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FORWARD GUIDANCE IN SMALL OPEN ECONOMY

Marine Charlotte André and Guido Traficante

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Arcelli Centre for Monetary and Financial Studies
Department of Economics and Finance
LUISS Guido Carli
Viale Romania 32, 00197, Rome - Italy
http://casmef.luiss.edu

Forward Guidance in Small Open Economy

Marine Charlotte André * Guido Traficante†

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Abstract

We examine forward guidance in a small open economy New Keynesian model. In a setup where forward guidance duration is known with certainty, we show that the elasticity of inflation with respect to the real exchange rate is a key variable in attenuating the forward guidance puzzle. Then we consider a credible forward guidance regime which is adopted stochastically, in normal times or under a liquidity trap. Compared to closed economy, forward guidance turns out to be more expansionary in open economy and the real exchange rate is a key variable driving this result. In particular, the response of output and inflation is amplified when aggregate supply is negatively related to the real exchange rate.

Keywords: Monetary policy, small open economy, forward guidance.
JEL Classification: E31, E52.

*Banco de México. The views and conclusions presented in this article are exclusively the responsibility of the authors and do not necessarily reflect those of Banco de México. Contact: mandree@banxico.org.mx

†Università Europea di Roma, Rome, Italy. Contact: guido.traficante@unier.it

1 Introduction

The global financial crisis has triggered a vivid interest in theoretical and empirical research on unconventional monetary policies that look for a substitute of the short-term nominal rate when the latter reaches the zero lower bound. A key example of such unconventional policies is given by forward guidance, through which policymakers announce a path of the nominal interest rate starting immediately or in the future for a particular duration. Through this policy, the central bank tries to manage expectations of the future policy rates once the zero lower bound is no longer binding in order to influence macroeconomic dynamics. In the basic New Keynesian DSGE model, an anticipated change in the policy rate produces an effect on output which is independent of the duration and timing: this is the forward guidance puzzle. As a consequence, the effects of a temporary variation in the policy rate that takes place very far in the future is the same if the variation were to take place immediately or in the near future. This puzzle, discussed by Del Negro et al. (2012), Carlstrom et al. (2015) and McKay et al. (2016), derives from the fact that, in a baseline New Keynesian DSGE model, the dynamic IS relationship has no discounting of the expected output gap and, in turn, of future real interest rates. Consequently, the literature introduced some discounting mechanism in the Euler equation so that aggregate demand responds less than one-to-one to its future expected changes. Examples include an overlapping-generations structure *à la* Blanchard and Yaari in the demand side (Del Negro et al. (2012)), heterogeneous agents and incomplete markets (McKay et al. (2016)), sticky information (Carlstrom et al. (2015)). McClung (2020) shows that a regime characterized by passive monetary policy and active fiscal policy does not imply forward guidance puzzle. With active fiscal policy, Ricardian equivalence does not hold and agents perceive government debt as net wealth. As a consequence, forward guidance announcements that lower the expectations of future interest rates produce negative wealth effects that counteract the monetary stimulus.

In this paper we analyze the theoretical implications of forward guidance in small open economy, focusing on the international transmission of such a policy. To the best of our knowledge, Galí (2020) is the only theoretical contribution about forward guidance in open economy. He shows that if the home central bank announces an increase (decrease) of the nominal interest rate of T periods, with no reaction from the foreign central bank, the exchange rate appreciation (depreciation) at the time of announcement is proportional to the duration and the size of the interest rate change, but it is independent on the duration of the forward guidance. Therefore forward guidance puzzle arises also in a small-open economy model. We follow Gali and Monacelli (2005) and Leitemo and Söderström (2008) in modeling a small country that freely trades with the rest of the world, constituted of a continuum of foreign economies. We evaluate the forward guidance policy in normal times and under a liquidity trap, induced by a negative shock to the natural rate, which will be the only shock present in our model. Forward guidance will be analyzed in a deterministic scenario, where its duration is known with certainty, and in a stochastic setting, modeled along the lines of Bilbiie (2019).

Our main results are the following ones. First, we show the analytical conditions that guarantee that the forward guidance puzzle does not hold in open economy. A key determinant for eliminating the puzzle is the elasticity of inflation with respect to the real exchange rate which could be either positive or negative. To that extent, an exchange rate depreciation increases consumer prices and therefore reduces households' purchasing power. The optimal labor supply choice will imply higher wages and, in turn, higher marginal costs and inflation. However, after an exchange rate depreciation, aggregate consumption could fall because imported goods are more expensive. In the latter case, the marginal rate of substitution then falls, leading to lower real wages and marginal cost. Our results point out that, for some empirically plausible values of the elasticity that determine a negative relationship between inflation and real exchange rate, we do not have forward guidance puzzle. Second, exchange rate pass-through in the Phillips curve is positively associated with the expansionary effects of forward guidance also in a stochastic setup, where the duration of the policy and the state of the economy ("normal times" versus liquidity trap) follow a Markov chain. Finally, compared to the closed-economy counterpart, forward guidance tends to be more expansionary in open economy: this is due to the combination the role played by the real exchange rate and to the better trade off between output and inflation (because of a larger Phillips' curve slope).

The paper is organized as follows: Section 2 describes the reference model, while Section 3 derives the analytical conditions for the presence of the puzzle in a deterministic setup, varying the frictions in international financial markets. In Section 4 we study stochastic forward guidance before concluding in Section 5. Appendix A provides some tedious computation not reported in the main text.

2 The model

We shortly summarize, with some slight changes in notation, the small open economy model of Gali and Monacelli (2005) and Leitemo and Söderström (2008). With the objective of deriving analytical solutions, the only shock (defined later) is a preference shock that drives the economy into a liquidity trap.

The small domestic country freely trades with the rest of the world (foreign country), constituted of a continuum of foreign economies. We assume that foreign and domestic countries share preferences and technology. Domestic and foreign firms produce traded consumption goods, using labor as the sole input. Households derive their utility from consuming both domestic and foreign goods, and have a marginal decreasing disutility in labor supply to firms.

Denoting by e_t the log-linearized real exchange rate, we have by definition

$$e_t = s_t + p_t^f - p_t, \tag{1}$$

with s_t being the nominal exchange rate (units of domestic currency against one unit of foreign currency), p_t^f the price level of the goods produced in the foreign country and p_t the price level of domestically produced goods.

The real exchange rate is directly related to the inflation rate in the domestic goods sector, π_t, via the New Keynesian Phillips curve:[1]

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t - \phi e_t, \tag{2}$$

where x_t denotes the output gap, $0 < \beta < 1$ the discount factor, and E_t the rational expectations formed by private agents (conditional on information set available at time t). The composite parameter $\kappa = \hat{\kappa}(\eta + \sigma)$ $\hat{\kappa} \equiv \frac{(1-\vartheta)(1-\vartheta\beta)}{\vartheta}$ is the output-gap elasticity of inflation and encompasses the effect of the output gap on inflation via real marginal costs. Phillips' curve slope depends on ϑ, the share of firms that do not optimally adjust but simply update in period t their previous price by the steady-state inflation rate, on η, which represents the steady-state Frisch elasticity of labor supply, and on $\sigma \equiv \frac{\hat{\sigma}}{1-\omega}$ with $\hat{\sigma}$ denoting the inverse of the elasticity of intertemporal substitution, and $0 \leq \omega \leq 1$ the share of foreign goods in domestic consumption. The real exchange rate enters the Phillips curve through the coefficient $\phi = \omega\hat{\kappa}\left[(2-\omega)\zeta\sigma - 1\right]$, where ζ stands for the elasticity of substitution across domestic and foreign goods. The economic intuition behind the relationship between inflation and real exchange rate is the following: when households choose labor supply, they care about the purchasing power of their wage deflated by the consumer price index that also includes prices of imported goods, implying that the equilibrium wage and hence the real marginal costs depend on the real exchange rate. As highlighted in Leitemo and Söderström (2008), there are two competing effects shaping the relation-

[1]Differently from Gali and Monacelli (2005), Leitemo and Söderström (2008) derive a Phillips curve including the real exchange rate. For the microfoundations of the model, see Leitemo and Söderström (2008). Notice that π_t is different from the inflation rate of the consumer price index that also takes into account the inflation of foreign goods consumed by residents. In the closed economy, π_t represents both producer and consumer price inflation rates.

ship between exchange rate and inflation. On the one hand, an exchange rate depreciation (i.e. an increase in the exchange rate) increases consumer prices and therefore reduces households' purchasing power. The optimal labor supply choice will imply higher wages and, in turn, higher marginal costs and inflation. On the other hand, an exchange rate depreciation leads to a decrease in the demand for imports and therefore a reduction in aggregate consumption. The marginal rate of substitution then falls, leading to lower real wages and marginal cost. The composite parameter ϕ is positive as long as $(2-\omega)\zeta\sigma > 1$: this condition holds in Leitemo and Söderström (2008), determining a negative relationship between inflation and exchange rate for their model calibrated to Sweden. However, for economies whose main exports are based on price competitiveness, generally the first effect dominates and an exchange rate depreciation induces higher inflation, which reduces domestic consumption. For instance, (Mihailov et al., 2011) show that for Spain a currency depreciation increases the possibility to export at the cost of a lower purchasing power for consumers. In general, there is not unanimous consensus about the sign of the relationship. Differently from Leitemo and Söderström (2008), Walsh (1999) and Razin and Yuen (2002) obtain a positive relationship between these two variables in theoretical models, while the estimates of Phillips curve in Mihailov et al. (2011) show that inflation can be either positively or negatively correlated with the expected change in the real exchange rate with the coefficients ranging from -0.26 to 0.47 for different European countries. Therefore, in our analysis we will consider both signs in the relationship.

