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LEARNING WITH UNCERTAIN INFLATION TARGET

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Learning with uncertain inflation target
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Abstract

In a standard new Keynesian framework we derive the conditions under which increasing the inflation target does not deliver expectational instability. We consider two monetary policy regimes with respect to information about the inflation target. Under transparency, there is full disclosure of the inflation target, while, under opacity, the private sector uses optimal Kalman filter to disentangle persistent and transitory increases in the inflation target. Interestingly, the analytical condition that guarantees asymptotical E-stability under transparency is the same that we obtain for a variety of calibrations under opacity. On the other hand, under transparency the transition to the long-run equilibrium is faster and the variance of inflation is lower. Our results are consistent with the view that increasing the inflation target can be used as a policy instrument without unanchoring expectations.

JEL classification: E52, E62, F41, F42.

Keywords: Kalman filter, Adaptive Learning, Policy Targets.

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1 Introduction

In the last years, most central banks have decided to increase transparency of monetary policy targets and strategies with the aim of anchoring private sector's expectations. Nevertheless, a central bank that changes its inflation target could not be successful in managing private sector's expectations. In particular, governor Bernanke[1] at the 2010 Jackson Hole Symposium underlined that a higher inflation target could unanchor inflation expectations. Ascari et al. (2017) have investigated Bernanke's concern in a DSGE model with trend inflation and adaptive expectations, showing that a higher inflation target tends to lower the speed of convergence of expectations. Both under transparency and opacity, the transitional period would last longer and would induce a more aggressive response to inflations in order to stabilize expectations. The cost of a hawkish attitude is that the response to output is quantitatively small, which contradicts the recent accommodative monetary policy to respond to output gap increases. Transparency can offset marginally the destabilizing effect of a high inflation target. Branch and Evans (2017) use a model where agents have imperfect information about the inflation target and form expectations using discounted least squares. They find that if inflation rises, agents will perceive this increase as permanent, leading to higher inflation expectations that feed back into higher effective inflation rates. An additional issue is the zero lower bound reached in most advanced economies after the Great Recession, which has stimulated the debate about the inflation target as a policy instrument. Both the academia, e.g. Blanchard et al. (2010) and Ball (2014), as well as policymakers such as Ben Bernanke during his mandate, discuss the desirability of increasing the central bank's inflation target in order to deal with the problem of the zero lower bound on interest rates. Another reason to raise the inflation target lies in the possibility of lowering the real cost of public debt, as explained in Krause and Moyen (2016).

[1] See Bernanke et al. (2010).

In this paper we analyze whether a higher inflation target may destabilize private sector's expectations and increase the volatility of inflation and output gap. We assume that the inflation target varies over time due to a combination of temporary and very persistent shocks. We consider two central bank's communication regimes, namely transparency and opacity. Under transparency, the central bank communicates if changes in inflation target are persistent rather than temporary, while under opacity the private sector does not know in real time how persistent is the change in inflation target. More in detail, the private sector will use a signal in order to estimate efficiently the unobservable inflation target. The extracted signal improves the adaptive learning process which, in turn, affects the economy's transitory equilibria.

Our research question is similar to Ascari et al. (2017), but we depart significantly from their definition of transparency.[2] In their paper, transparency is described as a regime where the private sector knows the interest rate as well as the policy rule; under opacity the interest rate is observed but the policy rule is not known to the private sector. In our analysis, transparency is referred to the possibility of observing the inflation target. Under opacity, the private sector cannot observe the components of the inflation target and, as a consequence, it must solve a signal extraction problem in order to forecast the inflation target.

Moreover, our work draws on the literature which uses Kalman filtering in presence of incomplete information about potential output. Orphanides (2001), Orphanides (2003a), Orphanides (2003b) and Cukierman and Lippi (2005) report evidence of a significant (real time) overestimation of potential output during the oil shocks of the 1970s, which induced policy errors even if the central bank efficiently estimates potential output. Some examples in the case of imperfectly observed inflation target are Erceg and Levin (2003) and Melecký et al. (2009). In both cases, the private sector does not distinguish between

[2] We also depart from Berardi and Duffy (2007), who adopt the adaptive learning approach to monetary policy transparency by focusing on the value of fully specified models to be learnt.

the permanent and the transitory component of the target.

Our model economy is a standard DSGE model with Rotemberg pricing and time-varying inflation target, where the central bank follows a classic version of the Taylor rule. The private sector has to learn the minimal state variable (MSV) solution of the economy on a recursive basis and, under opacity, it does not observe the permanent and the transitory component of the inflation target separately. Differently from previous contributions, we use a setup where adaptive learning and Kalman filter coexist. Technically, in our model, the central bank has full information about the economy's structure, while the private sector learns adaptively on a recursive basis, according to the approach developed in Marcet and Sargent (1989) and extensively treated in Evans and Honkapohja (2001). Under the opacity regime, in addition, the private sector does not distinguish the highly persistent component and the temporary component of the inflation target and it efficiently extracts information about the inflation target through Kalman-filtering the signal that combines both the components.[3] Both under transparency and opacity, we assume that the central bank neglects the role of the learning process on the monetary policy transmission mechanism. Indeed, this assumption allows us to focus on the role played by inflation target across different information regimes.

Our main findings are two. First, we show that the condition that guarantees long-run E-stability under opacity is the same that we have in the case of monetary policy transparency. Therefore, in terms of long-run expectational stability, being more transparent does not make significative difference. Second, the transition to the long-run equilibrium will be affected by the communication regime, and the central bank's choice to hide its true inflation target raises central bank's loss function.

The rest of the paper is organized in the following way. Section 2 presents the model. Section 3 discusses the asymptotic learnability conditions under transparency and opacity.

[3] This structure of the inflation target follows Erceg and Levin (2003), Ireland (2007) and De Michelis and Iacoviello (2016).

Section 4 reports loss evaluations based on numerical simulations and provides also the impulse responses to a persistent target shock. Section 5 concludes and an appendix shows some technical details about real-time learning.

