors that only use money (money users), and all the bonds to those that can use bonds (bond users).

In general, the interest rate makes a unit of bond more attractive than a unit of money. Moreover, money users' consumption is invariant to the relative supply of money because the price level adjusts proportionally. Given a path for inflation, the optimal policy follows: hike the interest rate up to the rate of time preference so as to increase bond users' consumption up to the efficient level.

In other words, welfare in Williamson (2012) increases with consumption inequality and is maximized in the equilibrium with plentiful high-yield assets. This mechanism implies that when the private sector does not produce enough interest-bearing assets, such that the interest rate is not the highest possible, then the authority must inject bonds and retire money.

My analysis reaches the opposite results of Williamson (2012) by only departing from his model in one dimension, namely the liquidity risk-sharing contract offered by banks. I design a different deposit contract that weakly dominates the original one in terms of ex-ante welfare. Precisely, I *restrict* the set of feasible contracts by requiring that the total claims on money and bonds at the depositor's disposal is equal to the initial amount deposited.

With respect to the original arrangement, the deposit is closer to a debt-like contract.

Under the optimal contract, a standard expansionary OMO has two opposite effects in equilibrium. On the one hand, a direct and negative consequence on the consumption of bond users, qualitatively as in Williamson (2012). On the other hand, an indirect and positive effect on the consumption of money users, despite flexible prices. In the end, consumption inequality shrinks and this ameliorates social welfare.

The expansionary channel of the bond purchase rests on the increase of real money balances, that is the price level does not rise proportionally to the supply of money, even if price are flexible. Where does this positive effect on the demand for real balances come from? It comes from the bond users: As bond users do not consume the efficient quantity anymore, bonds are worth more to them, and since they need money to buy bonds, in turn this increases the demand for money. In the end, money is valued not only as medium of exchange but also as means of payment.

The debt-like contract burdens the bank with the problem of making an efficient liquidity exchange for each type of depositor, and not just the problem of allocating assets where they are most valued in terms of consumption. Now efficiency requires that money and bonds must have the same marginal benefit in the depositor's portfolio, although the overall returns can differ across depositors.

In particular, the trade-off between money for consumption and bond for saving of the money user is not trivial. The bank must evaluate, and equate, the benefit from adding one unit of money to its account, and the cost of reducing its saving by the same amount – as the new contract imposes. At the margin, the cost of money is nothing but the interest rate on bonds. As portfolio-choice phenomenon, a lower return on bond corresponds to a lower opportunity cost of money; as general-equilibrium result, the marginal benefit of money must decreases as well, something that is accomplished with a higher monetary consumption. This link between the interest rate and the purchasing power of money in the portfolio of the money user simply mirrors the higher value of money as means of payment for bonds in the portfolio of the constrained bond users.

The result that society is better off under a restriction of individual chances is reminiscent of Kocherlakota (2003), where society benefits by restricting the liquidity of bonds. The parallelism with that result holds at the individual decision level. As any agent in that economy strictly prefers to be the sole one with liquid bonds, so here if only one agent was allowed to choose ex-ante between the two deposit contracts, while the rest of them tight at the optimal contract, this agent would certainly opt for the Williamson (2012)'s arrangement.

The next section introduces the environment and explains the new deposit contract. I describe the stationary equilibria and rank them according the welfare measure in Section III. A brief summary and some comments are left for Section IV.

2 The model

This section presents the baseline model of Williamson (2012) where fiscal policy is passive, there are no costs associated with currency, and private liquidity is absent. I work with the simplest environment because the generality of the policy implications is not compromised.

More precisely, a nonpassive fiscal policy regime is aimed at capturing the relationship in typical developed economies between the central bank and the fiscal authority; the costs associated with currency make the Friedman rule nonoptimal – and this assumption alone does not alter the welfare ranking of the equilibria where bonds are essential; private liquidity is perfect substitute for public bonds in terms of optimal policy.

The assumptions on fiscal policy and the costs of currency are relevant here in one respect. When the social costs of currency exchange are sufficiently high, a permanent purchase of bonds can be profitable also in Williamson (2012), but the mechanism is different from mine and seems less general[5].

In Williamson (2012), money and interest-bearing assets, generically modeled as bonds, have an essential role as medium of exchange. The former payment instrument is interest free, like cash or some forms of deposits, and can be used without restrictions. Bonds are partially liquid in the precise sense that not every agent can use them to buy consumption. When agents discover their payment technology, i.e. only money or money and bonds, an atomistic banking sector provides an essential mechanism to reallocate liquidity across depositors.

[5] On one hand, and because of the fiscal deficit, the central bank cannot increase inflation without rising permanently the proportion of money, which hurts the consumption of bond users. On the other hand, the central bank is willing to tax with higher inflation the undesirable consumption of some money users. Because of this trade-off, the equilibrium with plentiful bonds is no longer feasible and the optimal interest rate, below the rate of time preference, can be reached with a purchase of bonds.

2.1 The environment

Time is discrete and indexed by $t \in \mathbb{N}$. The economy is populated two types of infinitely-lived agents, namely seller and buyer. There is a $[0, 1]$ continuum of each type. In all periods a Walrasian (CM) and a frictional (DM) market open sequentially.

In the CM, agents can produce one unit of a non-storable good using one unit of labor, and they derive one unit of utility from its consumption. Define X_t the seller's demand and H_t the buyer's supply on the market for the CM good.

The DM is characterized by agents' anonymity and random meetings. It is assumed a measure one of meetings between a seller and a buyer, with the latter making a take-it-or-leave-it offer to the former. The seller can produce h_t units of a non-storable good with a one-to-one labor-input technology. The buyer can only demand x_t units of the same good for consumption. The buyer's utility function $u(x)$ fulfills $u'(0) = \infty$, $u''(x) < 0 < u'(x)$, $u(x) = x$ for some $x > 0$, and $\varepsilon = -x\frac{u''(x)}{u'(x)} < 1$ for $x > 0$. Call x^*, solving $u'(x) = 1$, the efficient consumption.

The life-time utilities of sellers and buyers are, respectively,

$$E_0 \sum_{t=0}^{\infty} \beta^t (X_t - h_t),$$

$$E_0 \sum_{t=0}^{\infty} \beta^t [-H_t + u(x_t)],$$

where E_0 is the expectation in $t = 0$ and $\beta \in (0, 1)$ is the discount factor.

The technological restriction on buyers determines a clear efficient dynamics for the economy. Sellers should produce for buyers in the DM, and vice versa. The decentralized trade requires a medium of exchange because anonymity and randomness only allow the trade to be quid pro quo. Indeed, the seller will not accept buyer's promise of future production – in the next CM – because the buyer cannot be punished for reneging on his debt.

To this purpose, in the CM the government issues two outside assets, i.e. money and nominal bond, which correspond to different payment technologies. Money is a tangible, perfectly divisible, and not counterfeitable object. Bonds are accounting items in the balance-sheet of the government, which is endowed with a record-keeping technology. After one period the bond is redeemed with money, hence it is also risk free. The government supplies bonds through an open market operations (OMO), defined as a swap of one unit of money with one unit of bond. The nominal interest rate $q \geq 1$ is determined endogenously by the agents' demand.

The government budget identity in period t is

$$M_t + B_t = M_{t-1} + q_t B_{t-1} + T_t,$$

where M identifies the outstanding money after the OMO, B is the supply of bonds, and $T > 0$ (< 0) is the lump-sum transfer (tax). On the RHS, the private agents hold assets from $t-1$, i.e. money after bond redemption. The total amount of money is then adjusted through the transfer[6]. On the LHS, the total amount of money in period t is then allocated between outstanding money and bonds.

Then two instruments define the economic policy: the gross growth rate of total assets,

$$\mu_t = \frac{M_t + B_t}{M_{t-1} + B_{t-1}}, \tag{1}$$

and the proportion of money to total assets,

$$\delta_t = \frac{M_t}{M_t + B_t}. \tag{2}$$

To be sure, if $q > 1$ then a unit of bond has a higher purchasing power then a unit of interest-free money because the former guarantees more consumption to the seller in the next CM. Evidently, without further restrictions the only equilibrium interest rate is $q = 1$ because any $q > 1$ would generate an infinite demand for bonds driving the interest rate down[7].

Crucially, bonds are partially illiquid in the sense that they cannot be used in a fraction ρ of the DM meetings, called *nonmonitored*. This information is public. Here the seller does not have the technology to verify the real transfer of bonds from the buyer's account on his own and only accepts money. In the complementary fraction of meetings, labeled *monitored*, bonds are liquid because the seller can verify the electronic payment. The payment friction creates a well-shaped demand for bonds.

The private agents enter the CM with heterogeneous amount of assets. The linear production and utility functions eliminate any wealth effect from asset holding. As result, the type-specific distribution of assets is degenerate at the end of the CM.

On the one hand, sellers have no incentive to accumulate assets – and the monetary economy is viable – as long as

$$\frac{\phi_{t+1}}{\phi_t} \leq r_{t+1} \leq \frac{1}{\beta} \tag{3}$$

[6] In $t = 0$, $M_0 + B_0 = T_0$ with $T_0 > 0$.

[7] Here the government controls the supply of bonds. In principle, it could set the interest rate and supply as many bonds as the public demands.

holds. In (3), ϕ is the price of money in terms of the CM good such that $\frac{\phi_{t+1}}{\phi_t}$ represents the real return on money, $r_{t+1} \equiv \frac{\phi_{t+1}q_{t+1}}{\phi_t}$ defines the real return on bonds, and β^{-1} is the gross rate of time preference. Condition (3) ensures that sellers have no incentive to accumulate assets in the CM and establishes a no-arbitrage condition on asset returns. The seller's problem is solved in appendix A.1.1.