The New Keynesian IS equation is given by

$$x_t = E_t x_{t+1} - \sigma^{-1}(r_t - E_t \pi_{t+1}) + \sigma^{-1}\rho_t - \delta\left(E_t e_{t+1} - e_t\right), \tag{3}$$

where r_t is the nominal short-term interest rate, ρ_t represents an exogenous disturbance that moves the natural interest rate, and δ a composite parameter defined by $\delta \equiv \frac{1}{\sigma}\left[\frac{\Omega}{(1-\omega)} - 1\right]$ with $\Omega \equiv (1-\omega)\left[(1-\omega) + (2-\omega)\omega\zeta\sigma\right]$. The composite parameter δ is the elasticity of the output gap with respect to the expected change in the real exchange rate, reflecting the substitution effect induced by such a change on the demand of domestically produced goods.[2] Also with respect to the output gap, there are two competing effects of exchange rate movement. On the one hand, an exchange rate depreciation increases consumer prices and reduces expected inflation; the resulting increase in the real interest rate reduces consumption and the output gap, given the expected future exchange rate. On the other hand, the exchange rate depreciation increases export demand, and therefore output. As shown in Leitemo and Söderström (2008), the same condition shown above for ϕ determines the type of relationship between exchange rate and output gap indirectly through the Phillips curve; ϕ determines whether a country would export more following a depreciation of its national currency.

Finally, the real UIP condition relates the real interest rate differential with the expected rate of real depreciation:

$$r_t - E_t\pi_{t+1} = E_t e_{t+1} - e_t, \tag{4}$$

where foreign variables are set to zero for simplicity. In the baseline version of the model, we will consider the simple UIP relationship, before introducing some frictions in terms of risk premiums or portfolio adjustments a la Wieland (2012).

3 Forward Guidance puzzle in open economy

One of the main results obtained in the literature is the so-called forward guidance puzzle, through which an anticipated change in the policy rate produces an effect on output which is independent of the duration and timing: this is the forward guidance puzzle. As a consequence, the effects of a temporary variation in the policy rate that takes place very far in the future is the same if the variation were to take place immediately or in the near

[2]Note that Ω and δ are positive for $(2-\omega)\zeta\sigma > 1$.

future. This puzzle derives from the fact that, in a baseline New Keynesian DSGE model, the dynamic IS relationship has no discounting of the expected output gap and, in turn, of future real interest rates.

In this section, we consider the case in which the central bank announces that the policy rate will be fixed for T periods, equal to $\bar{r}$. As to the natural rate, we assume that it is at its steady state value 0.[3] As it is well known, an exogenous interest rate path implies equilibrium indeterminacy, while this does not occur if the forward guidance period has a finite duration and it is followed by a rule that ensures determinacy. Knowing that, after T periods, monetary policy will be set in such a way (for example through a Taylor rule), we follow the methodology in Carlstrom et al. (2015) to characterize the dynamics of the economy under the forward guidance period. In particular, combining the IS curve, the NKPC and the UIP condition inflation dynamics can be expressed through the following second order difference equation during a period of constant interest rate:

$$\pi_t = -\frac{\kappa}{\sigma}\bar{r} + \Gamma\pi_{t+1} - \beta\pi_{t+2}, \qquad \Gamma \equiv 1 + \beta - \phi + \delta\kappa + \frac{\kappa}{\sigma} \tag{5}$$

with two terminal conditions

$$\pi_T = \left(\phi - \delta\kappa - \frac{\kappa}{\sigma}\right)\bar{r} \tag{6}$$

$$\pi_{T-1} = \left(2 + \frac{\kappa}{\sigma} + \beta - \phi + \delta\kappa\right)\left(\phi - \delta\kappa - \frac{\kappa}{\sigma}\right)\bar{r} \tag{7}$$

Notice that the value of Γ collapses to that of Carlstrom et al. (2015) if we consider a closed-economy. The condition for having a stable inflation dynamics is that the eigenvalues of (5) are less than one in absolute value[4].

[3] We could also assume that there is a preference shock such that the economy enters in a liquidity trap, as in the experiment of Carlstrom et al. (2015), but the qualitative results would not change.

[4] More recently, Gibbs and McClung (2020) show the sufficient conditions for when a rational expectations structural model predicts bounded responses of endogenous variables to forward guidance announcements. The conditions coincide with a special case of the well-known (E)xpectation-stability conditions that govern when agents can learn a Rational Expectations Equilibrium

Name	Value for Sweden	Value for Spain
β	0.99	0.99
ζ	1	0.25
$\hat{\sigma}$	1	0.78
η	3	3
ϑ	0.75	0.85
ω	0.4	0.25
κ	0.401	0.113
ϕ	0.057	-0.0038
δ	0.4	-0.1299
σ	1.667	1.045
α	0.25	0.25

Table 1: Calibration

As shown by Carlstrom et al. (2015), in closed economy there is only one eigenvalue which is inside the unit circle, so the inflation rate explodes exponentially in the duration of forward guidance. We compute these eigenvalues calibrating the model along the lines of Leitemo and Söderström (2008) and Mihailov et al. (2011), for a calibrated version of Sweden and Spain, respectively, shown in Table 1. We consider in our simulations a range of values for the elasticity ϕ in the interval between -0.26 – 0.47, according to the estimates in Mihailov et al. (2011) to assess if it affects the presence of forward guidance puzzle. The values of interest for our findings in this section are given by $\sigma = 1.67$, $\beta = 0.99$, $\kappa = 0.401$ and $\delta = 0.4$, while in the next section we will also consider the calibrated version for Spain, with $\sigma = 1.045$, $\beta = 0.99$, $\kappa = 0.1131$ and $\delta = -0.1299$ (see Table 1).[5]

Since after T all the three endogenous variables are equal to zero, we can solve the system backwards from the end of the forward guidance regime. It can be shown that the solution of the difference equation (5) is given by

$$\pi_t = \frac{\frac{\kappa}{\sigma}\bar{r}}{\frac{\kappa}{\sigma} + \phi + \delta\kappa} + m_1 z_1^{T+1-t} + m_2 z_2^{T+1-t} \tag{8}$$

[5]The exact value of ρ is not relevant for the evaluation of the stability of the equilibrium.

where z_1 and z_2 are the two eigenvalues and the constants m_1 and m_2 come from the terminal conditions (6) – (7).

Simulating the model for the values of ϕ estimated by Mihailov et al. (2011), we show in Figure 1 that for $\phi > 0.40$, the two roots of (5) are less than one so that inflation dynamics are stable.[6] Analytically, this result can be understood by writing the relationship (5) as a second-order polynomial

$$h(w) \equiv w^2 - \Gamma w + \beta.$$

It can be easily seen that $h(0) > 0$, while $h(1) = -\kappa\delta + \phi - \frac{\kappa}{\sigma} < 0 \iff \phi < \kappa\delta + \frac{\kappa}{\sigma}$. Therefore, if $h(1) < 0$, we know that one eigenvalue is larger than one in absolute value while the other is less than one. This occurs surely if the elasticity of inflation with respect to the real exchange rate is negative, but not for all the positive values that ϕ can hinge. In particular, for values of $\phi > 0.40$, the two roots are identical and equal to 0.995, hence we can conclude that the forward guidance puzzle does not arise.