2 The model

The model we use is a standard DSGE model with monopolistic competition and price setting *à la* Rotemberg. We assume that a representative household maximizes an expected utility function defined on consumption and leisure, subject to a sequence of budget constraints:

$$\max E_o \sum_{t=0}^{\infty} \beta^t \left[\ln(C_t) - \frac{1}{1+\psi} h_t^{1+\psi} \right] \tag{1}$$

$$s.t. \quad T_t + B_{t-1} + W_t h_t \geqslant P_t C_t + \frac{B_t}{R_t} \tag{2}$$

where β is the intertemporal discount factor, h_t are units of labor needed to get nominal salary W_t, ψ is the inverse Frisch elasticity of labor supply, T_t are nominal transfers from the central bank and B_t are one-period zero-coupon bonds returning gross interest rate R_t.

The first order conditions with respect to consumption, hours worked and bonds respectively are:

$$C_t : \quad \Lambda_t = \frac{1}{C_t}, \tag{3}$$

$$h_t : \quad 1 = \Lambda_t \frac{W_t}{P_t}, \tag{4}$$

$$B_t : \quad \Lambda_t = \beta R_t E_t \left(\frac{\Lambda_{t+1}}{\Pi_{t+1}} \right), \tag{5}$$

where Λ_t is the Lagrange multiplier on the budget constraint and Π_t is the gross inflation rate at time t. Combining all the first-order conditions and log-linearising around a zero-inflation steady state equilibrium, we get the IS relationship

$$x_t = E_t x_{t+1} - (i_t - E_t \pi_{t+1} + r_t^n)\,, \tag{6}$$

where $x_t = y_t - \overline{y}_t$ is the real output gap[4], i_t is the nominal interest rate and r_t^n is the natural interest rate, which acts as a demand shock in the model[5].

Production is conducted by monopolistic competitive firms according to a stochastic constant-returns-to-scale linear technology using labor, augmented by a technology shock A_t whose log a_t is an AR(1):

$$Y_t(i) = A_t h_t(i) \tag{7}$$

Consequently, real marginal costs in log-linearised terms, mc_t, can be expressed in equilibrium as follows:

$$mc_t = w_t - p_t - z_t = y_t - a_t. \tag{8}$$

We assume that firms adjust their prices in line with the central bank's inflation target π_t^* (defined below) and that prices are sticky *à la* Rotemberg. Defining with the parameter $\phi > 1$ the degree of price stickiness, with ε the price elasticity and with $\mathcal{F}_{t,t+1}$ the stochastic discount factor, the firm's pricing problem can be written as:

$$\max_{P_t(i)} E_0 \sum_{t=0}^{\infty} \mathcal{F}_{t,t+1} \left[\left(\frac{P_t(i)}{P_t} \right)^{1-\varepsilon} Y_t - \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} Y_t MC_t - \frac{\phi}{2} \left(\frac{P_t(i)}{P_{t-1}(i)\,(1+\pi_t^*)} - 1 \right)^2 Y_t \right], \tag{9}$$

while the following equation represents the first order condition:

$$(1-\varepsilon)\frac{Y_t}{P_t}\left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} + \varepsilon Y_t \frac{MC_t}{P_t}\left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon-1} - \phi \frac{Y_t}{(1+\pi^*)_t\, P_t(i)} \left(\frac{P_t(i)}{(1+\pi_t^*)\, P_{t-1}(i)} - 1 \right) +$$

[4] The output gap is defined as real output minus output under flexible prices $\overline{y}_t$.
[5] The lower-case variables denote percentage deviations from steady state.

$$+\phi E_t \left\{ \mathcal{F}_{t,t+1} \left(\frac{P_{t+1}(i)}{\left(1+\pi^*_{t+1}\right) P_t(i)} - 1 \right) \frac{P_{t+1}(i)}{\left(1+\pi^*_{t+1}\right) P_t(i)} \frac{Y_t}{P_t(i)} \right\} = 0.$$

By exploiting the fact that in equilibrium all firms charge the same price ($\frac{P_t(i)}{P_t} = 1$) and $\mathcal{F}_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{-1}$, it is possible to show that the pricing decision leads to the following log-linear Phillips curve:

$$\pi_t - \pi^*_t = \frac{(\varepsilon - 1)}{\phi} mc_t + \beta E_t \left(\pi^*_{t+1} - \pi^*_{t+1}\right) + u_t \tag{10}$$

where u_t is an AR(1) cost-push shock.

Expressing real marginal costs in terms of output gap, we have the usual relation between output gap and inflation present in the new keynesian Phillips curve (11).

$$\pi_t - \pi^*_t = \beta E_t \left(\pi_{t+1} - \pi^*_{t+1}\right) + \kappa x_t + u_t \tag{11}$$

With respect to monetary policy, we assume that the central bank follows a standard Taylor rule and responds to the deviation of inflation and output from their respective gaps as follows:

$$i_t = r^n_t + \phi_\pi \left(\pi_t - \pi^*_t\right) + \phi_x x_t. \tag{12}$$

Similarly to Erceg and Levin (2003), the inflation target is the sum of two stochastic components that represent a highly persistent component π^*_{pt}, with an autoregressive coefficient ρ_p close to unity, and a temporary component π^*_{qt}, with a much smaller autoregressive coefficient ρ_q that is close to zero:

$$\begin{bmatrix} \pi^*_{pt+1} \\ \pi^*_{qt+1} \end{bmatrix} = \begin{bmatrix} \rho_p & 0 \\ 0 & \rho_q \end{bmatrix} \begin{bmatrix} \pi^*_{pt} \\ \pi^*_{qt} \end{bmatrix} + \begin{bmatrix} \varepsilon_{pt+1} \\ \varepsilon_{qt+1} \end{bmatrix} \quad \text{or} \quad \Pi^*_{t+1} = R\Pi^*_t + \varepsilon_{t+1} \tag{13}$$

Thus, the shock ε_{pt} drives the inflation target away from steady-state for a very prolonged period, while the shock ε_{qt} has only a very transient effect on the target. Moreover, we assume that the inflation target innovations ε_{pt} and ε_{qt} are mutually uncorrelated with variances σ_p^2 and σ_q^2 respectively, and are not correlated with any other shocks to the model economy.