On the other hand, buyers produce the CM good in order to accumulate assets for the next DM. The buyer receives the private information on the payment shock after the OMO when, in general, he holds money and bonds. With probability ρ he is not going to use bonds and he would like to have more money; with probability $1-\rho$ he is going to use money and bonds, but he would prefer more bonds whenever $q > 1$. Hence, the payment friction also creates a Pareto-improving role for asset reallocation.

Financial intermediation is provided by banks, institutions endowed with record-keeping technology. A bank can be run by any individual, it forms in the CM before the arrival of the information on the payment shock, and dissolves in the CM of the next period. The market is subject to free entry.

The bank offers a deposit contract that maximizes the expected utility of the buyers, which are ex-ante identical. The buyer deposits the CM goods and the bank invests them in $m_t = \phi_t M_t$ units of money and $a_t = \phi_t B_t$ units of bonds, in real terms[8]. Each buyer holds a total amount of $m_t + a_t$ claims. Importantly, the depositor can meet the bank before the DM meeting and can withdraw money. By the law of large numbers, each bank will end up with a proportion ρ of nonmonitored depositors[9].

2.2 The new contract

The new contract allows the non-monitored depositor to save bonds. In particular, I *restrict* the set of feasible contracts by imposing that the claims the depositor is entitled of before DM transactions take place, is equal to the initial amount deposited, i.e. $m_t + a_t$. The restriction makes the arrangement more close to a debt contract than in Williamson (2012)[10].

[8]The environment determines an equivalent outcome if buyers deposit the nominal assets.

[9]Banks and bonds are *coessential.* Without bonds, reallocation is clearly not necessary. Without the financial sector, welfare is maximized with a zero supply of bonds under the sufficient condition that $u'u''' \leq (u'')^2$.

[10]The contract cannot involve the creation of liquidity within the bank as the latter is not entitled of full commitment or enforcement power on its clients. It is only assumed that in the CM "there is lack of recordkeeping, except for records of the ownership of claims to accounts with financial intermediaries and the government"(W12, p.2574).

Concretely, if the bank decides to allocate $m_{t,\ell}$ units of extra money to the non-monitored depositor, then the latter keeps $a_t - m_{t,\ell}$ claims on bonds. The total amount of money, i.e. $\rho m_{t,\ell}$, corresponds to a reduction of $\frac{\rho m_{t,\ell}}{1-\rho}$ units of money for each monitored depositor. At the same time, the bank have to increase his claims on bonds by the same amount. In what follows the time index for current period is suppressed and ± 1 have the intuitive meaning.

Label x_k, with $k = n, m$, the consumption quantity in the nonmonitored and monitored meeting, respectively. The new equilibrium deposit contract $(m,\, a,\, m_\ell,\, a')$ solves

$$\max_{m,a,m_\ell,a'} -m-a+\rho\left[u\left(x_n\right)+\beta r_{+1}\left(a-m_\ell\right)\right]+(1-\rho)\left[u\left(x_m\right)+\beta r_{+1}a'\right] \quad (4)$$

subject to the offers

$$x_n = \beta\frac{\phi_{+1}}{\phi}\left(m+m_\ell\right), \quad (5)$$

$$x_m = \beta\frac{\phi_{+1}}{\phi}\left(m-\frac{\rho m_\ell}{1-\rho}\right)+\beta r_{+1}\left(a+\frac{\rho m_\ell}{1-\rho}-a'\right), \quad (6)$$

and the constraints $a \geq 0$, $a' \geq 0$, and

$$a - m_\ell \geq 0, \quad (7)$$

$$m - \frac{\rho m_\ell}{1-\rho} \geq 0. \quad (8)$$

Expressions (7) and (8) are resource constraints and implicitly ensure that the buyers do not receive less than $m + a$. The constraint $a + \frac{\rho}{1-\rho}m_\ell - a' \geq 0$ is never binding because the monitored depositor has incentives to use at least a marginal quantity of bonds in transaction. $m > 0$ can be derived by $\lim_{x\to 0} u'(x) \to \infty$ for the nonmonitored depositor. Lastly, the monetary savings in both meetings are normalized to zero[11]. The bank's problem is derived in appendix A.1.2

2.3 Interpretation of the contract

Before looking at the stationary equilibria, I discuss the main implication of the debt contract. Let me anticipate that in steady state the value of total

[11] Once the depositors have withdrawn money, they have the incentive to save it if and only if $\mu = \beta$, that is when $x_n = x_m = x^*$. The bank has the incentive to save bonds if and only if $r = \beta^{-1}$, or $x_m = x^*$. In this case the monitored depositor can eventually save money too. In all cases, the actual amount of savings is irrelevant for welfare.

assets is constant across periods, i.e. $\phi(M+B)=\phi_{-1}(M_{-1}+B_{-1})$, such that the growth rate of total assets pins down the *monetary* inflation rate from (1):

$$\frac{\phi}{\phi_{+1}}=\mu. \tag{9}$$

In steady state also $r_{+1}=r$.

Now consider the general case with bonds used in transaction. The first order conditions of problem (4) for m and a are, respectively,

$$m:\rho\frac{\beta}{\mu}u_n'+(1-\rho)\left(\frac{\beta}{\mu}u_m'+\lambda_m\right)=1, \tag{10}$$

$$a:\rho\left(\beta r+\lambda_n\right)+(1-\rho)\,\beta r u_m'=1. \tag{11}$$

I have shortened $u(x_k)$ to u_k. In (10), the LHS hosts the marginal benefit of money. Money provides payment services in both meetings. For the *monitored* depositors, money can have a higher marginal value as means of payment for bonds whenever constraint (8) binds. λ_m is the associated Kuhn-Tucker multiplier. In (11), the *nonmonitored* depositors must use bonds as store of value and, potentially, also as means of payment for money. The second role increases the marginal value of bonds whenever (7) is binding, with λ_n the associated multiplier. Clearly, bonds buy consumption in the monitored meetings.
On both RHSs, the marginal cost of investing in money and bonds in the CM is 1. Therefore the marginal returns of assets across meetings must be the same:

$$\rho\frac{\beta}{\mu}u_n'+(1-\rho)\left(\frac{\beta}{\mu}u_m'+\lambda_m\right)=\rho\left(\beta r+\lambda_n\right)+(1-\rho)\,\beta r u_m'. \tag{12}$$

At the same time, the FOC with respect to m_ℓ, after some trivial algebra, reads

$$\frac{\beta}{\mu}u_n'+\beta r u_m'=\beta r+\lambda_n+\frac{\beta}{\mu}u_m'+\lambda_m, \tag{13}$$

The interpretation of condition (13) is crucial. It simply states that the marginl benefit of the liquidity exchange in terms of consumption, on the LHS, must equate the marginal cost. The RHS hosts the opportunity costs of bond and money for the nonmonitored and monitored depositor, respectively. Not surprisingly, a trade is efficient when benefit and cost equate at the margin.

The combination of (12) and (13) produces

$$\beta r u_m'=\frac{\beta}{\mu}u_m'+\lambda_m \tag{14}$$

and – if and only if –

$$\frac{\beta}{\mu}u_n' = \beta r + \lambda_n. \tag{15}$$

For the monitored depositor, in (14), the bank compares (and equates) the marginal value of bonds with its cost, that is the marginal value of money. For the non-monitored depositor, in (15), the bank considers the marginal benefit of money and its cost, the cost being anything else other than the marginal return from bond saving.

Here is the implication.

The next section describes the equilibria, their associated welfare, and determine the optimal policy.

3 Equilibria and welfare

In this section I present the equilibria and rank them according to the measure of social welfare:

$$\mathcal{W} = \rho\left[u\left(x_n\right) - x_n\right] + (1-\rho)\left[u\left(x_m\right) - x_m\right]. \tag{16}$$

This measure corresponds to the trade surplus in the DM because the surpluses of buyers and sellers in the CM offset each others. Moreover, (16) entirely refers to the buyers' surplus as they have all the bargaining power.

For what follows, define from (2) the relative supply of bonds issued by the government in terms of the numeraire:

$$a = m\left(\frac{1}{\delta} - 1\right). \tag{17}$$

Definition 1. *Given the policy parameters μ and δ, a stationary monetary equilibrium solves* (4) *subject to* (5)-(8)*, $a \geq 0$, and $a' \geq 0$. It also satisfies* (3)*,* (9)*, and* (17)*. The equilibrium characterizes x_n and x_m.*

I select monetary equilibria, i.e. with $\phi > 0$. Clearly, the equilibrium also defines a value for the endogenous interest rate. The model produces five types of equilibria.

3.1 Equilibria

Friedman rule

When the return on money satisfies $\frac{1}{\mu} = \frac{1}{\beta}$, consumption is efficient in both meetings, i.e. $x_k = x^*$, and bonds are not essential, i.e. $\delta \in (0, 1]$.

With no other frictions, the equilibria where $\frac{1}{\mu} < \frac{1}{\beta}$ – at least the trade with currency is not efficient – are dominated by the Friedman rule equilibrium and are not interesting. In that respect, W12 assumes that some monetary trades are a wasteful of resources from the society's point of view[12]. This assumptions makes $x_n = x^*$ and $\mu = \beta$ no longer optimal. The advantage of this approach is that it does not affect the individual allocations but only the welfare hierarchy of the equilibria. This consequence holds here, without adding anything. As result, only the following equilibria with $\mu > \beta$ will be considered for the welfare analysis.

Liquidity trap

Exactly as in W12, if the supply of bonds is relatively scarce, that is

$$\{\mu > \beta, \delta \geq \rho\},$$

then bonds are not essential, i.e. $q = r\mu = 1$, and allocations are symmetric and solve

$$\frac{\beta}{\mu} u'(x) = 1. \tag{18}$$

On the RHS, the marginal cost of money (or bonds) in real terms. On the LHS, the marginal benefit in terms of consumption in the DM. Call $\bar{x} < x^*$ the quantity that solves (18) when $\mu > \beta$[13]. The social welfare is simply $u(\bar{x}) - \bar{x}$.