We now consider the role of frictions in international financial markets in determining explosive dynamics in endogenous variables. The first friction we introduce is proportional to excess return on domestic real bonds, as in Wieland (2012). More specifically, the UIP condition modifies in the following way:

$$E_t e_{t+1} - e_t = r_t - E_t \pi_{t+1} - (f_t - E_t f_{t+1}). \tag{9}$$

where f_t is the friction depending on the excess return on domestic real bonds through a factor τ:

$$f_t = \tau (r_t - E_t \pi_{t+1}). \tag{10}$$

[6]More in detail, for $\phi > 0.40$ we find that the roots are complex and to evaluate the stability of (5) we need to evaluate if $\sqrt{\left(\frac{\Gamma}{2}\right)^2 + \left(\frac{\sqrt{|\Gamma^2 - 4\beta|}}{2}\right)^2} < 1$

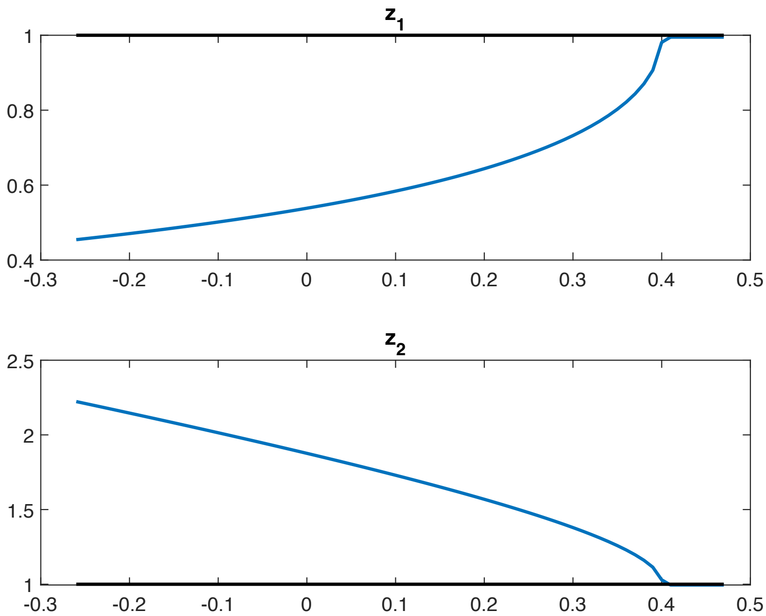

Figure 1: Roots of inflation varying the elasticity of inflation with respect to the real exchange rate

In the case of fixed interest rates and certainty, (9) therefore becomes

$$e_{t+1} = e_t + \bar{r} - (1-\tau)\pi_{t+1} - \tau\pi_{t+2}. \tag{11}$$

The latter equation, together with the IS and NKPC constitute a system with unknowns π_t, x_t and e_t. Following the methodology previously discussed, it is possible to derive the inflation dynamics during constant interest rate period driven by the following second order difference equation:

$$\pi_t = -\frac{\kappa}{\sigma}\bar{r} + \Gamma'\pi_{t+1} - [\beta + \tau(\phi - \delta\kappa)]\pi_{t+2} \quad \Gamma' \equiv 1 + \beta + (1-\tau)(-\phi + \delta\kappa) + \frac{\kappa}{\sigma}. \tag{12}$$

When $\tau > 0$ the friction will limit the movement of the terms of trade and thus the exchange rate relative to the baseline model. In Wieland (2012), the friction is dependent on central bank's response coefficient to inflation and on the probability of entering a zero lower bound episode, denoted by p. In particular, denoting with ϕ_π the response to inflation in a standard Taylor rule, the friction τ is given by

$$\tau = \frac{\phi_\pi - 1}{\phi_\pi - p}.$$

Therefore, we can think that in normal times ($\phi_\pi > 1$), the real exchange rate is expected to depreciate when the real interest rate increases, while it is expected to appreciate under a liquidity trap when $\tau = p^{-1}$. Notice that $(1-p)^{-1}$ can be interpreted as the duration of constant interest rate policy such as forward guidance. Here we consider a regime of fixed interest rate denoted so that ϕ_π is replaced by $\bar{r}$, while p is interpreted in terms of a known duration of forward guidance period. We set $\bar{r} = 10\frac{1-\beta}{\beta}$ (approximately equal to 10 basis points[7]) and we let $p \in [0.1, 0.99]$ corresponding to a forward guidance duration of $T = (1-p)^{-1}$. As a consequence, τ varies between $\frac{1-\beta}{\beta}$ and 111. The value of $\phi = 0.0572$ comes from the baseline calibration in Leitemo and Söderström (2008). Figure 2 shows that one root is always in the unit circle, while the second root is below unity approximately for $\tau > 20$, so that we can conclude that large frictions in the UIP do not induce forward guidance puzzle. In terms of duration of fixed interest rates, large frictions correspond to the case of a very short duration, slightly larger than one quarter. However, if we increase the size of inflation elasticity with respect to the real exchange rate, by taking the extremes of the interval used before, we find that the cases of stability increase dramatically, as shown in Figure 3. In particular, it can be seen that inflation is always explosive for an intermediate value of the elasticity (dashed line), while with a negative elasticity (solid line) it is always stable. Finally, for high and positive elasticity (circled line) inflation is stable, consistently with what shown in Figure 1.

Recently, Galí (2020) has used convex portfolio adjustments as in Bacchetta and Van Wincoop (2019) to explain forward guidance puzzle in open economy, assuming constant inflation rates. More in detail, the idea of Bacchetta and Van Wincoop (2019) is that no arbitrage condition in financial

[7]This value is chosen without loss of generality to get values easy to show graphically.

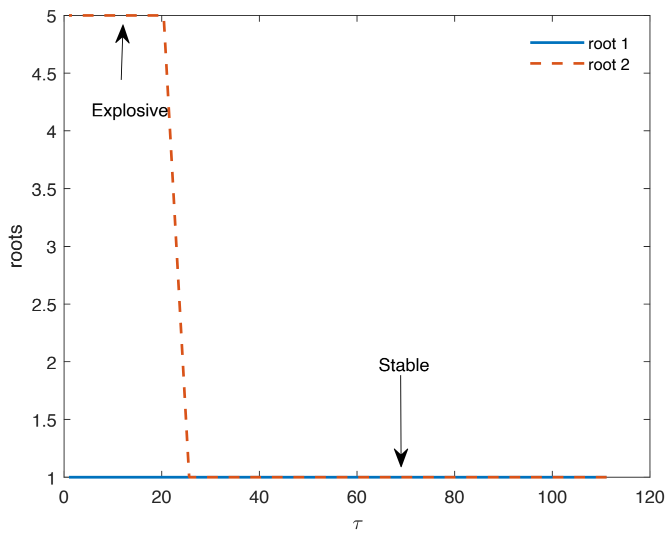

Figure 2: Roots of inflation in the model with frictions in the UIP.

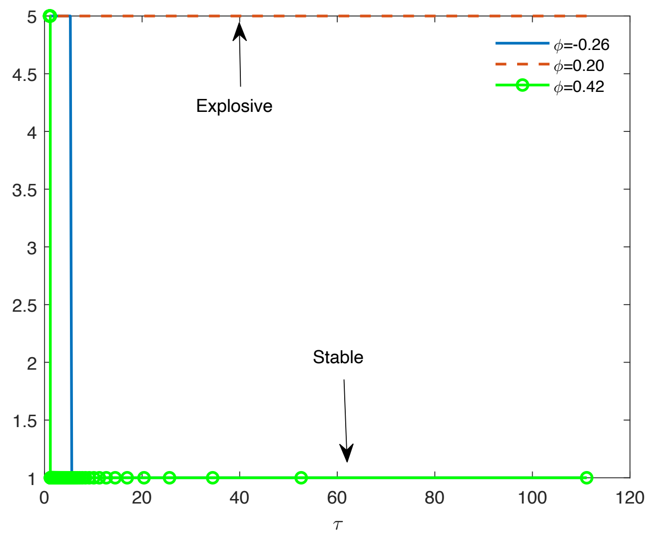

Figure 3: The potentially explosive root of inflation in the model with frictions in the UIP varying ϕ.

markets is a modified version of the UIP:

$$E_t e_{t+1} - \theta e_t + b\psi e_{t-1} + i_t - E_t \pi_{t+1} = 0, \tag{13}$$

where the lagged term in the real exchange rate depends on the adjustment costs (otherwise $\psi = 0$), $\theta = 1 + b\psi + \sigma\nu^2 b$ where ν^2 is the variance of the real exchange rate and $b \in [0.25, 0.1875]$ is a parameter related to the degree of home bias. Due to the adjustment costs, the solution during the period with forward guidance is characterized by four eigenvalues: according to our calibrated values, taken from the previous analysis and from Bacchetta and Van Wincoop (2019), two eigenvalues are outside the unit circle, hence in this case we have forward guidance puzzle.[8]

4 Forward guidance with stochastic duration

Now we consider a stochastic version of the model where also the duration of forward guidance is stochastic. In doing that, we will first consider a special case, isomorphic to closed economy and then we will study a more general version where analytical solutions become cumbersome.

4.1 Analytical solution for a special case

In this section we derive the solution for a special case of the model. In particular, we consider a version isomorphic to the closed–economy model, as in Gali and Monacelli (2005).[9] Analytically, this setup can be obtained by setting $\delta\hat{\sigma} = (1-\omega)(1-2\omega)$. In such a case we can focus only on the new Keynesian Phillips curve and the IS curve respectively

[8] In particular, the values taken from Bacchetta and Van Wincoop (2019) are $\psi = 15$, $b = 0.25$ and $\nu = 0.0271$. Also changing some of these values or the value of ϕ we have an explosive dynamics. Computational details are available upon request.

[9] More in detail, Gali and Monacelli (2005) consider such a setup to derive a computationally easy version of welfare-based loss function.