We consider two communication regimes: transparency and opacity. By transparency we mean a regime in which the central bank communicates the policy rule and the current realizations of the inflation target components. Under opacity, the private sector observes the interest rate set by the central bank, but it is unable to disentangle the permanent and the temporary components in the inflation target. This, in turn, hampers firms' pricing decision which is based on current as well expected inflation target, as shown in equation (11). In such a regime, the sum of the permanent and transitory components constitutes the signal s_t available to the private sector:

$$s_t = H\Pi_t^*, \quad H = \begin{bmatrix} 1 & 1 \end{bmatrix}. \tag{14}$$

The private sector will use the signal (14) to compute optimal estimates of the unobserved components π_{pt}^*, π_{qt}^* through the Kalman gain matrix K.[6] If we label with Σ the variance-covariance matrix and with $P_{t+1|t}$ the mean squared error in forecasting Π_t^*, then optimal forecasts of the future inflation targets and their real time estimates at time

[6] The Kalman gain matrix K and the mean-squared error are found numerically by iterating until convergence on

$$K_t = RP_{t|t-1}H\left(H'P_{t|t-1}H + \sigma_\nu^2\right)^{-1}$$

and

$$P_{t+1|t} = \left(R - K_t H'\right) P_{t|t-1} \left(R - K_t H'\right)' + \Sigma$$

where σ_ν^2 stands for the variance of a measurement error in the signal (14) and Σ is the variance-covariance matrix of Π_t^*. In this paper we generally assume no measurement error, but there is still the impossibility of distinguishing the two components in the inflation target.

t respectively can be derived as follows:

$$\begin{bmatrix} \pi^*_{pt+1|t} \\ \pi^*_{qt+1|t} \end{bmatrix} = (R - KH') \begin{bmatrix} \pi^*_{pt|t-1} \\ \pi^*_{qt|t-1} \end{bmatrix} + KH' \begin{bmatrix} \pi^*_{pt} \\ \pi^*_{qt} \end{bmatrix}, \tag{15}$$

$$\begin{bmatrix} \pi^*_{pt|t} \\ \pi^*_{qt|t} \end{bmatrix} = \begin{bmatrix} \pi^*_{pt|t-1} \\ \pi^*_{qt|t-1} \end{bmatrix} + KH' \begin{bmatrix} \pi^*_{pt} - \pi^*_{pt|t-1} \\ \pi^*_{qt} - \pi^*_{qt|t-1} \end{bmatrix}. \tag{16}$$

In the estimation of inflation target's components, the weight attributed to the signal (14) gets smaller as it gets older and, if there is also a measurement error ν_t, the signal will be less precise inducing agents to optimally assign a lower weight (in absolute value) to it. More specifically, in presence of higher volatility in the measurement error, the one-step-ahead forecast error for the policy targets increases, while the elements in the Kalman gain matrix, through which the private sector estimates the two components in the inflation target, diminish in absolute value.

3 E-stability under Transparency and Opacity

Under transparency, we can substitute the policy rule (12) into the IS equation (6) and obtain a system with two forward looking variables that can be summarized through the following relationship:

$$Z_t = AE^*_t Z_{t+1} + C\eta_t, \tag{17}$$

where $Z_t \equiv \begin{bmatrix} x_t & \pi_t \end{bmatrix}'$, $\eta_t \equiv \begin{bmatrix} \pi^*_{pt} & \pi^*_{qt} & u_t \end{bmatrix}'$ and the matrices A and C are, respectively:

$$A \equiv \Omega \begin{bmatrix} 1 & 1 - \beta\phi_\pi \\ \kappa & \kappa + \beta + \beta\phi_x \end{bmatrix} \quad \Omega \equiv [1 + (\phi_x + \kappa\phi_\pi)]^{-1}$$

$$C \equiv \Omega \begin{bmatrix} \beta\rho_p\phi_\pi & \beta\rho_q\phi_\pi & -\phi_\pi \\ 1-\beta\rho_p+\kappa\phi_\pi+\phi_x-\beta\rho_p\phi_x & 1-\beta\rho_q+\kappa\phi_\pi+\phi_x-\beta\rho_q\phi_x & 1+\phi_x \end{bmatrix}. \tag{18}$$

Given the structure of the economy, model (17) has the following perceived law of motion (PLM):

$$Z_t = \gamma\eta_t, \tag{19}$$

while the actual law of motion (ALM) of the economy is as follows:

$$Z_t = (AR\gamma + C)\,\eta_t \quad R \equiv \begin{bmatrix} \rho_p & 0 & 0 \\ 0 & \rho_q & 0 \\ 0 & 0 & \rho_s \end{bmatrix}. \tag{20}$$

Following Evans and Honkapohja (2001), the conditions for E-stability are that the matrices A and $R' \otimes A$ have real parts less than 1[7]. Since R is a diagonal matrix composed by the autoregressive coefficients of the shocks, it is sufficient to study the eigenvalues of A and check that they are less than one. The characteristic polynomial of A is:

$$\lambda^2 + a_1\lambda + a_0 \quad a_1 = -tr(A) \quad a_0 = det(A)$$

The two eigenvalues are inside the unit circle if the following conditions hold:

$$|a_0| < 1$$
$$|a_1| < 1 + a_0$$

The first condition is easily satisfied since $a_0 = \beta\Omega < 1$. The second condition holds if

$$\phi_\pi > 1 - \frac{1-\beta}{\kappa}\phi_x \tag{21}$$

[7]In Appendix A it is shown that E-stability implies convergence of the real-time learning algorithm.

Therefore, condition (21) guarantees that the equilibrium is E-stable.

Under the opacity regime, the private sector learns the policy targets using the Kalman gain matrix K which quantifies the weight attached to the signal every period. In this case the economy cannot be summarized using equation (17) as before, but we have to take into account also the perceived components of the inflation targets $\pi^*_{pt|t}$ and $\pi^*_{qt|t}$. Therefore, the system (17) reads as

$$Z_t = AE^*_t Z_{t+1} + CC\Xi_t \quad \Xi_t \equiv \begin{bmatrix} \pi^*_{pt} & \pi^*_{qt} & u_t & \pi^*_{pt|t} & \pi^*_{qt|t} \end{bmatrix}' \tag{22}$$

Accordingly, the matrix CC is

$$CC \equiv \begin{bmatrix} 0 & 0 & -\phi_\pi & \beta\rho_p\phi_\pi & \beta\rho_q\phi_\pi \\ 0 & 0 & 1+\phi_x & 1-\beta\rho_p+(\kappa\phi_\pi+\phi_x-\beta\rho_p\phi_x) & 1-\beta\rho_q+\kappa\phi_\pi+\phi_x-\beta\rho_q\phi_x \end{bmatrix} \tag{23}$$

while the vector of shocks evolves with the following structure:

$$\Xi_t = \Psi\Xi_{t-1} + \epsilon_t \quad \Psi = \begin{bmatrix} R & \emptyset \\ \begin{bmatrix} KH & \emptyset \end{bmatrix} & \mathbf{I} - KH \end{bmatrix}, \tag{24}$$

with $\emptyset$ and $\mathbf{I}$ indicating zero matrix and the identity matrix, both of appropriate dimensions, and ϵ_t including innovations to the true and contemporaneous perceived shocks.