The banks do not require any compensation for holding the scarce supply of bonds, i.e. $r = \frac{1}{\mu} < \frac{1}{\beta}$. Because of the scarcity of bonds, (i) the bank can transform the entire portfolio of the nonmonitored depositor in claims on money and (ii) the liquidity constraint of the buyer is binding, i.e. $m_\ell = a$ in (7). Thus, using the definition of a from (17), its offer (5) reads

$$x_n = \frac{\beta}{\mu}(m + m_\ell) = \frac{\beta}{\mu}(m + a) = \frac{\beta}{\mu}\frac{m}{\delta}.$$

At the same time, the bank allocates the supply of bonds to the monitored depositors. Since money is abundant, the buyers may maintain also currency, i.e. (8) is not binding. Because the trade is not efficient, savings are zero

[12] To be precise, the welfare in (16) becomes $\rho\left[(1-v)u(x_n) - x_n\right] + (1-\rho)\left[u(x_m) - x_m\right]$, where a portion v of monetary trades requires seller's production with no socially valuable utility.

[13] Expression (18) holds in the Friedman rule equilibrium with $\mu = \beta$.

($a' = 0$) and the offer reads

$$x_m = \frac{\beta}{\mu}\left(m - \frac{\rho m_\ell}{1-\rho}\right) + \frac{\beta}{\mu}\left(a + \frac{\rho m_\ell}{1-\rho}\right) = \frac{\beta}{\mu}\frac{m}{\delta}.$$

In this equilibrium, money does not provide liquidity services to the monitored depositor, $\lambda_m = 0$ in (14), while bonds provide liquidity services to the nonmonitored depositor, and $\lambda_n > 0$ in (15).

Plentiful bonds

This equilibrium emerges when the supply of bonds is very abundant. The parameter configuration is

$$\{\mu > \beta, \delta \leq \delta_I\},$$

where $\delta_I < \rho$. Bonds pay the highest admitted rate, i.e. $r = \frac{1}{\beta}$ or equivalently $q = \frac{\mu}{\beta}$. Consumption in the nonmonitored meeting solves (18), while the efficient return on bonds allows the monitored depositors to maximize trade surplus, i.e. $x_n = \bar{x} < x_m = x^*$[14]. Hence, the welfare is higher than in the Liquidity trap.

Here the supply of bonds is so abundant that banks require the highest compensation in order to hold it. Money is very scarce such that its supply can be transferred entirely to the nonmonitored depositors while they still holding some bonds, i.e. $m + m_\ell = \frac{m}{\rho}$ and $a > m_\ell$. Symmetrically, the portfolio of monitored depositors is entirely in bonds. The offers read

$$x_n = \frac{\beta}{\mu}\frac{m}{\rho} < x_m = \frac{m}{\delta} - a' = x^*.$$

As the Liquidity trap, also the Plentiful bonds equilibrium exists in W12, but with one difference: the threshold in the original contribution, call it δ_P, is higher than mine. This means that for a given supply $\delta \in (\delta_I, \delta_P]$, W12's banks require $r = \frac{1}{\beta}$ as compensation for partial liquidity, while banks here demand bonds at $r < \frac{1}{\beta}$ – as we will see in the next equilibrium. In other words, given the interest rate, the new arrangement increases the demand for bonds.

[14] Allowing for private liquidity within the bank, this allocation would emerge for any $\delta\,(0,1]$. Indeed, assume bank's full commitment. The banker can create efficient IOUs – that pay $r = \beta^{-1}$ – in electronic form and backed by its production, eventually redeemed to the monitored seller in the next CM. Or assume bank's enforcement power. The monitored depositors could simply make credible promises to the sellers. In both cases, $x_m = x^*$ irrespective of the quantity of government bonds, and money could be entirely allocated to the nonmonitored depositors. Interestingly, this allocation is not the first-best equilibrium away from the Friedman rule

Adequate bonds

The Adequate bonds equilibrium is determined by

$$\{\mu > \beta, \delta_I < \delta \leq \rho\}.$$

The real interest rate $\rho \in \left[\hat{r}, \frac{1}{\beta}\right)$, with $\hat{r} > \frac{1}{\mu}$, depends negatively on δ. Consumptions in both meetings are not efficient and fulfill $\bar{x} < x_n < x_m < x^*$. Evidently, the welfare is higher than in the Liquidity trap, but it is not obvious the comparison with the Plentiful bonds. For sure, this equilibrium reduces consumption inequality.

Precisely, the liquidity constraint of the nonmonitored depositor, (7), is not binding such that (15) determines x_n,

$$\frac{\beta}{\mu} u'(x_n) = \beta r < 1. \tag{19}$$

In (19), the marginal utility of money equates the opportunity cost of money, i.e. the interest rate on bonds. In other words, money and bonds must have the same marginal return. The inequality puts clear that with $r < \frac{1}{\beta}$, this equilibrium has higher monetary consumption than the Liquidity trap and the Plentiful bonds equilibrium. Condition (11) determines x_m,

$$u'(x_m) = \frac{1 - \rho\beta r}{(1-\rho)\beta r}. \tag{20}$$

Still, the amount of money is relatively scarce and (8) binds, i.e. $m = \frac{\rho m_\ell}{1-\rho}$. Hence, with $a' = 0$ the offers are

$$\bar{x} < x_n = \frac{\beta}{\mu}\frac{m}{\rho} < x_m = \beta r \frac{m}{\delta} < x^*. \tag{21}$$

Importantly, the real interest rate and the value of money are *negatively* correlated. Indeed, the value of money supply, in the hands of nonmonitored depositors, is determined in (19):

$$u'\left(\frac{\beta m}{\mu\rho}\right) = \mu r. \tag{22}$$

This equation is the main difference between W12's and my contract. Here as in W12, money delivers higher marginal consumption in the nonmonitored than monitored meetings. It is therefore efficient to allocate the supply of

money to the nonmonitored meetings. Equivalently, money provides liquidity services for the monitored depositors, i.e. $\lambda_m > 0$ in (10).
Moreover, in W12 assets are *allocated* between depositors. The marginal cost of this operation simply reflects the cost of money holding, i.e the inflation rate. As consequence, the value of money in W12 solves (18) in any equilibrium, irrespective of the interest rate.
In contrast, here assets are *exchanged* because of the wealth constraint for each depositor. In evaluating the reallocation of liquidity, the bank must consider the trade-off between consumption and saving of the nonmonitored depositors when changing its portfolio composition. Globally, a lower marginal cost of providing additional money (a lower interest rate) decreases the marginal utility of money (increases monetary consumption). Hence the value of money supply solves (22) and, in this equilibrium, increases when the interest rate falls.

It must be noted that the contract produces multiple equilibria for $\delta = \rho$, in that both the Adequate bonds with $r = \hat{r}$ and the Liquidity trap with $r = \frac{1}{\mu}$ can emerge. The only difference between the two cases is the binding constraint. If $\lambda_m > \lambda_n = 0$, then the former; if $\lambda_n > \lambda_m = 0$, so the latter. Indeed, the model allows for a continuum of multiple equilibria with both multipliers greater than zero, a situation presented in the next and last case. To be clear from now, multiplicity of equilibria is a standard feature of asset pricing model in the presence of liquidity constraints. Basak et al. (2008) explore the impact of one portfolio constraint on the general financial equilibrium and find that it may generate additional and inefficient equilibria other than the Pareto efficient.

Sufficient bonds

When the relative supplies of money and bonds reflect the measures of nonmonitored and monitored trades, respectively, such that

$$\{\mu > \beta, \delta = \rho\},$$

then the model gives rise to a multiplicity of equilibria. The ones with $r \in \left\{\frac{1}{\mu}, \hat{r}\right\}$ have already been analysed. In this case the equilibrium interest rate ranges in $\left(\frac{1}{\mu}, \hat{r}\right)$ and each r determines a unique consumption profile.
Those equilibria may emerge with $\lambda_m > 0$ in (14) – as in the Plentiful and Adequate bonds – and $\lambda_n > 0$ in (15) – as in the Liquidity trap. The offers can be derived by (21) with $\delta = \rho$. Using (14) in (10) (or (15) in (11)), the

allocations solve

$$\rho\frac{\beta}{\mu}u'\left(\frac{\beta m}{\mu\rho}\right)+(1-\rho)\beta r u'\left(\beta r\frac{m}{\rho}\right)=1,$$

subject to $1 < u'_m < u'_n < \frac{\mu}{\beta}$, and $u'_n > \mu r$.
In the appendix (A.2.2), Case 2.2.1, I show that the interest rate and the value of money are jointly determined and *positively* correlated, such that consumptions increase with the interest rate. Moreover, for $r \to \hat{r}$ the allocations converge to those in the Adequate bonds, hence the equilibria in the Sufficient bonds case are strictly dominated.
The case $\delta = \rho$, and $q = [\mu\hat{r}, 1]$, is interesting because the interest rate (q) and the value of money (ϕ) satisfy different equilibria given the same fundamentals. The multiplicity is an issue when $\hat{r}$ maximizes welfare, as it can happen.

The next part identifies the optimal policy.

3.2 Welfare

The analysis of the stationary equilibria has identified two parameter spaces in which the authority can maximize the welfare, given $\mu > \beta$. The candidates are $\delta \leq \delta_I$ and $\delta_I < \delta \leq \rho$, that is the Plentiful bonds and the Adequate bonds equilibrium, respectively. Alternatively, the government chooses between the Friedman rule on bonds, i.e. $r = \frac{1}{\beta}$, and a moderate interest rate, i.e. $\frac{1}{\beta} < r \leq \hat{r}$. The other types of equilibria are dominated.