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t, \tag{14}$$

$$x_t = E_t x_{t+1} - \frac{1}{\sigma} \left(i_t - E_t \pi_{t+1} - \rho_t \right). \tag{15}$$

while the UIP condition (4) can be treated separately from the two other equations above. As already said before, with the objective of characterizing a relatively simple solution, we are not considering other shocks different from shocks to the natural rate ρ_t. Due to the isomorphism of the model to closed economy, we can solve for inflation and output gap separately from the real exchange rate.

We assume that the central bank performs a forward guidance exercise where it is fully credible. As in Bilbiie (2019), we model forward guidance stochastically through a Markov chain as a state of the world with a probability distribution of p for the liquidity trap to happen. Consequently the expected stochastic duration of the liquidity trap is $T_L = (1-p)^{-1}$ which is the stopping time of the Markov chain. ρ_t is following Markov chain of 3 states, one first state is the steady state S where $\rho_t = \rho$ and once reached, there is a probability 1 of staying there. The second state is the liquidity trap, being transitory, denoted by L where $r_t = 0$ and $\rho_t = \rho_L < 0$ with persistence probability p. After this time T_L, the CB sets $r_t = 0$ while $\rho_t = \rho > 0$, with probability q. The probability to move back to steady state from F is $1-q$. We denote this state F, with expected duration $T_F = (1-q)^{-1}$. Therefore, we have the following three states of the world:

1. Liquidity trap L, with $r_t = 0$ and $\rho_t = \rho_L$. The economy remains in this state with probability p and arrives to the state of forward guidance with probability $(1-p)q$.

2. Forward guidance F, with $r_t = 0$ and $\rho_t = \rho$. The economy is in this state with probability q and goes back to the steady state with probability $1-q$.

3. Steady state S with $r_t = \rho_t = \rho$ (absorbing state).

Given these assumptions, we can write the following expectations for our endogenous variables:

$$E_t x_{t+1} = p x_L + (1-p) q x_F,$$
$$E_t \pi_{t+1} = p \pi_L + (1-p) q \pi_F.$$

Following the methodology in Bilbiie (2019), we first solve for the state F and then for the state L. In state F we must solve the following system in x_F and π_F:

$$\pi_F = \beta q \pi_F + \kappa x_F, \tag{16}$$

$$x_F = q x_F - \frac{1}{\sigma}\left(-q\pi_F - \rho\right). \tag{17}$$

From (16) we can obtain π_F which, combined with (17) gives us

$$x_F = q x_F - \frac{1}{\sigma}\left(-\frac{q\kappa}{1-\beta q} x_F - \rho\right).$$

Solving the previous expression and remembering the relationship with π_F we get the following pair of values for the state F:

$$x_F = \frac{1-\beta q}{\sigma(1-q)(1-\beta q) - q\kappa}\rho, \tag{18}$$

$$\pi_F = \frac{\kappa}{\sigma(1-q)(1-\beta q) - q\kappa}\rho. \tag{19}$$

Now we consider the state of the world L. In this case the system to solve becomes:

$$x_L = p x_L + (1-p) q x_F - \frac{1}{\sigma}\left(-p\pi_L - (1-p) q \pi_F - \rho_L\right),$$
$$\pi_L = \beta p \pi_L + \beta(1-p) q \pi_F + \kappa x_L.$$

From the second equation we can get $\pi_L = \frac{\beta q(1-p)\pi_F + \kappa x_L}{1-\beta p}$, plug it into x_L and solve for it to get

$$x_L = \frac{(1-\beta p)}{[\sigma(1-p)(1-\beta p)-\kappa p]}\rho_L + \frac{q(1-p)[\kappa+\sigma(1-\beta q)(1-\beta p)]}{[\sigma(1-p)(1-\beta p)-\kappa p][\sigma(1-\beta q)(1-q)-q\kappa]}\rho. \tag{20}$$

After finding the value of x_L, we can compute π_L

$$\pi_L = \frac{1-p}{\sigma(1-p)(1-\beta p)-p\kappa}\left\{\frac{\kappa}{1-p}\rho_L + \frac{1-\beta q+\kappa[\sigma\beta q(1-p)+\kappa p]}{\sigma(1-\beta q)(1-q)-q\kappa}\rho\right\}, \tag{21}$$

and work on the UIP equation to find the solution for the real exchange rate. In particular, since we are considering a model with full–risk sharing and unitary elasticity of substitution between domestic and foreign goods, we can decouple the solution for the real exchange rate from the solution for inflation and output gap. The solution for the real exchange rate in the two states come from the following system:

$$e_F = qe_F + \pi_F, \tag{22}$$

$$e_L = pe_L + (1-p)qe_F + p\pi_L + (1-p)q\pi_F. \tag{23}$$

It is possible to show the solutions for the exchange rate in the forward guidance state and under a liquidity trap respectively[10]:

$$e_F = \frac{\kappa}{\sigma(1-q)^2(1-\beta q)-q\kappa}\rho, \tag{24}$$

$$e_L = \frac{q\kappa}{\sigma(1-q)^2(1-\beta q)-q\kappa}\rho + \frac{p}{\sigma(1-p)(1-\beta p)-p\kappa}\left\{\frac{\kappa}{1-p}\rho_L + \frac{1-\beta q+\kappa[\sigma\beta q(1-p)+\kappa p]}{\sigma(1-\beta q)(1-q)-q\kappa}\rho\right\} + \frac{q\kappa}{\sigma(1-q)(1-\beta q)-q\kappa}\rho, \tag{25}$$

In order to derive sharper analytical conditions, we consider the same special

[10]In the L state, we use the fact that $e_L = qe_F + \frac{p}{1-p}\pi_L + q\pi_F$.

case of Bilbiie (2019), namely $\beta = 0$. In particular, the effect of forward guidance duration on output gap is positive for a critical value of q:

$$\frac{\partial x_F}{\partial q} = \frac{\sigma + \kappa}{(\sigma(1-q) - q\kappa)^2}\rho = \frac{\sigma + \kappa}{\sigma(1-q) - q\kappa} x_F > 0 \quad \iff q < \frac{\sigma}{\sigma + \kappa}. \tag{26}$$

Taking into account the relationship of π_F with x_F, we can conclude that forward guidance determines output and inflation expansion, together with real depreciation. In open economy movements in the exchange rate have two competing effects. On the one hand, a real exchange rate depreciation increases CPI inflation. The consequent reduction in real wage induces firms to increase nominal wages, which, in turn, determines an increase in marginal costs and inflation. On the other hand, an exchange rate depreciation leads to a decrease in the demand for imports and therefore a reduction in aggregate consumption. Since the marginal rate of substitution falls, real wages and marginal cost decrease. The final effect on inflation will depend on which of the two effects is stronger. However, in the case analyzed here, the two effects offset and the exchange rate movements do not affect inflation. Similarly, there are two competing effects that exchange rate produces on output. On the one hand, an exchange rate depreciation raises real interest rate, decreasing consumption. On the other hand, an exchange rate depreciation increases exports and therefore output. Again, for the case under scrutiny here, these two effects are the same and the exchange rate dynamics do not affect output. In this model, therefore, even if we observe real depreciation, there is not a transmission of it to the real variables and closed-economy results apply also in open economy. However, if we abandon the assumption of unitary elasticity of substitution between domestic and foreign goods, this mechanism does not work: real exchange rate becomes another endogenous variable that cannot be decoupled from output and inflation. Therefore, there is an open economy channel that affects monetary policy transmission.

As to the capacity of forward guidance to reduce the effects of liquidity trap, we have:

$$\frac{\partial x_L}{\partial q} = \frac{\sigma(1-p)}{\sigma(1-p) - p\kappa}\left[\frac{(1+\kappa)}{\sigma(1-q) - q\kappa}\rho + q\left(\frac{\partial x_F}{\partial q} + \frac{\partial \pi_F}{\partial q}\right)\right]. \tag{27}$$

To understand the effect of forward guidance on output in state L, provided that(26) holds, a similar condition on p ($p < \frac{\sigma}{\sigma+\kappa}$) is sufficient to guarantee that output gap in state L is increasing in q. In such a case, more forward guidance leads to larger output gap, inflation and real exchange rate depreciation (the latter two move in the same direction as output). Moreover, the model exhibits forward guidance puzzle since[11]

$$\frac{\partial^2 x_L}{\partial q \partial p} = \frac{\sigma^2(\kappa+\sigma)}{[\sigma(1-q) - q\kappa]^2[\sigma(1-p) - p\kappa]^2}\rho > 0. \tag{29}$$

The previous analysis shows that, in presence of unitary substitution between domestic and foreign goods, forward guidance implies a rise in domestic output gap accompanied by a depreciation in the real exchange rate if condition (26) holds.