The ALM will be

$$Z_t = (A\gamma\Psi + CC)\,\Xi_t \tag{25}$$

implying that the matrices A and $\Psi' A$ must have eigenvalues inside the unit circle to have analytical E-stability. Under the condition (21), A has eigenvalues inside the unit circle, so it is sufficient to analyze matrix Ψ. Since Ψ is a block-triangular matrix, its determinant will be equal to $det(R)det\,(\mathbf{I} - KH') = \prod eigs(R) \prod eigs\,(\mathbf{I} - KH')$, where we label with det the determinant and with $\prod eigs$ the product of eigenvalues of matrices R and $(\mathbf{I} - KH')$ respectively. We know that the eigenvalues of R are inside the unit circle, while the eigenvalues of $\mathbf{I} - KH'$ are equal to 1 and to the scalar product of K and H.[8]

Fully analytical results are not available in this case, so we proceed using baseline calibrated values of the parameters in order to compute the E-stability condition. Given a Dixit-Stiglitz price elasticity of demand ε equal to 10 and an inverse Frisch elasticity $\psi = 1.5$, price rigidity *à la* Rotemberg is set in order to get a Phillips' curve slope $\kappa = 0.21$[9]. The Taylor-rule coefficientss $(\phi_\pi,\ \phi_x)$ are such that determinacy is ensured in the standard setup of the model. An autoregressive coefficient of 0.95 is attached to the persistent[10] component of the inflation targets, whereas the value of the temporary component is $\rho_q = 1 - \rho_p = 0.05$. The volatility of the components in the inflation target follows Melecký et al. (2009): with the idea that permanent changes in the inflation target do not occur very often, the standard deviation of the more persistent component σ_p is lower than the standard deviation of the temporary component σ_q. Finally, the cost-push shock persistence ρ_u and standard deviation σ_u are calibrated as in Schaumburg and Tambalotti (2007). Table 1 presents the calibration used in this paper.

In figure 1 we present evidence of these results, showing the scalar product of K and H, which determines analytically if under opacity the equilibrium is E-stable. As discussed before, matrix K depends on the signal available to the private sector, represented by matrix H, hence it is not possible to derive a closed-form solution. We compute $K.H$ varying the response to inflation and for different values of measurement error. In the top panel we present the scalar product in the case of zero measurement error: in this case

[8]We thank Roberto Costas for this point. The interested reader could check Costas-Santos (2006) and Costas-Santos (2009) for further details.

[9]This value of the Phillips' curve slope is consistent with a Calvo degree of price stickiness equal to 0.75.

[10]Such a calibrated value takes into account the computational issues that could arise for values closer to unity (e.g. 0.99). This issue is clarified in Section 4.

the signal available to the private sector has the highest precision, hence it is strongly considered in the estimation of policy targets. As a consequence, the elements of the Kalman gain matrix are higher and the $K.H$ elements again guarantee that the E-stability condition holds. Notice that $K.H$ is increasing in the interest rate response to inflation. However, even in the case of a huge and unrealistic response to inflation, the system is E-stable under opacity.[11] The bottom panel of figure 1 repeats the exercise for two values of the measurement error different from zero.[12] In presence of a measurement error, the signal becomes less precise and the figure confirms that agents optimally attach a lower weight to it in estimating the components of inflation target. In both panels, with zero and positive measurement error, the value of $K.H$ is positively correlated to the measurement error but even for very (implausibly) high values of ϕ_π it goes above unity.

Summing up, for our calibration, the condition that guarantees E-stability under transparency turns out to be valid also under opacity. Therefore, in terms of long-run expectational stability, opacity produces the same asymptotic convergence of a standard DSGE economy with adaptive learning. On the other hand, the information regime affects the transition to the long-run equilibrium in our model, as shown in the simulations in the next section.

4 Numerical analysis

In this section, using Montecarlo simulations, we show that the choice of the central bank to hide the inflation target has a significant detrimental impact on the volatility of inflation and output gap. Under the transparency regime, the propagation of the target shock is (on average) smaller in magnitude and in duration than under opacity. The information regime also affects the dynamics of the adjustment after a change in the

[11] We also tried with even bigger and more unrealistic values for ϕ_π and our results do not change.

[12] We do not plot in only one graph $K.H$ because we would have a figure distorted by the scale of numerical values.

permanent policy target.

To that extent, we evaluate the performance of monetary policy using the loss function (26), depending on inflation and output gap [13].

$$\mathcal{L}_t = \frac{1}{2}\left[(\pi_t - \pi_t^*)^2 + 0.5x_t^2\right] \tag{26}$$

Losses are evaluated by discounting equation (26) to the present time and computing the expected value as the average of the Montecarlo outcomes. We simulate the economy under the assumption of constant gain learning rather than decreasing gain. The implication of this assumption is that newer information is weighted more than information gathered far back in the past, which is crucial in assessing the impact of an unexpected structural change like an increase of inflation target. In particular, we assume that the highly persistent component in the inflationary target is raised by 1 percentage point in the midst of our simulation horizon (i.e. at t = 250). After the shock, we compare the outcomes of stochastic simulations of the economy, both under transparency and opacity. In each run of the experiment, the economy is simulated 5'000 times over a horizon of 500 periods (namely quarters, according to the calibration). We control for the pseudo randomness of the stochastic innovations such that (pseudo) random shocks differ in each simulation, although they are the same across the different regimes. Hence, what actually changes between experiments is only the information regime. Agents start extracting information from the observed variables (namely past inflation, output gap and interest rate[14]) since the beginning of the simulation. Nevertheless, we exclude the first hundred observations that are generated. The learning algorithm is initialised according to the coefficients of the MSV solution under rational expectations, such that the initial value of

[13] Under the assumption of Rotemberg pricing, the welfare losses derive from the inefficient diversion of resources from aggregate consumption to the price-revision technology. Both the Calvo and the Rotemberg models imply the same relative weights in the welfare criterion. See Lombardo and Vestin (2008) and Nistico (2007).