The analysis so far can be summarized in the following way. In general, the demand of bonds by banks increases with the interest rate as these assets are risky, in the sense that they are partially illiquid. When the government issues an abundant quantity, i.e. $\delta \leq \delta_I$, banks clear the supply by requiring $r = \frac{1}{\beta}$. The monitored depositor can consume the efficient quantity $x_m = x^*$, and the return on savings of the nonmonitored depositor is the highest. The trade-off between consumption and saving in the portfolio of the latter depositor keeps low the value of money, which can afford $x_n = \bar{x}$. Indeed, (18), which defines $\bar{x}$, is nothing more than (19) with $r = \frac{1}{\beta}$. The welfare of this policy is defined by

$$\mathcal{W}_{PB} = \rho\left[u(\bar{x}) - \bar{x}\right] + (1-\rho)\left[u(x^*) - x^*\right]. \tag{23}$$

When the supply of bonds is adequate, i.e. $\delta_I < \delta \leq \rho$, the market requires a lower interest rate to hold the illiquidity risk. Compared to the previous case, consumption in the monitored meeting, and saving in the nonmonitored one, are no longer efficient. To act as a counterbalance, monetary trades are favored because the value of money must increase – as the general-equilibrium outcome of liquidity reallocation. As result, consumptions inequality shrinks. Defining x_k^A the equilibrium quantities solving (19) and (20), the welfare in the Adequate bonds region is

$$\mathcal{W}_{AB} = \rho \left[u(x_n^A) - x_n^A \right] + (1-\rho) \left[u(x_m^A) - x_m^A \right]. \tag{24}$$

The first derivative of (24) with respect to $r \in (\beta^{-1}, \hat{r}]$ reads

$$\begin{aligned} \mathcal{W}'_{AB} &= \rho \left[\frac{\partial u(x_n^A)}{\partial x_n^A} - 1 \right] \frac{\partial x_n^A}{\partial r} + (1-\rho) \left[\frac{\partial u(x_m^A)}{\partial x_m^A} - 1 \right] \frac{\partial x_m^A}{\partial r} \\ &= \rho\mu \frac{u'(x_n^A) - 1}{u''(x_n^A)} - \frac{u'(x_m^A) - 1}{\beta r^2 u''(x_m^A)}. \end{aligned} \tag{25}$$

Since x_n^A and x_m^A converge to $\bar{x}$ and x^*, respectively, as $r \to \frac{1}{\beta}$, then

$$\lim_{r \to \frac{1}{\beta}} \mathcal{W}'_{AB} = \rho\mu \frac{u'(\bar{x}) - 1}{u''(\bar{x})} < 0.$$

This limit proves that welfare in the Adequate region decreases as the interest rate converges toward $\frac{1}{\beta}$. In turn, this is sufficient to prove that the Plentiful bonds is dominated by the Adequate bonds at least in a neighborhood of δ_I. In other words, reducing consumption inequality increases the social welfare. The next proposition details the optimal policy.

Proposition 1. *Call δ^* the policy that maximizes the welfare when $\mu > \beta$, and r^* the corresponding real interest rate. Thus $r^* \in \left(\frac{1}{\beta}, \hat{r} \right]$, with the optimal policy $\delta^* \in (\delta_I, \rho]$. Moreover, $\delta^* \neq \rho$ for μ sufficiently close to β and μ sufficiently high.*

Proof. In the appendix A.3. □

The proposition says that the authority must never set the supply of bonds so abundant to generate the highest return on bonds, because such a policy exacerbates consumption inequality between monetary and interest-bearing assets transactions. Social welfare is maximized in the Adequate bonds equilibrium. When the inflation rate is very low, or sufficiently high, the optimal policy satisfies $\delta \in (\delta_I, \rho)$.

For moderate inflation, it can be that welfare maximization requires $r = \hat{r}$, corresponding to set $\delta = \rho$. However, in this case the policy generates additional Pareto-inefficient equilibria that cover all the possible welfare configurations, from the highest with $r = \hat{r}$ to the lowest with $r = \frac{1}{\mu}$. To be sure, an allocation will also replicate the welfare in the Plentiful bonds equilibrium.

In this environment an easy solution to the multiplicity of equilibria is changing the policy instrument. The authority should set the interest rate in the OMO instead of the quantity, that would be endogenously determined by the private sector.

A brief summary and some final remarks are left for the conclusion.

4 Conclusion

Monetary policy is effective in the long run by changing the relative supply of liquidity aggregates. Williamson (2012) shows that open-market operations are non-neutral under flexible prices when interest-bearing assets are used as medium of exchange in some trades. In particular, an OMO that increases the relative supply of money is never optimal. The optimal policy is to retire money until the real interest rate equals the rate of time preference, the highest possible in a monetary economy.

In the same environment, but augmented with an efficient contract to insurance agents against asset illiquidity, I find that OMOs are optimal also in the other direction, namely the purchase of assets through injection of money – what is usually defined as expansionary monetary policy.

Away from the Friedman rule, the equilibrium with plentiful high-yield assets turns to be an *inequality* trap where real interest rate and consumption inequality are extreme, and OMOs are irrelevant at the margin (trap). A permanent injection of money, that helps the economy to escape from this trap, increases the social welfare.

In this article the standard expansionary OMO works through a non-standard channel. The money injection lowers the asset return and generates a real-balance effect: although the supply of money increases, the demand for money does by more. The scarcity of low-yield assets determines a higher demand by financially sophisticated agents, i.e. those that can use the assets in exchange. As money is necessary to buy additional assets from savers, the means-of-payment function of money adds to the real value of the money stock. With a lower interest rate, the purchasing power of money increases and consumption inequality shrinks.

The real-balance effect which holds at the general equilibrium level, is mirrored by the decentralized liquidity exchange at the bank level. In general, efficient re-allocation of liquidity implies the equivalence between the marginal values of assets in the portfolio of each individual. In terms of the model, the equivalence of marginal returns must hold *within* each meeting buyer/seller (ex-post) and not only *across* meetings (ex-ante).

The optimal combination of liquidity aggregates is influenced by the rate of price change. If inflation is sufficiently high or close to the Friedman rule, the relative amount of money must be lower than the fraction of monetary trades. For moderate values of inflation, the optimal interest rate can be associated with a relative supply of money equal to the fraction of monetary trades. This policy generates a compact set of multiple equilibria where the nominal interest rate and the price level are together indeterminate.

When the optimal policy generates the multiplicity of equilibria, those maps all the possible values of welfare. Since the nominal interest rate and the price level are jointly determined, the central bank should set the former in order to pin down one allocation. This multiplicity is a rationale for the practice of central banks to control the interest rate instead of the nominal aggregates in the New Monetarist approach.

The last comments are due to the role of financial intermediation. First, in this environment the reallocation of liquidity is essential in a very general sense: interest-bearing assets and banks are coessential. In other words, the society is better off with zero interest-bearing assets if liquidity risk-sharing is not possible. In this case the optimal policy by the central bank is to acquire the entire supply of assets in exchange for money in order to maximize welfare. Second, banks are endowed with recoordkeeping technology and cannot create liquidity. But, interestingly, allowing for either full commitment or enforcement power does not ameliorate welfare per se. With those assumptions the society can at best replicate the equilibrium with plentiful high-yield assets, but cannot reach the first best implied by the optimal contract, as the latter can only be obtained when there is a link between the purchasing power of money and the interest rate.

A Appendix

A.1 Individual choices

The variable $Y^i_{k,t}$ is chosen in the CM and $Y^{i,\prime}_{k,t}$ in the DM. $i = b, s$ refers to buyer or seller type, $k = m, n$ refers to monitored and non-monitored type, and t is the time index. When k refers to the previous period it becomes k'. I drop the time index for current periods using ± 1 with obvious meaning. Call $W(\cdot,\cdot)$ the value function in the CM and $V(\cdot,\cdot) \equiv \rho V_n(\cdot,\cdot) + (1-\rho)V_m(\cdot,\cdot)$ the expected value function from the DM meetings, with money and bonds holding as arguments. I start by solving the seller's problem.

A.1.1 Seller

I maintain the superscript $i = b$ and drop $i = s$ to alleviate the notation. The seller's problem in the CM reads

$$W(M'_{k',-1}, B'_{k',-1}) = \max_{X_{k'},M_{k'},B_{k'}} X_{k'} + V(M_{k'}, B_{k'})$$

with the individual budget constraint

$$X_{k'} + \phi(M_{k'} + B_{k'}) = \phi(M'_{k',-1} + qB'_{k',-1} + T).$$

Using the latter, the problem becomes

$$\begin{aligned} W(M'_{k',-1}, B'_{k',-1}) &= \phi(M'_{k',-1} + qB'_{k',-1} + T) + \max_{M_{k'},B_{k'}} [-\phi(M_{k'} + B_{k'}) + V(M_{k'}, B_{k'})] \\ &= \phi(M'_{k',-1} + qB'_{k',-1} + T) + W(0,0). \qquad \text{(A.1)} \end{aligned}$$