We compute some simulations for this version of the model isomorphic to closed economy. In doing that, we consider a value for the Phillips curve slope $\kappa = 0.02$, which is more or less between the two values for Sweden and Spain summarized in Table 1. Such a value is similar to what used by Bilbiie (2019). Moreover, we compute the equilibrium with $\beta = 0$ or $\beta = 0.99$ for the more general case where analytical results are more complicated. Finally, we set the probability of being in a liquidity trap $p = 0.8$, $\rho = 0.01$ and $\rho_L = -0.01$. Given our calibration, the threshold condition (26) requires $q < 0.98$, hence we can say that for all the values considered in our simulation for q we obtain, for the case $\beta = 0$, that the three endogenous variables are increasing in the size of the forward guidance duration. In the more general case of

[11] This result derives from another way to derive the effect of larger q on x_L

$$\frac{\partial x_L}{\partial q} = \frac{(\kappa+\sigma)(1-p)\sigma\rho}{[\sigma(1-p) - \kappa p][\sigma(1-q) - q\kappa]^2} > 0. \tag{28}$$

Then you can compute from (28) the effect of forward guidance in a liquidity trap.

β different from zero, and more specifically $\beta = 0.99$, the condition that guarantees expansionary effect of forward guidance becomes more complex. In particular, it can be easily shown that expansionary effects occur whenever the following condition holds

$$\sigma(1-q)(1-\beta q) > q\kappa.$$

The latter inequality implies that analytically there are two values for q that guarantee that forward guidance raises output, inflation and real exchange rate. However, provided that q must be lower than one, we find numerically that the only admissible value for q is $q < 0.87$ for Sweden, while for Spain we have $q < 0.75$.[12]

For the case $\beta = 0.99$, represented in figures 4–5, in the state F, we can observe that the variables barely move while they respond much more for large value of q (specifically, for $q > 0.8$). Under a liquidity trap, we observe a similar path, with a larger response of the three variables when q is close to 0.8. Interestingly, forward guidance is not monotonically expansionary, in fact after $q = 0.87$, we observe deflation associated with an appreciation and a recession.

This may suggest that there could be a level of forward guidance under a liquidity trap that closes the gaps in the endogenous variables. In particular, it is possible to derive the value of q, labelled q^0, that implies zero output gap. Given the isomorphic structure to the closed economy that we are considering, the value that closes the output gap is very close (up to some different definition in the structural equations) to that derived in Bilbiie (2019) in the case of $\beta = 0$[13]. Even if q^0 is able to close also the inflation

[12] This value corresponds to the lowest solution of the expression, given by $\frac{\kappa+(1+\beta)\sigma-\sqrt{[\kappa+(1+\beta)\sigma]^2-4\beta\sigma^2}}{2\beta\sigma}$.

[13] In particular, this value for q^0 is given by

$$q^0 = \frac{\sigma\Delta_L}{1-p+\Delta_L(\kappa+\sigma)} \quad \Delta_L \equiv -\frac{\rho_L}{\rho}$$

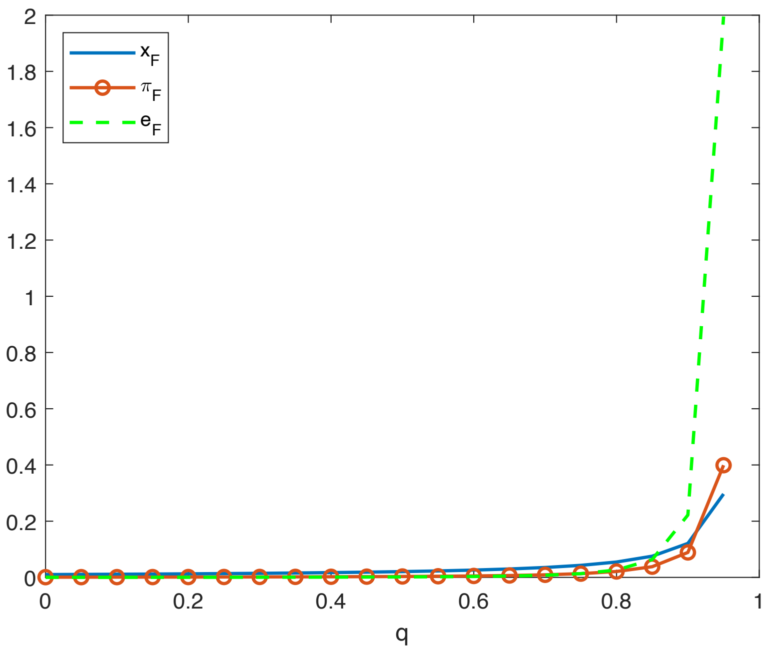

Figure 4: Output gap (solid line), inflation (circle line) and real exchange rate (dashed line) during forward guidance ($\beta = 0.99$).

gap, this arrives at the cost of fluctuations in the real exchange rate. This analysis confirms that the value derived for q^0 is not optimal, because it does not take into account future costs of forward guidance in state F, as in closed economy, but also because closing inflation and output gaps come at the cost of haing real exchange rate volatility. As long as we have complete markets, fluctuations in the real exchange rate probably do not affect welfare losses. As highlighted by De Paoli (2009), under complete markets and unitary elasticity of substitution between domestic and foreign goods, the dynamics of the small open economy are independent of the asset market structure. In such a case, the flexible price equilibrium is optimal and, absent markup shocks, a policy of complete domestic price stabilization closes the welfare relevant output gap. However, this is not the case when we abandon the assumption of unitary elasticity of substitution: in such a case the central bank should care about real exchange rate variability. In Section 4.3, we will focus on a more general version of the small-open economy model, breaking

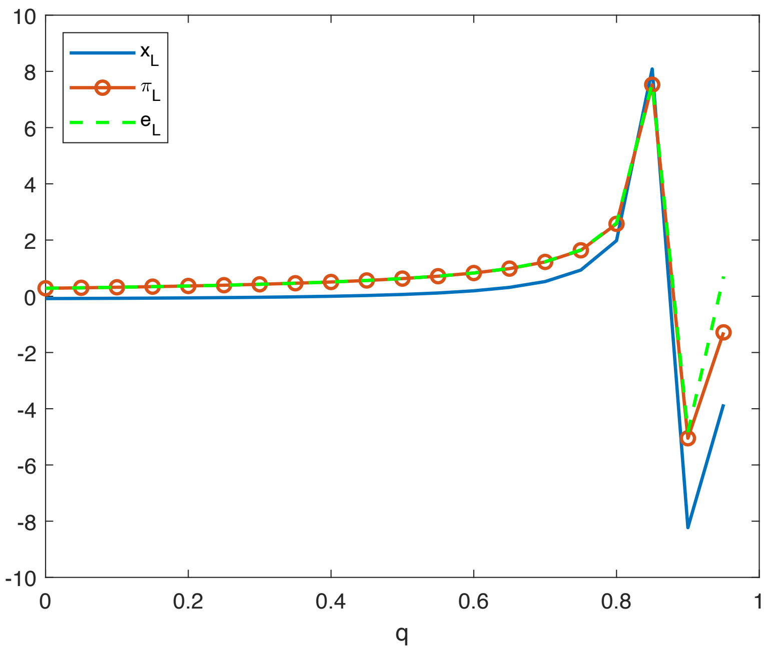

Figure 5: Output gap (solid line), inflation (circle line) and real exchange rate (dashed line) during LT ($\beta = 0.99$).

the isomorphism assumed here. In doing this, we highlight the different transmission mechanism through *i)* closed versus open economy, and *ii)* on the nature of the exchange-rate pass-through on inflation.

4.2 Observing marginal effects: A special case with a contemporaneous Phillips curve

Here we consider a model with contemporaneous Phillips curve (i.e. $\beta = 0$) to characterize analytically how exchange rate passthrough and duration of the policy affect the transmission of forward guidance.

These three above equations remain positive for $q \leqslant 0.84$ for the Swedish economy setup, where $\phi = 0.057$, and for $q \leqslant 0.91$ for the Spanish economic setup, where $\phi = -0.0038$. To observe, respectively, the net effect of ϕ and q on inflation, the output gap and the exchange rate, we calculate the derivatives[14].

Marginal effect of pass-through. Here we study the effect of an increase in the exchange-rate pass-through in the Phillips curve. We show analytically that an increase in the exchange-rate pass-through in the Phillips curve, ϕ, always yields higher inflation, a higher output gap and the depreciation of the exchange rate, everything else being equal.

$$\frac{\partial \pi_F}{\partial \phi} = \frac{\kappa q \sigma}{\left[\sigma\left(1-q\right)-\kappa q+\phi q \sigma\right]^2}\rho > 0,$$
$$\frac{\partial x_F}{\partial \phi} = \frac{q \sigma}{\left[\sigma\left(1-q\right)-\kappa q+\phi q \sigma\right]^2}\rho > 0,$$
$$\frac{\partial e_F}{\partial \phi} = \frac{\kappa q^2 \sigma}{\left[\sigma\left(1-q\right)-\kappa q+\phi q \sigma\right]^2}\rho > 0.$$

In state L, we have $\frac{\partial \pi_L}{\partial \phi} = \frac{\sigma \beta q (1-p)^2}{\sigma(1-p)(1-\beta p)-\kappa p}\frac{\partial \pi_F}{\partial \phi} > 0$ if and only if $\sigma\left(1-p\right)\left(1-\beta p\right)-\kappa p > 0$. The same condition guarantees that also output gap and exchange rate are increasing in ϕ[15]. Also in state L, a higher exchange rate pass-through in absolute value leads to stronger response of the variables, confirming the above observation: forward guidance is more expansionary in open economy, when the condition $\sigma\left(1-p\right)\left(1-\beta p\right)-\kappa p > 0$ holds, and its effect becomes even larger if the exchange rate pass-through is high in absolute value, due to the greater effect of depreciation on activity and thus on aggregate demand.