[14] In these simulations we assume standard coefficients in the Taylor rule: $\phi_\pi = 1.5$ and $\phi_x = 0.5$.

the guessed MSV solution is equal to the true parameters of the data generating process (DGP) of the economy. The initial moment matrix, instead, is purely random. As for the initialisation of the Kalman algorithm, even in the adaptive learning framework, agents have enough information about the target processes to correctly build the Kalman gain matrix, which is applied to a limited subset of variables.[15] Importantly, the Kalman gain matrix does not depend on the periodic estimates of other parameters, but it is computed only once at the beginning of the simulation and in each period it is used to infer the values of the target. Due to the constant gain hypothesis we embody a projection facility in the simulation algorithm. Such a procedure is necessary to rule out coefficient and moment matrices that include infinite or NaN elements.[16]

The simulation goes through three steps. First, once the deep parameters are declared, we find the rational expectation equilibrium (REE) of the economy, jointly with the parameters of the Kalman filter. The REE parameters constitute the benchmark values and are used to initialise the coefficient matrix in the learning algorithm. The elements in the Kalman gain matrix are used at each iteration of the learning algorithm to filter the interest rate value and infer the evolution of the inflation target. The second step concerns the simulation of the shock realizations and the subsequent formation of the agents' information set. Within this step we declare the variables that agents use in order to perform the adaptive improvement of their estimated VAR. If agents operate under transparency, then their information set includes actual realizations of the inflation target, as well as the cost-push shock. On the contrary, if agents operate under opacity, their information set differs because it includes the Kalman-filtered values of the inflation target, rather than actual realizations. In the third and final step agents

[15] Erceg and Levin (2003) also estimates the actual signal to noise ratio associated to the persistent parameter, obtaining a Kalman gain of 0.13. According to our calibration the implied Kalman gain associated to the persistent component is 0.115.

[16] See Branch and Evans (2017) for an application or Carceles-Poveda and Giannitsarou (2007) for a discussion of projection facilities in practice.

perform their parameter-update (accordingly with their information set) which yields the expected values of the state variables. Therefore, the expected values are enclosed into the structural equations and the economy generates the actual realizations of inflation, output and interest rate according to the ALM of the system. At this stage we compute the period loss associated with deviation from targets. Once the three steps are complete, the algorithm has generated a complete set of realizations of the state variables that are used to produce descriptive statistics and loss.
The stochastic simulations of inflation and output gap are depicted in figures 2 and 3, where the two series show the median realization (among 5'000) of inflation (π) and output gap (x) in each period. It can be observed the effect of a one percent impulse on π_p^* at $t = 250$.

In the previous section we have shown that permanent shocks to inflation target do not affect the asymptotic learnability of the REE when the algorithm encompasses a decreasing gain. The stochastic simulations provide some support to the conjecture that this is also the case if a constant gain is used in order to update the coefficients. In this case the convergence of the parameters is in distribution and consists in stationary oscillations of the estimated parameters around (and close to) the REE values. This implies some noise added to the actual values of the state variables, because parameters do not converge to a single point value. Their oscillations, indeed, pass on to actual values through expectations that enter the ALM. It follows that, contrary to the decreasing gain case, a constant gain implies that the economy is permanently in a transitional equilibrium, such that the endogenous state variables never catch up the exact equilibrium trajectory of the RE solution, but they are stationary and oscillate in their surroundings. It can be observed that the median series shown in figures 2 and 3 do not diverge and remain stationary around the equilibrium trajectory, after the target shock occurring in period 250. Inflation is hit by the shock more heavily with respect to output-gap, as shown by the variances reported in table 2. The value of the persistent shock is zero in each period except in period 250 (when it is 1%). Therefore, given the dynamics of π and x shown in figures 2 and 3, it is apparent that the main trends of inflation and output gap are driven by the target values because their trend mimic the dynamics of the targets.[17] Having persistent rather than permanent shocks explains the mean reversion to the initial zero trend (i.e. the trend before the impulse). In fact, the persistent shock fades away after less than a hundred years. The important feature of the series in both figures 2 and 3 is that the system keeps converging after the shock, because agents keep learning effectively, both under transparency and under opacity. The peak value of inflation after the persistent shock to the target is higher under transparency than under opacity. In fact, under opacity it takes some time for the private sector to understand the relevance as well as the nature of the shock. The same reason explains that the peak under opacity is hit some period after the shock (which is consistent with the impulse responses shown below).

Table 2 reports the variances and the corresponding expected loss, normalized to the expected loss of the REE. It follows that EL under transparency is more than 15% smaller than with rational expectations. Under opacity, EL is 10% higher than with RE. The examination of expected loss across different regimes suggests that agents are better off under transparency when they are adaptive learners and this result is mainly driven by the smaller variance of inflation, given the similar output gap variance.[18] Interestingly,

[17]Notice that we analyze the case of a persistent, rather than a permanent, component of the shock essentially for computational reasons. If we had set the autoregressive parameter to a value very close to unity, we would get a much more persistent effect of the one-shot change, at the cost of simulations based on almost unit-root series. These would lead to non significant estimations because the learning algorithm is essentially a recursive version of the OLS estimator, which on the contrary must rely on stationary processes. Having near unit-root processes would imply moment matrices to occasionally diverge and that would, in turn, make the whole system divergent.

[18]Lower expected losses under learning than under full information is a well known effect of learning. It arises from the tighter anchoring of expectations to central banks inflation targets due to underestimation of the persistence of the ALM. See Dennis and Ravenna (2008) for further discussion and for a comparison with central bank learning together with private sector .

the simulated values show that opacity weakens the anchoring of expectations to targets, and more than offsets the gains from learning.