The linear preference in the CM removes the wealth effect of assets and the agent's trading history. Therefore, $M \equiv M_{k'}$ and $B \equiv B_{k'}$. The choice of initial holdings reduces to

$$\max_{M,B} -\phi(M + B) + V(M, B). \qquad \text{(A.2)}$$

In the CM the seller receives the shock and in the DM he decides whether to accept or reject the buyer's offer, and also decides about savings for the next CM. Seller's problem in the DM is

$$V_k(M, B) = \max_{h_k,M'_k,B'_k} -h_k + \beta W_{+1}(M'_k, B'_k) \qquad \text{(A.3)}$$

$$\text{s.t. } \phi(M'_k + q_{+1}B'_k) = \varphi_k h_k + \phi(M + q_{+1}B), \qquad \text{(A.4)}$$

in which φ_k is the price (in terms of the numeraire) of the DM good in the k-meeting. In case of rejection $h_k = 0$ and $V_k(M,B) = \beta W_{+1}(M,B)$. At this point we must consider the buyer's offer.
The buyer makes the offer by considering his budget constraint and the seller's incentive constraint, that is

$$\begin{aligned} V_k^b(M_k^b, B_k^b) = & \max_{x_k, M_k^b, B_k^b} u(x_k) + \beta W_{+1}^b(M_k^{b,\prime}, B_k^{b,\prime}) \\ & \text{s.t. } \varphi_k x_k + \phi(M_k^{b,\prime} + q_{+1}B_k^{b,\prime}) = \phi(M_k^b + q_{+1}B_k^b), \\ & \quad - h_k + \beta W_{+1}(M_k', B_k') \geq \beta W_{+1}(M,B). \end{aligned} \tag{A.5}$$

The TIOLI offer implies that (A.5) holds as equality, that is

$$h_k = \beta[W_{+1}(M_k', B_k') - W_{+1}(M,B)]. \tag{A.6}$$

Using (A.1) one period ahead in (A.6), and noting that the seller's asset holding also depends on the buyer's offer, i.e. $M_k' = M + M_k^b - M_k^{b,\prime}$ and $B_k' = B + B_k^b - B_k^{b,\prime}$, we can write

$$h_k = \beta\phi_{+1}[M_k^b - M_k^{b,\prime} + q_{+1}(B_k^b - B_k^{b,\prime})]. \tag{A.7}$$

From A.7 and (A.4), the price of the DM consumption in both meetings is $\varphi_k = \frac{\phi}{\beta\phi_{+1}}$. We can substitute A.6 in problem (A.3) and get

$$V_k(M,B) = \beta W_{+1}(M,B) = \beta\phi_{+1}(M + q_{+1}B + T_{+1}) + \beta W_{+1}(0,0). \tag{A.8}$$

Now the DM value function is independent by the type of meeting. Using (A.8), the initial problem (A.2) reduces to

$$\max_{M,B} \ \{(-\phi + \beta\phi_{+1})M + (-\phi + \beta\phi_{+1}q_{+1})B\} + \beta\left[\phi_{+1}T_{+1} + W_{+1}(0,0)\right].$$

The seller's problem has finite solutions if $-\phi + \beta\phi_{+1} \leq 0$ and $-\phi + \beta\phi_{+1}q_{+1} \leq 0$, that is condition (3). When sellers do not value assets more than consumption, then buyers can be accumulate liquidity and the monetary economy is viable.

When the subjective return on money (bond) is zero, we take $M = 0$ ($B = 0$) among the solutions $M \geq 0$ ($B \geq 0$). This is without loss of generality because savings of the asset with efficient return do not affect real quantities. Thus sellers' asset holding can be normalized to zero without loss of generality. Moreover, given the contemporaneity of choices implied by the Walrsian auctioneer, assuming $T = 0$ for the seller is without consequence for the analysis of social welfare, as T only affects buyers' welfare in the CM by the same amount.

At the end of the game the economic problem is that of buyers, which holds the entire stock of assets at the end of the CM.

A.1.2 Buyer/bank

The buyer's problem is partially solved by the bank. In the DM, the buyer maximizes his utility given the assets provided by the bank. In the CM, the bank decides the asset holdings given the exogenous measure ρ and the optimal DM choices of the buyers. Because of the continuous measure of banks and buyers, it is possible to reduce notation and useless complexity. We can think of the bank's problem in the CM as a buyer's problem facing uncertainty ρ, given the optimal DM choices provided by the bank's reallocation. From a mathematical point of view, the problems coincide.

I drop the superscript $i = b$. $m \equiv \phi M$ and $a = \phi B$ define the real value of nominal holdings. The buyer's problem in the CM reads

$$W(m'_{k',-1}, a'_{k',-1}) = \max_{H_{k'}, m_{k'}, a_{k'}} -H_{k'} + V(m_{k'}, a_{k'})$$

with the individual budget constraint

$$-H_{k'} + m_{k'} + a_{k'} = \frac{\phi}{\phi_{-1}}(m'_{k',-1} + qa'_{k',-1}) + \tau,$$

where τ is the real value of transfer (> 0) or tax (< 0). Using the latter expression, the CM problem becomes

$$\begin{aligned} W(m'_{k',-1}, a'_{k',-1}) &= \frac{\phi}{\phi_{-1}}(m'_{k',-1} + qa'_{k',-1}) + \tau + \max_{m_{k'}, a_{k'}} [-m_{k'} - a_{k'} + V(m_{k'}, a_{k'})] \\ &= \frac{\phi}{\phi_{-1}}(m'_{k',-1} + qa'_{k',-1}) + \tau + W(0,0). \qquad \text{(A.9)} \end{aligned}$$

The linear preference in the CM removes the assets' wealth effect and the agent's trading history. Therefore current choices do not depend on past history, i.e. $m \equiv m_{k'}$ and $a \equiv a_{k'}$. The choice of initial holdings reduces to

$$\max_{m,a} -m - a + V(m, a). \qquad \text{(A.10)}$$

At the end of the CM the buyer deposit $m + a$, then he receives the shock and the bank allocates funds. The new amount of assets id $m_k + a_k$. The contribution of the paper is to constrain the contract with $m_k + a_k = m + a$. After reallocation, in the meeting the buyers can decide savings m'_k and a'_k. The payment friction implies $a'_n = a_n$.

The value function in the specific meeting is, using (A.9) one period ahead,

$$\begin{aligned} V_k(m_k, a_k) &= u(x_k) + \beta W_{+1}(m'_k, a'_k) \qquad \text{(A.11)} \\ &= u(x_k) + \beta \left(\frac{\phi_{+1}}{\phi} m'_k + r_{+1} a'_k + \tau \right) + \beta W_{+1}(0,0). \qquad \text{(A.12)} \end{aligned}$$

At the end of the CM, the buyer receives the shock, meets th bank, then enters the meeting with the seller and makes the offer. The offers in (A.7), without the superscript b, are in the nonmonitored and monitored meeting, respectively:

$$x_n = \beta \frac{\phi_{+1}}{\phi}(m_n - m'_n), \tag{A.13}$$

$$x_m = \beta \frac{\phi_{+1}}{\phi}(m_m - m'_m) + \beta r_{+1}(a_m - a'_m). \tag{A.14}$$

In other words, the buyer decides about savings. Following W12, I show that it is possible to generalize $m'_k = 0$.
The non-monitored buyer solves

$$\max_{m'_n} u(x_n) + \beta \left(\frac{\phi_{+1}}{\phi} m'_n + r_{+1} a_n \right) \tag{A.15}$$

subject to (A.13) and $m'_n \geq 0$. The FOC is

$$-\beta \frac{\phi_{+1}}{\phi} u'(x_n) + \beta \frac{\phi_{+1}}{\phi} + \theta_1 = 0, \tag{A.16}$$

with θ_1 the Kuhn-Tucker multiplier for $m'_n \geq 0$. Because $u'(0) = \infty$, then $m_n - m'_n > 0$ and also $m > 0$. Hence either $u'(x_n) = 1 \Leftrightarrow \theta_1 = 0$, with $x_n = x^*$ and $m'_n \geq 0$, or $u'(x_n) > 1 \Leftrightarrow \theta_1 > 0$, with $x_n < x^*$ and $m'_n = 0$.
The monitored buyer solves

$$\max_{m'_m, a'_m} u(x_m) + \beta \left(\frac{\phi_{+1}}{\phi} m'_m + \beta r_{+1} a'_m \right) \tag{A.17}$$

subject to (A.14), $0 \leq m'_m \leq m_m$, and $0 \leq a'_m \leq a_m$. The FOC for monetary saving is

$$-\beta \frac{\phi_{+1}}{\phi} u'(x_m) + \beta \frac{\phi_{+1}}{\phi} + \theta_2 - \theta_3 = 0, \tag{A.18}$$

with $\theta_2 m'_m = 0$ and $\theta_3(m_m - m'_n) = 0$. In this case: either $u'(x_m) > 1 \Leftrightarrow \theta_2 > \theta_3 \geq 0 \Rightarrow m'_m = 0$, or $u'(x_m) = 1 \Leftrightarrow \theta_2 = \theta_3$ and either $\theta_2 = \theta_3 = 0 \Rightarrow m'_m \geq 0$ or $\theta_2 = \theta_3 > 0 \Rightarrow m'_m = 0$.

We are close to simplify the problem as in (4). After having proved $m > 0$ and generalized $m'_k = 0$, we only have to define m_k and a_k in terms of the initial holdings $m + a$ and the bank's reallocation. Concretely, if the bank decides to allocate m_ℓ units of extra money to the non-monitored depositor, then the latter keeps $a - m_\ell$ claims on bonds. The total amount of reallocated money, i.e. ρm_ℓ, corresponds to a reduction of $\frac{\rho m_\ell}{1-\rho}$ units of money for each monitored depositor. At the same time, the bank have to increase his claims on bonds by the same amount, i.e. $a_n = a + \frac{\rho m_\ell}{1-\rho}$. Hence, problem (4).