Marginal effect of forward guidance duration. We now want to see in this simplified model with contemporaneous Phillips curve if the forward guidance duration also produces different effects depending on the state the economy is currently in. The longer the forward guidance, the higher inflation and output gap are, where $\frac{\partial \pi_F}{\partial q}$ and $\frac{\partial x_F}{\partial q}$ are positive if and only if $\sigma(1-\phi)+\kappa$

[14] Further explanations can be found in section 2 before the IS equation.

[15] The derivatives are $\frac{\partial x_L}{\partial \phi} = \frac{p}{\sigma(1-p)}\frac{\partial \pi_L}{\partial \phi} > 0$ and $\frac{\partial e_L}{\partial \phi} = \frac{p}{(1-p)}\frac{\partial \pi_L}{\partial \phi} + \frac{\sigma \kappa q^2 (1-q)}{(1-q)^2[\sigma(1-q)+q(\kappa-\sigma\phi)]^2}\rho > 0$.

is positive, which is always true for our calibration setups. As to the effect on the exchange rate, we will have a depreciation when $\sigma + q^2 (\phi\sigma - \sigma - \kappa) > 0$, which occurs for $q < 0.92$ in our set-up for Sweden with negative exchange-rate pass-through and for $q < 0.96$ in the set up for Spain. Therefore, we could conclude that this effect is the one observed in our analysis.

This would be equivalent to a depreciation after 25 quarters of forward guidance in countries (like Spain) characterized by a positive exchange rate pass-through and 12.5 quarters (over 3 years) for countries like Sweden with a negative exchange rate pass-through. We can thus deduce that staying too long in a forward guidance can trigger a long period of currency depreciation for the economy, but it takes more time to observe the currency depreciation when the exchange rate pass-through is positive.[16]

We now focus our interest on the liquidity trap state and the effect of a movement in q on the state variables. We use the expressions for inflation, the output gap and the exchange rate given in Appendix (30-32). We first compute the derivatives of equations (30)-(32) with respect to q. In the Appendix we present the marginal effects analytically: in general terms we can conclude that the necessary condition for $\frac{\partial \pi_F}{\partial q}$ and $\frac{\partial x_F}{\partial q}$ to be positive is sufficient for $\frac{\partial \pi_L}{\partial q}$ and $\frac{\partial eL}{\partial q}$ to be positive whereas $\frac{\partial x_L}{\partial q} > 0$ is always positive for any calibration. A positive $\frac{\partial x_F}{\partial q}$ is sufficient for $\frac{\partial \pi_L}{\partial q} > 0$ and $\frac{\partial eL}{\partial q} > 0$ to be positive whereas $\frac{\partial x_L}{\partial q}$ is always positive for any calibration when $\beta = 0$[17].

To sum up, in the simplified version of the model, according to our calibrated exercises, forward guidance in a liquidity trap always has a positive effect on inflation and output gap and it induces a real depreciation.

16 $\frac{\partial \pi_F}{\partial q} = \frac{\kappa[\sigma+\kappa-\phi\sigma]}{[\sigma(1-q)-\kappa q+\phi q\sigma]^2}\rho > 0$, $\frac{\partial x_F}{\partial q} = \frac{\sigma+\kappa-\phi\sigma}{[\sigma(1-q)-\kappa q+\phi q\sigma]^2}\rho > 0$ and $\frac{\partial e_F}{\partial q} = \frac{\kappa\left\{\sigma+q^2(\phi\sigma-\sigma-\kappa)\right\}}{(1-q)^2[\sigma(1-q)-\kappa q+\phi q\sigma]^2}\rho > 0$.

17 $\frac{\partial \pi_L}{\partial q} > 0$ if and only if $\sigma(1-\phi) + \kappa(1+\delta\sigma) > 0$, then $\frac{\partial x_L}{\partial q} > 0 \Leftrightarrow q^2 > -\frac{\sigma}{[\kappa-\sigma(1+\phi)]}$, which is always true and finally $\frac{\partial e_L}{\partial q} > 0 \Leftrightarrow \kappa > \sigma\phi$. If the condition on the sign of $\frac{\partial e_L}{\partial q}$ holds, then the sign of $\frac{\partial \pi_L}{\partial q}$ is always positive.

4.3 Forward Guidance for a general version of the model

Here we compute the value of inflation, output gap and real exchange rate for a general version of the model, since in the previous sections we made some simplifying assumptions in terms of contemporaneous Phillips curve or isomorphism to closed–economy. Even if it is still possible to solve the model analytically, computation becomes cumbersome, and we decide just to describe the equilibrium outcomes without reporting here the exact analytical expressions.

A natural question that arises is if forward guidance is more or less effective in open economy. Figure 6 shows the value of inflation and output gap, both in state F and in state L, for the open–economy case (with $\phi = 0.0572$, as for Sweden, circled lines) and for the closed–economy case (solid line), varying the probability of forward guidance q. The graph shows that in state F forward guidance is more expansionary in open economy: we interpret this result by looking at the Phillips' curve slope and interest rate elasticity of the output gap (σ and κ respectively), which are larger than in closed economy. The other factor explaining the more expansionary effect in open economy is the exchange rate depreciation which boosts aggregate demand. Moreover, in closed economy a shorter period of forward guidance ($0 < q < 0.5$) is sufficient for the economy to reach the largest expansion for output in normal times.

On the other hand, in state L, the path followed by output gap and inflation presents more differences across open and closed economy. While in closed economy the effect is almost muted up to approximately before $q = 0.5$ and then we observe a through followed by a peak, in open economy the effect is globally more expansionary (as already discussed for state F) and the troughs are sensibly lower. More in detail, the economy experiences a peak for $q = 0.65$, then we observe a decrease with inflation and output gap going into negative territory. Again, there is a key contribution of the real exchange rate: when it appreciates the economy enters in a deflation and a recession. We now consider the role of exchange rate pass-through in

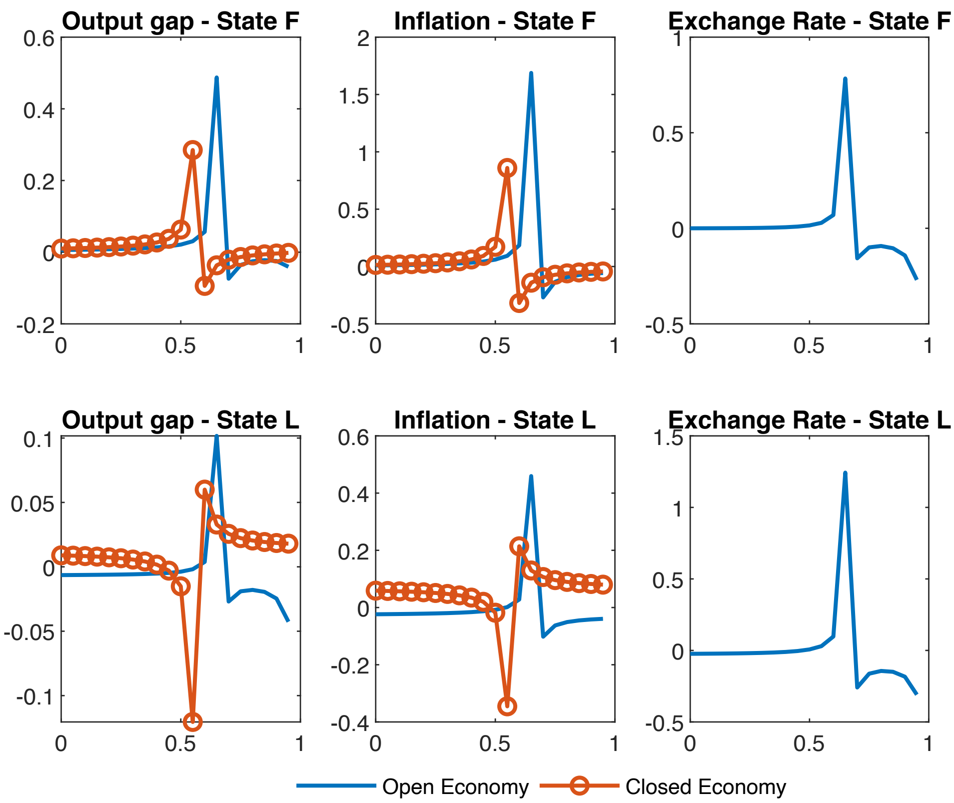

Figure 6: Comparison of forward guidance between open and closed economy.