The impact of a shock is analyzed more in detail by means of simulated impulse responses that are shown in figures 4 and 5. In each of the impulse response figure, the red continuous line represents the impulse under rational expectations. Since the economy under rational expectations is not affected by the information regime, the red lines are the same in the two figures. The dashed lines represent, respectively, the 15^{th} and the 85^{th} percentile of the endogenous state variables. We follow the methodology used in Orphanides and Williams (2007a) and Orphanides and Williams (2007b) and simulate the short run response of the economy starting from different steady states, selected with the objective to mimic their true distribution. This procedure is necessary because, although the economy's structural equations are log-linearized around the zero inflation steady state, the adaptive learning procedure used to update information determines some degree of non-linearity. Hence, as suggested by Orphanides and Williams (2007a) and Orphanides and Williams (2007b), the initial states are simulated according to a long run simulation, rather than computed theoretically, as in the case of rational expectations. The economy has been simulated for $10'000$ periods. Then, $1'001$ periods are selected randomly and the relevant data in each selected period has then been stored. In particular, all the current and past information necessary to update the parameters and compute the expectation are stored separately, e.g. coefficient matrix in current and one period before, previous period exogenous and endogenous state variables, previous period moment matrix and expectations, as well as previous period Kalman estimations. The stored values of these variables are then used as the starting points of the economy. Using the collected data, the economy is then simulated $1'001$ times. In each of these simulations, the economy is hit by a one percent impulse to the persistent target inflation during the first iteration (i.e. in its second period). Thus, it is possible to get the simulated distribution of the economy's reaction to the shock conditioned to the initial state. It is worth to remark that just like for the stochastic simulations, this procedure has been implemented under all the information regimes by controlling for the random innovation. In particular, the whole set of innovations used to simulate the initial points are the same under all the informative regimes.

Our simulations illustrate that the initial median response of the economy is not very different across the two regimes. Nevertheless, differences arise as for the reversion to the initial steady state and the variability of the response. The economy under transparency converges faster to the initial conditions than under opacity. Moreover, the dispersion of the endogenous state variables around the median value is higher under opacity, which can be accounted for the higher variability of inflation in the stochastic simulation under opacity. The target shock affects inflation more heavily than output gap. According to the simulations, the median response of the output gap barely moves, if compared to a more evident increase in inflation.
In our model there are two channels that determine that. The first channel works through a a monetary policy loosening that implies a rise in inflation, while the second channel is due to the direct effect in the Phillips curve under the assumption that firms revise their prices in line with the central bank's inflation target. Since learning (under both information regimes) generally occurs with a one period lag, such a direct effect can be accounted for the initial response of inflation, which indeed is similar under both the regimes. The difference between the two regimes arises in the follow up of the impulse, since information is processed differently. Despite the optimality property of the Kalman filter, opacity makes agents worst off since information flows more sluggishly trough the economy than under transparency. The optimality of the Kalman filter probably allows the system to remain learnable, since estimated values of the target components do not

depart significantly from the true values[19]. Nevertheless, transparency allows a more rapid transition after a persistent change in inflation target. This result supports the view according to which transparency fosters expectations coordination.

5 Concluding comments

In this paper we address the issue of whether random changes in the inflation target levels can be destabilizing for the economy and whether such changes should be disclosed or not to the public. The analysis proposed in this paper hinges on the expectational stability of a small-scale linear DSGE model with Rotemberg pricing and time varying inflation target, a workhorse model that allows a comparison with an extensive literature. We define opacity as the regime where the private sector is not able to distinguish between a highly persistent and a temporary increase in the inflation target and it estimates the two components by Kalman filtering the information provided by the central bank. Such estimations are also used to compute expectations.
Our analysis is both analytical and numerical. We work out the analytical condition for E-stability under transparency and under opacity and we find the necessary E-stability condition under transparency does not differ under both the regimes. We find that he standard asymptotic E-stability conditions under transparency continue to hold under opacity, although in the latter regime the result is numerical and not analytical.

Through numerical stochastic simulations, we find evidence that the economy does not diverge with a constant gain (such that a convergence in distribution is achievable) and that changing the persistent component of the inflation target does not unanchor the expectations. Moreover, we find that transparency reduces expected loss because it supports a faster coordination of expectations than under opacity. To that extent,

[19] Plots of the two filtered series are not shown but exhibit an apparent stationary trend. They are available upon request.

inflation target can be effectively controlled as a policy instrument available even when changing the policy interest rate is not an option.

Our paper has nonetheless some elements that can be modified to assess different transmission mechanisms of learning. First, a central bank that does not disclose the origin of target changes in full detail induces agents to extract information by the signal and to include this into the expectations formation procedure. Second, as emphasized by Marcet and Nicolini (2003) and Gaus (2013), an endogenous learning algorithm may be more suitable if agents believe they face structural breaks such as hyperinflations, as in Marcet and Nicolini (2003), or a persistent deflation after the Great Recession. Technically, the endogenous learning algorithm is a switching rule between a constant and a decreasing gain in learning. These issues are left as a direction for future research.

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A Real-time convergence

Learning in the real time is implemented according to the guidelines provided in chapters 6 and 10 in Evans and Honkapohja (2001) and in Evans and Honkapohja (1998). Under the assumption of gains from learning decreasing along time, we provide the sufficient condition for them to converge to OLS values.

The following matrix represents the moment matrix updating rule:

$$R_t = R_{t-1} + t^{-1}\left(v_{t-1}v_{t-1}^{'} - R_{t-1}\right), \tag{27}$$

where $v_t^{'} = (\pi_t^*, \bar{y}_t, u_t)$. In order to convert the system in a convenient form, I make a timing change on R_t, i.e. $S_{t-1} = R_t$, so that

$$S_t = S_{t-1} + t^{-1}\left(v_t v_t^{'} - S_{t-1}\right) + t^{-2}\left(\frac{-t}{t+1}\right)\left(v_t v_t^{'} - S_{t-1}\right). \tag{28}$$

The parameters updating equation is

$$\xi_t = \xi_{t-1} + t^{-1}R_t^{-1}v_{t-1}\left(T\left(\xi_t\right)^{'}v_t - \xi_{t-1}^{'}v_{t-1}\right)^{'}, \tag{29}$$

The parameter ξ_t includes the matrix of parameters to be estimated, such that $\xi_t^{'} = (c)$. In order to simplify notation, let $y_t = (\pi_t, x_t)^{'}$, $v_t = (\pi_{pt}^*, \pi_{qt}^*, u_t)^{'}$ and ε_t is a vector containing white noise innovations to v_t. The real-time learning algorithm above are in the form

$$\theta_t = \theta_{t-1} + t^{-1}\mathcal{H}\left(\theta_{t-1}, X_t\right) + t^{-2}\rho_t\left(\theta_{t-1}, X_t\right), \tag{30}$$

where $\theta_t = (\xi_t, S_t)$ and $X_t = (x_t, v_t, \varepsilon_t)^{'}$, whose convergence has been demonstrated to be driven by E-stability under the sets of conditions A and B in Section 6.2.1 in Evans and Honkapohja (2001). In order to prove that also the algorithm (28-29) converges, and that the convergence is driven by the E-stability conditions provided in Section 3, we need to analyze the equations governing vector X_t, that we can put in the form $X_t = A(\xi_{t-1}) X_{t-1} + B(\xi_{t-1}) W_t$, where $W_t = \varepsilon_t$, and

$$A(\xi_{t-1}) = \begin{pmatrix} 0 & T_c(\xi_{t-1}) & 0 \\ 0 & R & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$B(\xi_{t-1}) = (0, I, I)'$$

and zeros and I represent respectively matrices of zeros and identity matrices of appropriate dimensions.