A.2 Equilibria

We solve the system for steady-state monetary equilibria, where $\phi > 0$. In steady state the value of assets is constant across periods, i.e. $\phi(M + B) = \phi_{-1}(M_{-1} + B_{-1})$ and, from (1), the growth rate of total assets determines the inflation rate:

$$\mu = \frac{\phi_{+1}}{\phi}.$$

The deposit contract in steady state (m, a, m_ℓ, a') solves

$$\max_{m,a,m_\ell,a'} -m - a + \rho\left[u\left(x_n\right) + \beta r\left(a - m_\ell\right)\right] + (1-\rho)\left[u\left(x_m\right) + \beta r a'\right] \quad \text{(A.19)}$$

subject to the offers

$$x_n = \frac{\beta}{\mu}\left(m + m_\ell\right), \quad \text{(A.20)}$$

$$x_m = \frac{\beta}{\mu}\left(m - \frac{\rho}{1-\rho}m_\ell\right) + \beta r\left(a + \frac{\rho}{1-\rho}m_\ell - a'\right), \quad \text{(A.21)}$$

and the Kuhn-Tucker constraints

$$\theta a = 0, \quad \text{(A.22)}$$

$$\lambda_1\left(a - m_\ell\right) = 0, \quad \text{(A.23)}$$

$$\lambda_2\left(m - \frac{\rho}{1-\rho}m_\ell\right) = 0, \quad \text{(A.24)}$$

$$\lambda_3\left(a + \frac{\rho}{1-\rho}m_\ell - a'\right) = 0, \quad \text{(A.25)}$$

$$\lambda_4 a' = 0. \quad \text{(A.26)}$$

Expression (A.22) applies at bank's level, (A.23) at the nonmonitored depositor's level, the remainders are for the monitored buyer. (A.23) and (A.24) are resource constraints and implicitly ensure that the buyers do not receive less than $m + a$[15].

[15]Both resource constraints are important. In general, call Q a mass $q > 0$ of agents endowed with $x \geq 0$ and $y \geq 0$ goods each; call P a mass $p > 0$ endowed with the same amounts; consider the social planner's problem to transfer $\Delta x \geq 0$ to each agent in Q by reducing his amount of y by $\Delta y \geq 0$ under the constraint that $\Delta x = \Delta y$, i.e. no individual resource reduction occurs or the relative price is 1. To this end, each P's agent must give up $-\frac{q}{p}\Delta x$ of goods x in exchange for $\frac{q}{p}\Delta x$ of goods y. The resource constraints require $x - \frac{q}{p}\Delta x \geq 0$ and $y - \Delta x \geq 0$, and are not equivalent. Indeed, for instance, $y - \Delta x < 0$ and $x - \frac{q}{p}\Delta x \geq 0$ does not violate neither $x \geq 0$ nor $y \geq 0$, but only imply $x \geq \frac{q}{p}\Delta x > \frac{q}{p}y \geq 0$.

The FOCs at lenght are

$$m : -1 + \rho\beta\mu^{-1}u'_n + (1-\rho)\left(\beta\mu^{-1}u'_m + \lambda_2\right) = 0, \quad (A.27)$$

$$a : -1 + \theta + \rho\left(\beta r + \lambda_1\right) + (1-\rho)\left(\beta r u'_m + \lambda_3\right) = 0, \quad (A.28)$$

$$m' : \rho\left(\beta\mu^{-1}u'_n - \beta r - \lambda_1\right) + \frac{(1-\rho)\,\rho}{1-\rho}\left[\beta\left(r - \mu^{-1}\right)u'_m - \lambda_2 + \lambda_3\right] = 0, \quad (A.29)$$

$$a' : -\beta r u'(x_m) + \beta r - \lambda_3 + \lambda_4 = 0. \quad (A.30)$$

The first set of equilibria involves the cases with no bonds, i.e. $\delta = 1$. The equilibria satisfy $\theta \geq 0$. The second set concerns equilibria with a positive supply of bonds, i.e. $\delta < 1$ and $\theta = 0$.

In general, consider that

$$\delta = \frac{\beta r m}{x_m - x_n - \beta\left(\rho r - \mu^{-1}\right)\frac{m_\ell}{1-\rho} + \beta r a' + \beta r m} \quad (A.31)$$

from (A.20), (A.21), and (17).

A.2.1 $\delta = 1$.

With only money in the economy, consumption is identical in both meetings and it is determined by the inflation process. If the return on money is efficient, i.e. $\mu = \beta$, then $x_k = x^*$ otherwise $x_k < x^*$.

If $\delta = 1$, then $a = a' = 0$. Absent bonds, it must be $m_\ell = 0$ and $x_k = \frac{\beta}{\mu}m$. Also $\lambda_2 = 0$. The FOCs (A.28)-(A.30) became

$$a : -1 + \theta + \rho\left(\beta r + \lambda_1\right) + (1-\rho)\left(\mu r + \lambda_3\right) = 0,$$
$$m_\ell : r\left(\mu - \beta\right) = \lambda_1 - \lambda_3,$$
$$a' : r\left(\mu - \beta\right) = \lambda_4 - \lambda_3.$$

Hence $\lambda_4 = \lambda_1$ from the last two equations. Using $\lambda_3 = \lambda_1 + r(\beta - \mu)$ from the third FOC into the second, the latter reads $1 - \beta r = \theta + \lambda_1$.

Consumption $x_k = \bar{x}$ satisfies (A.27):

$$\frac{\beta}{\mu}u'(\bar{x}) = 1.$$

Defining $v(y) \equiv u'^{-1}(y)$ the function inverting the slope of the utility in the correspondent consumption, then

$$\bar{x}(\mu) = v\left(\frac{\mu}{\beta}\right). \quad (A.32)$$

If $\mu = \beta$, then $x_k = v(1) = x^*$ with $\lambda_1 = \lambda_3 = \lambda_4 \geq 0$ and $r \leq \beta^{-1}$. If $\mu > \beta$, then $x_k = v\left(\beta^{-1}\mu\right) < x^*$ with $\lambda_1 = \lambda_4 \geq r(\mu - \beta) > 0$ and $r < \beta^{-1}$. Call $\bar{x} = v\left(\beta^{-1}\mu\right)$ when $\mu > \beta$.

Lastly, note that the interest rate is irrelevant for the allocations and $\theta \geq 0$ is always a solution.

A.2.2 $\delta < 1$.

Now we look at equilibria with bonds, i.e. $a > 0$ and $\theta = 0$. The simplified FOCs are:

$$m : -1 + \rho\left[\beta\mu^{-1}u'_n\right] + (1-\rho)\left[\beta\mu^{-1}u'_m + \lambda_2\right] = 0, \tag{A.33}$$

$$a : -1 + \rho\left[\beta r + \lambda_1\right] + (1-\rho)\left[\beta r u'_m + \lambda_3\right] = 0, \tag{A.34}$$

$$m_\ell : \left[\beta\mu^{-1}u'_n - (\beta r + \lambda_1)\right] + \left[(\beta r u'_m + \lambda_3) - \left(\beta\mu^{-1}u'_m + \lambda_2\right)\right] = 0, \tag{A.35}$$

$$a' : -\beta r u'_m + \beta r - \lambda_3 + \lambda_4 = 0. \tag{A.36}$$

First of all, I show that $\lambda_3 = 0$, then I proceed by cases.

Proof $\lambda_3 = 0$. Subtracting (A.34) from (A.33), we obtain

$$\rho\left[\beta\mu^{-1}u'_n - (\beta r + \lambda_1)\right] - (1-\rho)\left[(\beta r u'_m + \lambda_3) - \left(\beta\mu^{-1}u'_m + \lambda_2\right)\right] = 0.$$

A trivial comparison with (A.35) implies that the term in the second square bracket must be zero:

$$\beta\left(r - \mu^{-1}\right)u'_m - \lambda_2 + \lambda_3 = 0.$$

Of course, $\beta\mu^{-1}u'_n = \beta r + \lambda_1$ too. Assume $\lambda_3 > 0$: $\lambda_2 = 0$ means $r < \mu^{-1}$, but $r \geq \mu^{-1}$ when $a > 0$ from (3); $\lambda_2 > 0$ is impossible because implies $x_m = 0$ from (A.24), (A.25), and (A.21). Hence $\lambda_3 = 0$.

The system (A.33)-(A.36) becomes

$$-1 + \rho\beta\mu^{-1}u'_n + (1-\rho)\left(\beta\mu^{-1}u'_m + \lambda_2\right) = 0, \tag{A.37}$$

$$\beta\left(r - \mu^{-1}\right)u'_m - \lambda_2 = 0, \tag{A.38}$$

$$\beta\mu^{-1}u'_n - \beta r - \lambda_1 = 0, \tag{A.39}$$

$$-\beta r u'_m + \beta r + \lambda_4 = 0. \tag{A.40}$$

Case 1: $\lambda_4 = 0$, or $\mathbf{x_m} = \mathbf{x}^*$. If $\lambda_4 = 0$, then $u'_m = 1 \Leftrightarrow x_m = x^*$ from (A.40), and $a' \geq 0$ from (A.26). Using the expressions for λ_2 and βr from (A.38) and (A.39), respectively, in (A.37), we obtain

$$\beta\mu^{-1}u'_n = 1 + (1-\rho)\lambda_1, \tag{A.41}$$

which used in (A.39) returns

$$1 - \beta r = \rho\lambda_1. \tag{A.42}$$

Case 1.1: $\lambda_1 = 0$. We obtain $a \geq m_\ell$ from (A.23), $r = \beta^{-1}$ from (A.42), and $x_n = v(\beta^{-1}\mu)$ from (A.41). Moreover, (A.38) reads

$$\lambda_2 = 1 - \beta\mu^{-1} \geq 0.$$

Case 1.1.1: Friedman rule. If $\mu = \beta$, then $x_n = v(1) = x^*$ and $\lambda_2 = 0$, or $m - \frac{\rho}{1-\rho}m_\ell \geq 0$ from (A.24). With the Friedman rule on money both buyers consume the efficient quantity. No resource constraint is binding because the proportion of assets does not matter, i.e. $\delta < 1$. This case complements $\mu = \beta$ when $\delta = 1$.