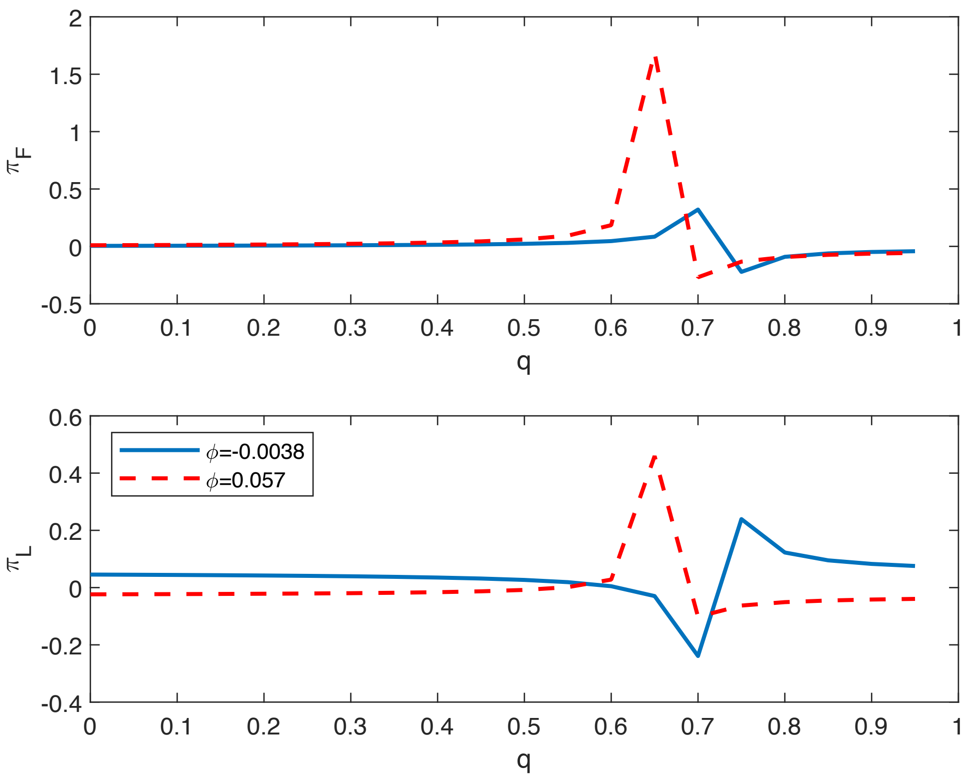

Figure 7: Inflation level for different ϕ corresponding to distinct economies: Spain ($\phi = -0.0038$, solid line), and Sweden ($\phi = 0.057$, dashed line).

determining these findings.

In the forward guidance experiment shown Figures 7–9, the variables evolve in the same direction whether the economy has a positive or a negative exchange rate pass-through. In state F, the largest exchange rate depreciation, obtained between $0.6 \leq q < 0.7$ goes hand-in-hand with a peak in inflation and output gap for the case of a positive exchange rate pass-through (dashed line). For the case of Spain, i.e negative exchange rate pass-through, we observe that we should at least engineer a forward guidance duration of 2.5 quarters to obtain a response of the exchange rate that depreciates in a small interval between $0.6 < q < 0.7$. Also in this case, the depreciation is associated with an expansion of output gap and inflation. Overall, the effect of forward guidance policy for Spain are lower compared to Sweden because

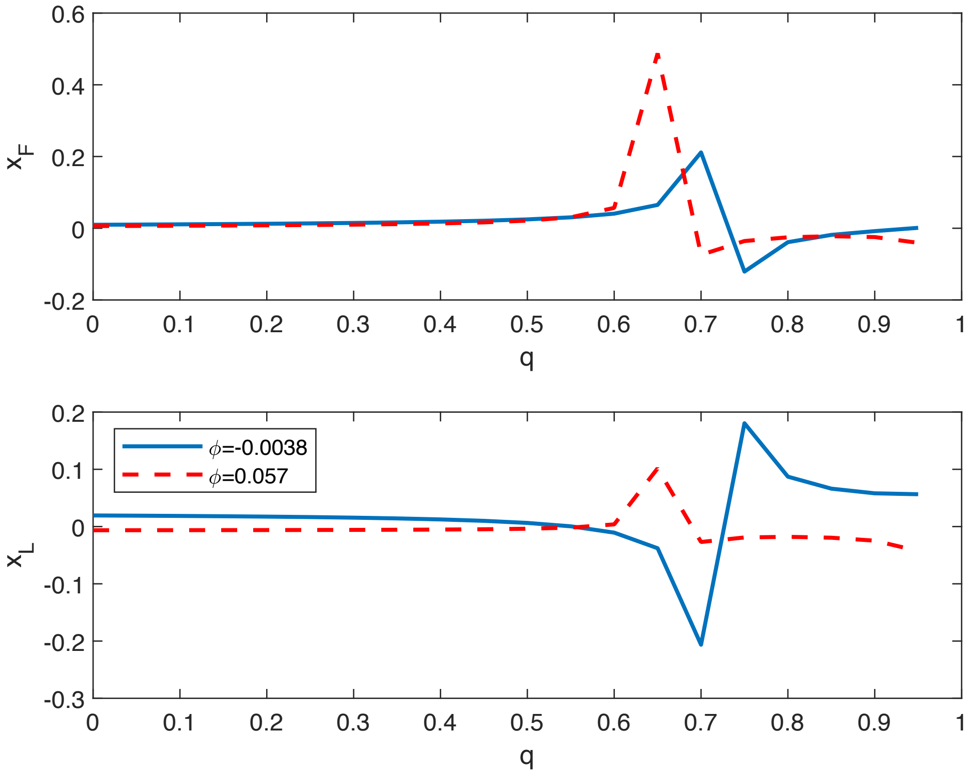

Figure 8: Output gap level for different ϕ corresponding to distinct economies: Spain ($\phi = -0.0038$, solid line), and Sweden ($\phi = 0.057$, dashed line).

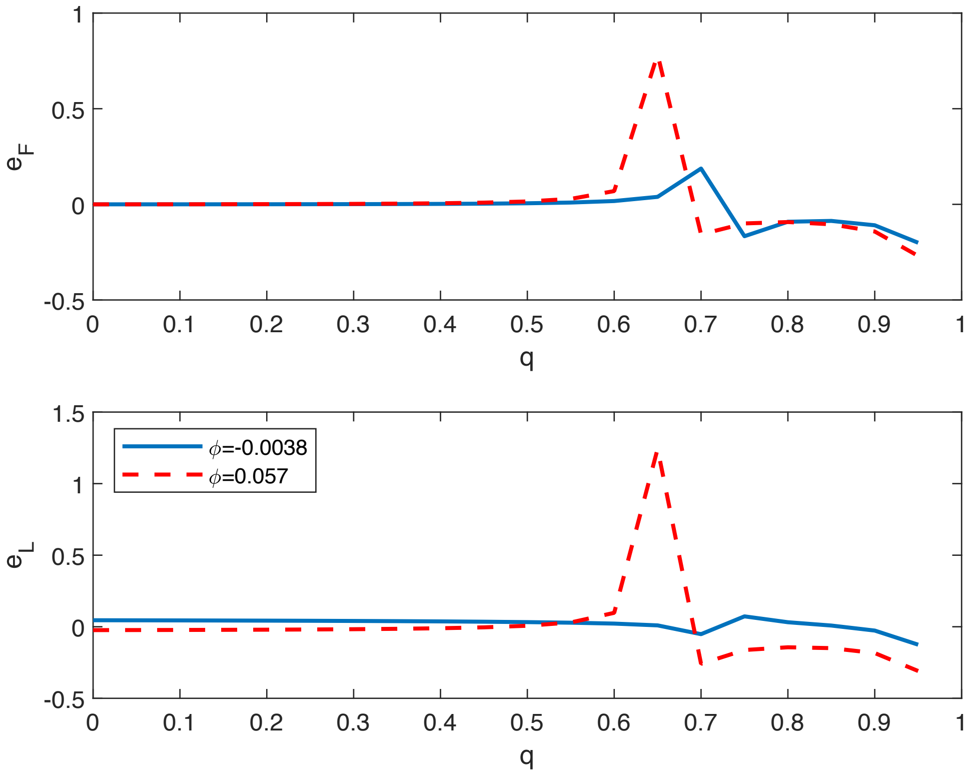

Figure 9: Exchange rate level for different ϕ corresponding to distinct economies: Spain ($\phi = -0.0038$, solid line), and Sweden ($\phi = 0.057$, dashed line).

of the much lower exchange rate pass-through[18].

As to the effects of forward guidance in a liquidity trap, as in the previous case, the response of the three variables becomes sizable for values of $q \geq 0.6$. However, under liquidity trap, inflation in Sweden is characterized by a peak and then it does not move substantially, while for Spain we observe that there is a trough (more or less when there is the peak for Sweden), followed by a peak and then inflation remains positive. Output gap moves in the same direction for both countries, while the real exchange rate depreciates significantly but temporarily only for the case of a larger exchange rate pass-trough.