In general, provided that assumptions A and B of Section 6.2.1 in Evans and Honkapohja (2001) are satisfied (which is the case for the model under examination), the dynamic properties of the system can be studied in terms of a difference equation associated to the model around a fixed point $\bar{X}_t(\theta)$ obtained for a fixed θ. The ODE associated to the process $X_t(\theta)$ is obtained as the limit for $t \to \infty$ of the mean of $\mathcal{H}(\theta_{t-1}, X_t)$. Hence, provided that $h(\theta) = \lim_{t\to\infty} E\mathcal{H}\left(\theta, \bar{X}_t(\theta)\right)$, the associated ODE is

$$\frac{d\theta}{d\tau} = h(\theta). \tag{31}$$

In practice, given a fixed point $\bar{\xi}$[20] of $T(\xi)$ such that the corresponding REE is asymptotically stationary, the process $X(\xi)$ is stationary for ξ sufficiently close to $\bar{\xi}$. On top of that, conditions A in Section 6.2.1 in Evans and Honkapohja (2001) hold provided that we choose an open set $\hat{D}$ around $(\bar{\xi}, \bar{S})$ such that $\forall \quad \xi, S \in \hat{D}$ there exists one fixed point only (i.e. $\bar{\xi}$) and S is always invertible. For bounded moments of W and for sufficiently

[20]The most convenient assumption is that the fixed point correspond to the REE values of the parameters.

small values of $\hat{D}$, the set of conditions B holds as well.

It follows that the asymptotic convergence of θ can be studied through Equation (31) as follows:

$$\frac{d\xi}{d\tau} = S^{-1} M_z \left(T\left(\xi, \right) - \xi \right),$$

$$\frac{dS}{d\tau} = M_z - S.$$

The above equations show that, as long as $S \to M_z$, convergence is driven by conditions $T\left(\xi\right) - \xi$, which turns out to be the condition for expectational stability. Hence, if the model is e-stable, the real-time adaptive learning algorithm locally converges to the REE.

As for the real-time learning under opacity, the information set is different than with respect to the transparency regime. In fact, the target shocks that structurally affect the economy are not directly observed by agents, but estimated by Kalman-filtering the signal by central bank. Hence, the estimations of the two components of the target (i.e. $\pi^*_{pt|t}$, $\pi^*_{qt|t}$) add to the minimum set of fundamental variables that drive the MSV solution under rational expectations. Then, under opacity, the vector v_t, enclosing the set of relevant variables contained used by agents, is defined ad follows: $v_t = \left(\pi^*_{pt}, \pi^*_{qt}, u_t, \pi^*_{pt|t}, \pi^*_{qt|t}\right)$.

The moment matrix updating rule is the same than in Equation 27, and the parameters updating equation is the same than in Equation 29. The process for parameter update can therefore be expressed as in Equation 30. The A matrix in the process $X_t = A\left(\xi_{t-1}\right) X_{t-1} + B\left(\xi_{t-1}\right) W_t$, where $W_t = \varepsilon_t$, is as follows:

$$A\left(\xi_{t-1}\right) = \begin{pmatrix} 0 & T_c(\xi_{t-1}) & 0 \\ 0 & R & 0 \\ 0 & \begin{bmatrix} KH & \emptyset \end{bmatrix} & \mathbf{I} - KH \end{pmatrix}.$$

Let matrix Ψ be as follows:

$$\Psi = \begin{pmatrix} R & 0 \\ \begin{bmatrix} KH & \emptyset \end{bmatrix} & \mathbf{I} - KH \end{pmatrix}.$$

As long as Ψ has eigenvalues within the unit circle, the stability of X_t is determined by $T_c(\xi_{t-1})$ only. Therefore, the same stability conditions apply with respect to the transparency regime. The discussion of the eigenvalues of matrix Ψ are provided in Section 3.

Table 1: Calibration

β	0.99
ψ	1.5
ε	10
κ	0.21
ρ_p	0.95
ρ_q	0.05
ρ_u	0.4
σ_p	0.00017
σ_q	0.001
σ_u	0.001

Table 2: Loss results

Opacity	Transparency
$\sigma_\pi^2 = 0.0034$	$\sigma_\pi^2 = 0.0028$
$\sigma_x^2 = 0.0018$	$\sigma_x^2 = 0.0018$
$EL = 1.1$	$EL = 0.864$

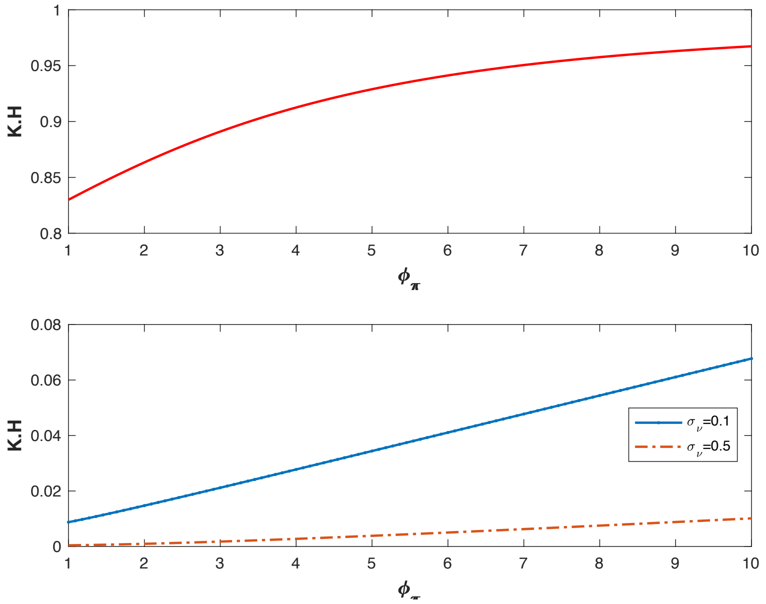

Figure 1: E-stability under opacity: the scalar product between K and H should be less than one.