Case 1.1.2: Illiquidity trap, or Plentiful bonds case. If $\mu > \beta$, then $x_n = \bar{x}(\mu) < x^*$ and $\lambda_2 > 0$, which implies $m_\ell = \frac{1-\rho}{\rho}m$ from (A.24). The offers (A.20)-(A.21) read

$$x_n = \frac{\beta}{\mu}\frac{m}{\rho} = \bar{x}(\mu) < x^*, \tag{A.43}$$

$$x_m = a + m - a' = \frac{m}{\delta} - a' = x^*. \tag{A.44}$$

On the one hand, the supply of money is transferred to the nonmonitored depositor, and in his meeting the value of money is determined. On the other hand, the monitored depositor has his claims entirely in bonds, and equal to m/δ from (17). Since $a' \geq 0$, then

$$\delta \leq \delta_I \equiv \frac{m}{x^*} = \rho\frac{\frac{\mu}{\beta}\bar{x}(\mu)}{x^*} < \rho. \tag{A.45}$$

The last inequality can be derived by $\frac{\mu}{\beta}\bar{x}(\mu) = \frac{\mu}{\beta}v(\frac{\mu}{\beta}) < v(1) = x^*$. Indeed, $\frac{\partial yv(y)}{\partial y} = v + yv'$ is equivalent to $x + u'(u'')^{-1}$, which is negative by assumption, i.e. $\varepsilon = -x\frac{u''}{u'} < 1$[16].

Case 1.2: $\lambda_1 > 0$, no equilibria. If $\lambda_1 > 0$, then $a = m_\ell$ from (A.23), $r < \beta^{-1}$ from (A.42), and

$$u'_n = \beta^{-1}\mu\left[1 + \frac{1-\rho}{\rho}(1 - \beta r)\right] > 1 \tag{A.46}$$

[16]TO SCRUTINIZE: If $\varepsilon = 1$, then $\delta_I = \rho$. As the coefficient of relative risk aversion increases, agents require higher compensation to hold bonds. With $\varepsilon = 1$, the demand for bonds of the agents is flat at $r = \frac{1}{\beta}$ for any supply $0 < \delta \leq \rho$. As ε is sufficiently high such that $\delta_I \approx 1$, agents demand bonds if only if $r = \frac{1}{\beta}$, such that the only equilibrium is the Plentiful case. With W12's contract this is good. Under the optimal contract this is not.

from (A.41), with

$$x_n = \frac{\beta}{\mu}(m+a) = \frac{\beta}{\mu}\frac{m}{\delta} < x^*. \tag{A.47}$$

Moreover, $a = m_\ell$ and (17) transform (A.24) in

$$\lambda_2 \left(m - \frac{\rho}{1-\rho} a \right) = \lambda_2 \left[1 - \frac{\rho(1-\delta)}{(1-\rho)\delta} \right] m = 0, \tag{A.48}$$

which implies $\delta \geq \rho$.

Case 1.2.1: $\lambda_2 = 0$. If $\lambda_2 = 0$, then $r = \mu^{-1} < \beta^{-1}$ from (A.38). Comparing (A.47) and (A.21), remembering that $r = \mu^{-1}$, we get

$$x_n = \frac{\beta}{\mu}\frac{m}{\delta} < x_m = \frac{\beta}{\mu}\left(\frac{m}{\delta} - a'\right) = x^*,$$

which is not possible.

Case 1.2.2: $\lambda_2 > 0$. If $\lambda_2 > 0$, then $\mu^{-1} < r < \beta^{-1}$ from (A.38) and $\delta = \rho$ from (A.48). The relation between (A.47) and (A.21) reads

$$x_n = \frac{\beta}{\mu}\frac{m}{\rho} < x_m = \beta r \left(\frac{m}{\rho} - a'\right) = x^*.$$

Looking at the real value of money, the allocations in the nonmonitored and monitored meetings require, respectively

$$m = \rho\frac{\mu}{\beta} v\left(\frac{\mu}{\beta\rho} - \frac{1-\rho}{\rho}\mu r\right) \geq \rho\frac{v(1)}{\beta r}.$$

Such an $r \in \left(\frac{1}{\mu}, \frac{1}{\beta}\right)$ does not exist if

$$\mu r v\left(\frac{\mu}{\beta\rho} - \frac{1-\rho}{\rho}\mu r\right) < v(1).$$

Indeed, it is possible to prove the above relations in two steps. First, consider the limit for $r \to \beta^{-1}$. The LHS becomes $\beta^{-1}\mu v(\beta^{-1}\mu)$, lower than $v(1)$ because $\varepsilon < 1$. Second, the LHS is increasing in r. The derivative reads $\mu v + \mu^2 r(1-\rho^{-1})v' = \mu\left[u' + \mu r(1-\rho^{-1})(u'')^{-1}\right] > 0$.

This concludes Case 1 under $\delta < 1$. To sum up, when $\lambda_4 = 0$ and $a' \geq 0$, the equilibria involve $x_m = x^*$ and $r = \beta^{-1}$. If $\mu = \beta$, then $x_n = x^*$

with $\lambda_1 = \lambda_2 = 0$. If $\mu > \beta$ and $\delta \leq \delta_I$, then $x_n = \bar{x}(\mu) < x^*$, with $\lambda_2 > \lambda_1 = 0$. In other words, the Friedman rule on money is compatible with any supply of bonds because they are irrelevant. Out of the Friedman rule on money, a sufficiently high supply of bonds can generate the highest interest rate, i.e. the Friedman rule on bonds. In this last case, the resource-of-money constraint of the monitored depositor binds, while the nonmonitored depositor save bonds.

Case 2: $\lambda_4 > 0$, or $x_m < x^*$. If $\lambda_4 > 0$, then $u'_m > 1 \Leftrightarrow x_m < x^*$ from (A.40), and $a' = 0$ from (A.26). The system includes (17), (A.20)-(A.21), (A.23)-(A.25), and (A.37)-(A.39).

Case 2.1: $\lambda_1 = 0$. The resource-of-bond constraint of the nonmonitored buyer is not bonding, i.e. $a \geq m_\ell$, from (A.23). Moreover, from (A.39), the marginal utility of nonmonitored consumption is determined by the nominal interest rate:

$$u'_n = \mu r. \tag{A.49}$$

This expression used in (A.37) defines

$$u'_m = \frac{1 - \rho\beta r}{(1-\rho)\beta r}, \tag{A.50}$$

which is bigger than 1 iff $r < \frac{1}{\beta}$.

Case 2.1.1: $\lambda_2 = 0$, no equilibria. With $\lambda_2 = 0$, $m \geq \frac{\rho m_\ell}{1-\rho}$ from (A.24), $r = \frac{1}{\mu}$ from (A.38), and $u'_n = 1 \Leftrightarrow x_n = x^*$ from (A.49). The offers give rise to the following inequality:

$$x_n = \frac{\beta}{\mu}(m + m_\ell) = x^* > x_m = \frac{\beta}{\mu}(m + a) \Leftrightarrow m_\ell > a.$$

The second inequality is infeasible.

Case 2.1.2: $\lambda_2 > 0$, Adequate bonds. With $\lambda_2 > 0$, $m_\ell = \frac{1-\rho}{\rho} m$ from (A.24), $r > \frac{1}{\mu}$ from (A.38), and $u'_n > 1 \Leftrightarrow x_n < x^*$ from (A.49). Given (17), then $a \geq m_\ell$ implies $\delta \leq \rho$. Hence

$$x_n = \frac{\beta}{\mu}\frac{m}{\rho} = v(\mu r) < x_m = \beta r \frac{m}{\delta} = v\left[\frac{1 - \rho\beta r}{(1-\rho)\beta r}\right] < x^*. \tag{A.51}$$

From the above expression, by equating the real value of money,

$$\frac{\delta}{\rho} = \frac{\mu r v(\mu r)}{v\left[\frac{1-\rho\beta r}{(1-\rho)\beta r}\right]} > \frac{\delta_I}{\rho} = \rho \frac{\mu\beta^{-1} v(\mu\beta^{-1})}{v(1)}. \tag{A.52}$$

The first equality determines the equilibrium interest rate given $\delta \leq \rho$ and $\mu > \beta$. The inequality narrow the domain of δ to $(\delta_I, \rho]$. The equilibrium exists for $r \geq \hat{r}$, where $\mu^{-1} < \hat{r} < \beta^{-1}$.

Let's study the fraction as function of $r \in (\mu^{-1}, \beta^{-1})$. The numerator decreases from $v(1)$ to $\mu\beta^{-1}v(\mu\beta^{-1})$, both not included, because $\varepsilon < 1$. The denominator increases up to $v(1)$, not included, because the derivative is $\frac{-v'}{(1-\rho)\beta r^2} > 0$. Hence, δ uniquely determines r and it must exist $\hat{r} \in (\mu^{-1}, \beta^{-1})$ such that (A.52) holds when $\delta = \rho$; if $\delta < \rho$, then $r > \hat{r}$; for $\delta \to \delta_I$, of course $r \to \frac{1}{\beta}$. To be exhaustive, the inequality in (A.52) is implied by $r < \beta^{-1}$ plus the numerator decreasing and the denominator increasing in $r \in (\mu^{-1}, \beta^{-1})$.

Case 2.2: $\lambda_1 > 0$. The resource-of-bond constraint of the nonmonitored buyer is bonding, i.e. $a = m_\ell$, from (A.23). (A.20) reads

$$x_n = \frac{\beta}{\mu}\frac{m}{\delta}, \tag{A.53}$$

with

$$u'_n > \mu r \Leftrightarrow x_n < v(\mu r) \tag{A.54}$$

from (A.39).