Overall, these results confirm that the exchange rate pass-through is a key variable. A forward guidance policy determines a depreciation of domestic currency which raises domestic consumer prices and reduces the real wage for a given nominal wage. Given households' marginal rate of substitution between leisure and consumption, households supply less labor and enjoy more leisure. Therefore firms must increase the real wage to offset the reduction in the households' real wage, leading to higher marginal cost and inflation. Meanwhile, the depreciation increases the relative price of foreign goods in terms of domestic goods, which makes domestic goods more attractive. Domestic activity is stimulated, which strengthens inflationary pressures in the case of a positive (and relatively much larger in absolute value) exchange rate pass-through. Under liquidity trap, we have cases with small pass-through in which output gap expansion is larger than the case of high pass-through but the role played by exchange rate movements is not relevant.

5 Conclusion

This paper studies forward guidance in a theoretical DSGE small open economy. We show that the elasticity of inflation to the real exchange rate is a

[18]Moreover, if we calibrate the economy using the data for for Spain, we could show that forward guidance turns out to be more expansionary in closed economy.

key variable in limiting the implausible overreaction of inflation and output gap to periods of fixed interest rate. When the duration of forward guidance is stochastic, the expansionary effect of the policy is positively related to the exchange rate pass-through and larger than in the closed economy counterpart because of a better inflation-output trade-off and the exchange rate channel. These findings generally hold also in the case in which forward guidance is implemented during a liquidity trap.

Our analysis suggests that a small-open economy model can produce more reasonable dynamics without recurring to the assumptions typically used in the literature to correct the forward-guidance puzzle, such as sticky information, incomplete markets and perpetual youth. Several extensions to our setup can be considered. First, we do not analyze optimal forward guidance and in particular how it is related to the open economy dimension. Second, with incomplete information set available to the central bank, there might be an attenuation of the forward guidance puzzle. Finally, we have abstracted from fiscal shocks and on how the interaction between monetary and fiscal policy modifies the transmission of forward guidance. We leave these questions for future research.

A Effects of forward guidance in liquidity trap

Calculations in liquidity trap

$$\begin{aligned}\pi_L &= \frac{\kappa q\left(1-p\right)^2\left[\sigma\left(1-\phi\right)+\kappa\left(1+\delta\sigma\right)\right]}{\left\{\sigma\left[\left(1-q\right)+q\phi\right]-\kappa q\right\}\left\{\sigma\left(1-p\right)\left[1-p\left(1-p\phi\right)\right]-\kappa p\left(1-p\right)+\delta\kappa p^2\sigma\right\}}\rho \\ &+\frac{\kappa\left(1-p\right)\left\{\sigma\left[\left(1-q\right)+q\phi\right]-\kappa q\right\}}{\left\{\sigma\left[\left(1-q\right)+q\phi\right]-\kappa q\right\}\left\{\sigma\left(1-p\right)\left[1-p\left(1-p\phi\right)\right]-\kappa p\left(1-p\right)+\delta\kappa p^2\sigma\right\}}\rho_L\end{aligned}$$

$$\begin{aligned}x_L &= \frac{q\left(1-q\right)\left\{\kappa\left(1-p\right)^2+\sigma\left(1-p\right)^2\right\}+q(1-q)\delta\kappa\sigma\left[\left(1-p\right)^2+p\phi\right]}{\left(1-q\right)\left[\sigma(1-q)-\kappa q+\phi q\sigma\right]\left[\sigma\left(1-p\right)^2-\kappa p\left(1-p\right)+p\phi\sigma\left(1-p\right)+\delta\kappa p^2\sigma\right]}\rho \\ &+\frac{\phi pq\left\{\kappa(1+p-q)+\sigma\left[-p\left(1-q\right)^2-3q+\phi q\left(1-p\right)+\delta\kappa pq\right]\right\}+\phi p^2\sigma}{\left(1-q\right)\left[\sigma(1-q)-\kappa q+\phi q\sigma\right]\left[\sigma\left(1-p\right)^2-\kappa p\left(1-p\right)+p\phi\sigma\left(1-p\right)+\delta\kappa p^2\sigma\right]}\rho \\ &+\frac{\left[\sigma\left(1-q\right)^2+\kappa q^2\right]\left(1-p\right)+\kappa pq+\phi\left(1-q\right)\left[\sigma p\left(1-q\right)+\sigma q\left(1-p\right)-pq\left(\kappa-\phi\sigma\right)\right]}{\left(1-q\right)\left[\sigma(1-q)-\kappa q+\phi q\sigma\right]\left[\sigma\left(1-p\right)^2-\kappa p\left(1-p\right)+p\phi\sigma\left(1-p\right)+\delta\kappa p^2\sigma\right]}\rho_L\end{aligned}$$

$$\begin{aligned}\Rightarrow e_L &= \frac{\kappa q}{\left(1-q\right)\left[\sigma\left(1-q\right)+q\left(\kappa-\sigma\phi\right)\right]}\rho \\ &+\frac{\kappa pq\left(1-p\right)\left[\sigma\left(1-\phi\right)+\kappa\left(1+\delta\sigma\right)\right]}{\left[\sigma\left(1-q\right)+q\left(\kappa-\sigma\phi\right)\right]\left\{\sigma\left(1-p\right)\left[1-p\left(1-p\phi\right)\right]-\kappa p\left(1-p\right)+\delta\kappa p^2\sigma\right\}}\rho \\ &+\frac{\kappa p}{\left\{\sigma\left(1-p\right)\left[1-p\left(1-p\phi\right)\right]-\kappa p\left(1-p\right)+\delta\kappa p^2\sigma\right\}}\rho_L\end{aligned}$$

Derivatives in liquidity trap We now focus our interest on the liquidity trap state and the effect of a movement in q or ϕ on the state variables, in the absence of shocks. We use the definitions for inflation, the output gap and the exchange rate given in Appendix.

$$\begin{aligned}\pi_L &= \frac{\kappa q\left(1-p\right)^2\left[\sigma\left(1-\phi\right)+\kappa\left(1+\delta\sigma\right)\right]}{\left\{\sigma\left[\left(1-q\right)+q\phi\right]-\kappa q\right\}\left\{\left(1-p\right)\left[\sigma\left(1-p\right)+\sigma p^2\phi-\kappa p\right]+\delta\kappa p^2\sigma\right\}}\rho \\ &+\frac{\kappa\left(1-p\right)\left[\sigma\left(1-q\right)-q\left(\kappa-\phi\sigma\right)\right]}{\left\{\sigma\left[\left(1-q\right)\left(1-\beta q\right)+q\phi\right]-\kappa q\right\}\left\{\left(1-p\right)\left[\sigma\left(1-p\right)+\sigma p^2\phi-\kappa p\right]+\delta\kappa p^2\sigma\right\}}\rho_L\end{aligned} \tag{30}$$

$$
\begin{aligned}
x_L &= \frac{q}{\sigma(1-q)}\left(\frac{\kappa+\sigma(1-q)+q(\kappa-\sigma\phi)}{[\sigma(1-q)+q(\kappa-\sigma\phi)]}\right)\rho \\
&\quad +\frac{p}{\sigma(1-p)}\pi_L+\frac{1}{\sigma(1-p)}\rho_L \qquad (31)
\end{aligned}
$$

$$
e_L = \frac{p}{(1-p)}\pi_L+\frac{\kappa q}{(1-q)[\sigma(1-q)+q(\kappa-\sigma\phi)]}\rho \qquad (32)
$$

$$
\begin{aligned}
\frac{\partial \pi_L}{\partial q} &= \frac{\sigma\kappa(1-p)^2[\sigma(1-\phi)+\kappa(1+\delta\sigma)]}{\{\sigma[(1-q)+q\phi]-\kappa q\}^2\{\sigma(1-p)[1-p(1-p\phi)]-\kappa p(1-p)+\delta\kappa p^2\sigma\}}\rho \\
&> 0 \quad \text{if and only if} \\
&\sigma(1-\phi)+\kappa(1+\delta\sigma)>0
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial x_L}{\partial q} &= \frac{1}{\sigma(1-q)^2}\left(1+\frac{\kappa[\sigma+q^2(\kappa-\sigma(1+\phi))]}{[\sigma(1-q)+q(\kappa-\sigma\phi)]^2}\right)\rho \qquad (33) \\
&\quad +\frac{p}{\sigma(1-p)}\frac{\partial \pi_L}{\partial q} \\
\frac{\partial x_L}{\partial q} &> 0 \quad \Leftrightarrow \quad \sigma+q^2\kappa>q^2\sigma(1+\phi)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial e_L}{\partial q} &= \frac{p}{(1-p)}\frac{\partial \pi_L}{\partial q}+\kappa\frac{\sigma(1-q^2)+q^2(\kappa-\sigma\phi)}{(1-q)^2[\sigma(1-q)+q(\kappa-\sigma\phi)]^2}\rho \qquad (34) \\
\frac{\partial e_L}{\partial q} &> 0 \Leftrightarrow \quad \kappa>\sigma\phi
\end{aligned}
$$

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