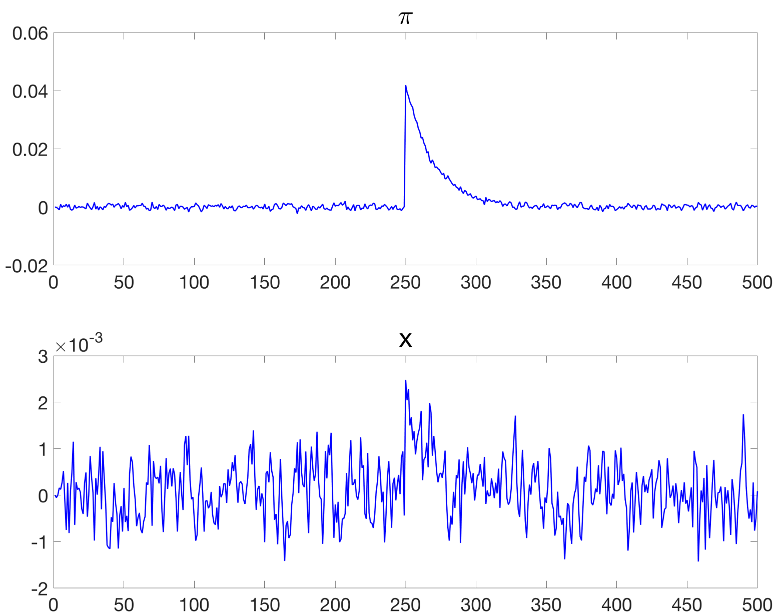

Figure 2: Inflation and output gap under transparency.

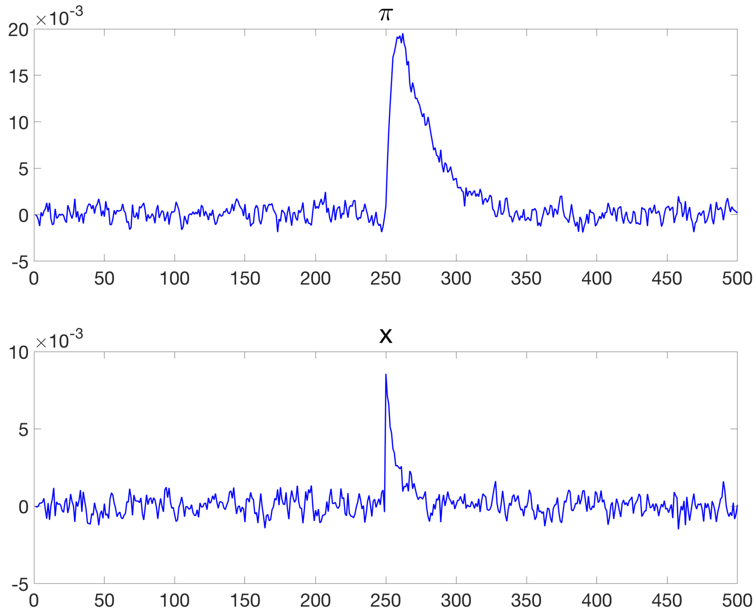

Figure 3: Inflation and output gap under opacity.

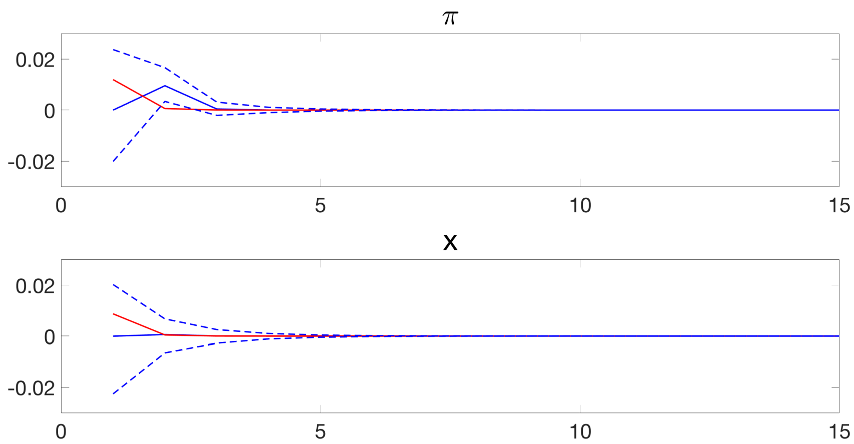

Figure 4: Impulse responses of inflation and output gap under transparency to a 1% change in persistent inflation target. Solid blue line: median values. Dashed lines: 85^{th} and 15^{th} percentiles. Red solid line: response under rational expectations.

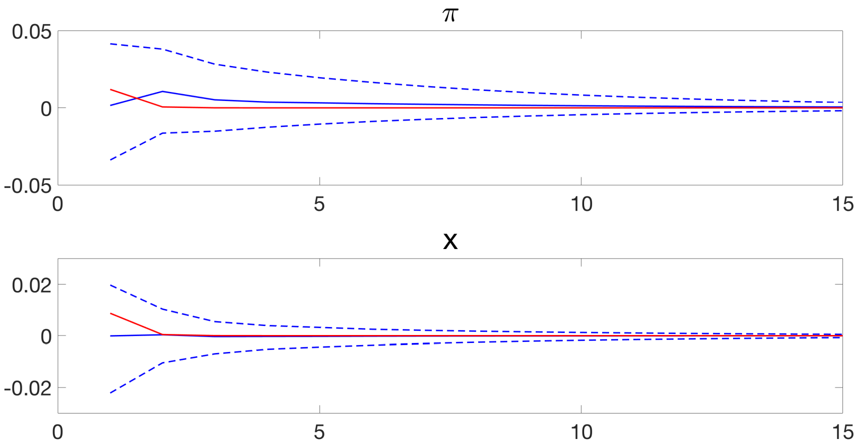

Figure 5: Impulse responses of inflation and output gap under opacity to a 1% change in persistent inflation target. Solid blue line: median values. Dashed lines: 85^{th} and 15^{th} percentiles. Red solid line: response under rational expectations.