Case 2.2.1: $\lambda_2 > 0$, multiple equilibria. If $\lambda_2 > 0$, $r > \frac{1}{\mu}$ from (A.38) and $\delta = \rho$ from (A.24). The offers fulfill

$$x_n = \frac{\beta}{\mu}\frac{m}{\rho} < x_m = \beta r \frac{m}{\rho} < x^*,$$

which correspond to (A.51) and produces

$$1 = \frac{\mu r x_n}{x_m}, \tag{A.55}$$

which resembles (A.52), with $\delta = \rho$. Using the value of λ_2 from (A.38), and solving for u'_m, condition (A.37) reads:

$$u'_m = \frac{1 - \rho\beta\mu^{-1}u'_n}{(1-\rho)} < \frac{1-\rho\beta r}{(1-\rho)} \Leftrightarrow x_m > v\left[\frac{1-\rho\beta r}{(1-\rho)}\right]. \tag{A.56}$$

Now remember the definition of $\hat{r}$ as the interest rate that solves (A.52) when $\delta = \rho$, and $r > \hat{r}$ if $\delta < \rho$ – the fraction in (A.52) is decreasing with r. Then we can show that conditions (A.54)-(A.56) are not compatible with $r \geq \hat{r}$:

$$\frac{\mu \hat{r} v(\mu \hat{r})}{v\left[\frac{1-\rho\beta\hat{r}}{(1-\rho)\beta\hat{r}}\right]} = 1 = \frac{\mu r x_n}{x_m} < \frac{\mu r v(\mu r)}{v\left[\frac{1-\rho\beta r}{(1-\rho)\beta r}\right]}, \tag{A.57}$$

where the last inequality is impossible. Hence the domain of the interest rate can be narrowed to $r \in \left(\frac{1}{\mu}, \hat{r}\right)$. The interest rate is determined jointly with m from condition (A.37):

$$\rho\frac{\beta}{\mu}u'\left(\frac{\beta m}{\mu\rho}\right) + (1-\rho)\beta r u'\left(\beta r\frac{m}{\rho}\right) = 1.$$

Partial differentiation according to r and m returns

$$\frac{\partial r}{\partial m} = -\frac{1}{\beta\rho}\frac{\rho\beta^2\mu^{-2}u_n'' + (1-\rho)\beta^2 r^2 u_m''}{(1-\rho)(u_m' + x_m u_m'')} > 0, \tag{A.58}$$

and implies a unique $m \in \left(\rho\frac{\mu}{\beta}v\left(\frac{\mu}{\beta}\right), \rho\mu r v(\mu r)\right)$. This means that x_n, x_m, r, and m are all positively correlated. Equivalently, there is a unique $x_n \in (\bar{x}, v(\mu r))$ and a unique $x_m = \beta r x_n$ for each $r \in \left(\frac{1}{\mu}, \hat{r}\right)$.

Case 2.2.2: $\lambda_2 = 0$, Liquidity trap. If $\lambda_2 > 0$, $r = \frac{1}{\mu}$ from (A.38) and $\delta \geq \rho$ from (A.24). Hence the monitored offer in (A.21) becomes identical to the nonmonitored offer in (A.53), resulting in $x_n = x_m$. Consumption is determined in (A.37) and corresponds to $\bar{x}(\mu)$ in (A.32).

A.3 Welfare

The welfare is decreasing for $r \to \frac{1}{\beta}$. I evaluate (25) in $r = \hat{r}$. To this purpose, manipulate (25) using $\varepsilon = -x\frac{u''}{u'}$:

$$\begin{aligned}
\mathcal{W}'_{AB} &= \rho\mu\frac{u'(x_n) - 1}{u''(x_n)} - \frac{u'(x_m) - 1}{\beta r^2 u''(x_m)} \\
&= \frac{\rho\mu x_n}{\varepsilon}\left(-1 + \frac{1}{u_n'}\right) - \frac{x_m}{\beta r^2\varepsilon}\left(-1 + \frac{1}{u_m'}\right);
\end{aligned}$$

then substitute the values of u_k' from (19) and (20):

$$\mathcal{W}'_{AB} = \frac{\rho\mu x_n}{\varepsilon}\left(-1 + \frac{1}{\mu r}\right) - \frac{x_m}{\beta r^2\varepsilon}\left(\frac{\beta r - 1}{1 - \rho\beta r}\right);$$

from (21) it is true that $x_m = \frac{\rho}{\delta}\mu r x_n$:

$$\mathcal{W}'_{AB} = \frac{\rho\mu x_n}{\varepsilon r}\left[-r + \frac{1}{\mu} - \frac{1}{\delta\beta}\left(\frac{\beta r - 1}{1-\rho\beta r}\right)\right];$$

note that δ varies with r; finally, evaluating the derivative in $\hat{r}$ allows to use $\delta = \rho$, and after some algebra:

$$\mathcal{W}'_{AB}(\hat{r}) = \frac{\rho\mu x_n}{\beta\varepsilon\hat{r}}\left[\frac{\beta}{\mu} + \frac{\rho^2(\beta\hat{r})^2 - (1+\rho)\,\beta\hat{r} + 1}{(1-\rho\beta\hat{r})\rho}\right]. \tag{A.59}$$

If the term in square brackets is lower than or equal zero, then $\hat{r}$ maximizes the social welfare.

In the second fraction, the denominator is positive while the numerator depends. Define z_1 the solution of the equation associated with the numerator with variable $\beta\hat{r}$: when $\frac{1}{\mu} < \frac{z_1}{\beta}$, the numerator is positive if $\hat{r} \in \left(\frac{1}{\mu}, \frac{z_1}{\beta}\right)$ and negative if $\hat{r} \in \left(\frac{z_1}{\beta}, \frac{1}{\beta}\right)$; when $\frac{1}{\mu} \in \left[\frac{z_1}{\beta}, \frac{1}{\beta}\right)$, the numerator is negative because $\frac{z_1}{\beta} \le \frac{1}{\mu} < \hat{r}$[17]. z_1 is only function of ρ.
Two results can be stated, by also consider that $\hat{r}$ depends negatively on μ from (A.52)[18].

First, suppose $\mu \to \infty$ so that $\hat{r} \to \frac{1}{\mu} \to 0$. The derivative turns to be positive, precisely $\mathcal{W}'_{AB}(\hat{r}) \to \infty$, which means that the optimal interest rate belongs to $\left(\hat{r}, \frac{1}{\beta}\right)$, associated to $\delta \in (\delta_I, \rho)$. In general, this happens when μ is sufficiently high such that $\frac{1}{\mu} < \hat{r} \le \frac{z_1}{\beta}$.

Second, I study the welfare for μ sufficiently close to β. In this case the sign of the term in square brackets is ambiguous. Note that when $\mu \to \beta$, also $\hat{r} \to \frac{1}{\beta}$ and $\mathcal{W}'_{AB}(\hat{r}) \to 0$. In general, the derivative of term in the square brackets w.r.t. μ reads

$$-\frac{\beta}{\mu^2} + \frac{-\rho^3(\beta\hat{r})^2 + 2\rho^2\beta\hat{r} - 1}{(1-\rho\beta\hat{r})^2\rho}\beta\frac{\partial\hat{r}}{\partial\mu},$$

and in the limit for $\mu \to \beta$ it becomes

$$\frac{1}{\beta}\left[-1 + \frac{-\rho^3 + 2\rho^2 - 1}{-\rho^3 + 2\rho^2 - \rho}\cdot\frac{x^*\left(1-\frac{1}{\varepsilon}\right)}{x^*\left(1-\frac{1}{\varepsilon}\right) + \left[u''(x^*)(1-\rho)\right]^{-1}}\right] < 0,$$

[17]The roots of the numerator, for $z \equiv \beta\hat{r}$, are $z_{1,2} = \frac{1}{\rho}\left[\frac{1+\rho}{2\rho} \pm \sqrt{\Delta}\right]$, where $0 < z_1 < 1 < z_2$ and $\Delta \equiv (1+\rho)^2/(2\rho)^2 - 1 > 0$. Hence the numerator is negative for $\beta\hat{r} \in (z_1, 1)$.

[18]Using partial differentiation of (A.52) in $\delta = \rho$, and remembering that $v(\cdot) = u'^{-1}(\cdot)$:

$$\frac{\partial\hat{r}}{\partial\mu} = -\frac{\hat{r}\left[v(\mu\hat{r}) + \mu\hat{r}v'(\mu\hat{r})\right]}{\mu\left[v(\mu\hat{r}) + \mu\hat{r}v'(\mu\hat{r})\right] + \frac{v'\left[\frac{1-\rho\beta\hat{r}}{(1-\rho)\beta\hat{r}}\right]}{(1-\rho)\beta\hat{r}^2}} = -\frac{\hat{r}x_n\left(1-\frac{1}{\varepsilon}\right)}{\mu x_n\left(1-\frac{1}{\varepsilon}\right) + \left[u''_m(1-\rho)\beta\hat{r}^2\right]^{-1}} < 0.$$

because the second term is the product of two positive numbers lower than 1.
Here is the conclusion for μ close to β. In the limit for $\mu \to \beta$ the derivative of the welfare function in $\hat{r}$ is zero, but this value is decreasing, meaning that it must have a positive value in a neighborhood of β. Hence, for values of μ close to β, the derivative of the welfare function in $\hat{r}$ is positive. In turn, this implies that the optimal interest rate belongs to $\left(\hat{r}, \frac{1}{\beta}\right)$, associated to $\delta \in (\delta_I, \rho)$.

For intermediate values of μ, the relation between μ and $\hat{r}$ complicates the analysis. In particular, the first term in the square brackets of (A.59) is positive and monotonically decreasing in $\mu \in [\beta, \infty)$ from 1 to 0, while the second term is monotonically increasing from -1 to ρ^{-1}. It cannot be excluded that $\mathcal{W}'_{AB}(\hat{r}) \leq 0$ for some μ. In this case, the optimal interest rate is $\hat{r}$ associated to $\delta = r$.

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