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SHOCKS TO THE LENDING STANDARDS AND THE MACROECONOMY

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Shocks to the Lending Standards and the Macroeconomy

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Abstract

This paper presents a model in which firms have endogenously-persistent lending relationships with banks which compete both on interest rates and collateral requirements. The economy features an endogenously-evolving lending standard which is subject to an exogenous shock. A shock to bank lending standards in this model leads to a spike in spread, drop in bank credit and amplification of macroeconomic volatility. These effects are higher at greater intensity and persistence of the lending relationships. This work shines a spotlight on how shocks to lending standards can have wider macroeconomic implications and shows how financial shocks can affect real economy.

Keywords: Lending Standards, Deep Habits in Banking, Macroeconomic Fluctuations
JEL Classification: E32, E44

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1 Introduction

The contribution of this paper is to uncover how a shock to lending standards can have larger macroeconomic implications and parse the mechanism through which a shock originating in financial sector can affect real economy. To this aim, this paper builds a framework in which lending standards are rooted in credit relationships between banks and firms and the credit standards are subject to an exogenous shock[1]. The idea here is to capture sudden developments in financial sector that negatively affect lending standards in the economy. I show that these shocks have important implications for the financial sector and the wider macroeconomy.

This is the first paper that examines the effects of changes in credit standards by looking at shocks that affect the probability of loan repayment. Though other papers such as De Veirman (2023) and Sharma (2023b) have investigated the impact of changes in credit conditions by looking at shocks to loan-to-value (LTV) ratios or more general shocks to collateral which have wider interpretation than LTV ratios and can stand in for any shocks that reduce the value of collateral, impair the ability of banks to assess sufficiently borrower's collateral or any other situation that leads banks to offer lesser credit per unit of collateral than before (Becard and Gauthier, 2022; Sharma, 2023a), none of the extant papers specifically look at changes in credit standards by focusing on unexpected changes in probability of loan repayment, nor have they attempted to tease out the implications of borrower-lender relationships and collateral competition[2] for shocks to lending standards.

There is a growing body of work on lending standards and macroeconomic outcomes. Of late, some of the papers that have employed variations in credit standards to explain macroeconomic outcomes and aggregate dynamics include works of Gete (2018) and Ravn (2016), among others. These papers feature financial frictions and a mechanism to characterise changes in lending standards over the business cycles. The common finding from this literature is that there is sizable variation in credit standards over the business cycles and these variations matter because they have important effects on investment, output and consumption. The extant literature, however, ignores exogenous shocks to lending standards and their effects on macroeconomic activity in an environment featuring competition on both interest rates and collateral requirements. Col-

[1] I will be using the terms 'credit standards' and 'lending standards', and 'credit relationships' and 'lending relationships' interchangeably throughout this work.

[2] It is worth pointing out here that most of the extant literature focuses on changes in credit conditions or credit standards by looking at shocks to collateral and ignores any role for collateral competition. As I show in this paper, competition on collateral requirements has interesting implications for macroeconomic dynamics.

lateral competition is important since it leads to an endogenously-evolving lending standard in this model. Though one may conjecture that a negative shock to lending standards might lead to a fall in bank credit and a subsequent slump in economic activity, it's not clear what happens when the environment is characterized by presence of credit relationships betweeen borrowers and lenders. Do these credit relationships moderate or amplify shocks to the lending standards? What role do intensity and presistence of lending relationships play and what is the underlying mechanism behind these effects? This work seeks to answer these questions.

It's important to consider these questions and take into account presence of lending relationships when studying shocks to lending standards because lending relationships can be found across a number of economies and their presence has important implications for macroeconomic dynamics. A model that ignores the existence of these credit relationships may not accurately capture the macroeconomic effects of shocks to credit standards and may underestimate the ensuing economic effects and volatility. This has consequences not only for study of these economic shocks but also macroeconomic policy and management. These arguments support the case for study of shocks to lending standards in an environment which explicitly acknowledges the presence of credit relationships between banks and firms. This is further supported by recent empirical studies such as Ricci, Soggia, and Trimarchi (2023) who have shown the negative effects of shocks to bank lending standards on Italian firms and find that these shocks have sizable and persistent effects on credit and production.

In order to study macroeconomic effects of a shock to bank lending standards, I develop a Two-Agent New Keynesian (TANK) model which features collateral-constrained entrepreneurs who own and run the firm in the economy. These entrepreneurs borrow from banks which are owned and funded by households. Banks accept deposits from households which is their only source of funding and make loans to firms which have lending relationships with them. Following Ravn (2016), the banks in this model compete not only on interest rates but also the collateral requirements they impose on their borrowers. To the best of my knowledge, the only paper that studies implications of lending relationships in a DSGE model with collateral competition is Ravn (2016) who builds a model in which firms have lending relationships with banks which compete on both interest rates and collateral requirements. This comptition on collateral requirements leads to an endogenously-evolving credit standard in the economy. Ravn (2016) then studies the implications of this environment for effect and transmission of macroeconomic shocks. My contribution relative to this work is formulation of a process for lending standard that is subject

to an exogenous shock. This is to capture the idea that lending standards in an economy can get hit by unexpected negative shocks for a number of reasons such as higher uncertainty or a general deterioration of credit conditions in the economy. I then use this setup to study the economic implications of a shock to the credit standards. My goal in this work is to unveil both the effects and the transmission mechanism in a model that features lending relationships between firms and banks and that considers competition on both interest rates and collateral requirements. In order to do this, I abstract from other macroeconomic shocks and focus exclusively on unexpected changes in credit standards[3].

I find that after a shock to credit standards, spread rises and bank credit falls. This effect is much greater when lending relationships are present versus when they are abesent. The fall in bank loans leads to a drop in investment, capital, consumption and output. Employment falls on impact and wages fall quickly after an initial increase on impact. In case of no lending relationships, after an initial fall, these variables return to their steady-state value and show comparatively little amplification. However, when lending relationships are present, the shocks to credit standards lead to significant macroeconomic amplification and volatility. After an initial spurt, the spread returns to its steady-state value and then overshoots it. It stays below its steady-state value for an extended period. This reflects the fact after spread rises and bank loans fall, banks gradually lower their spread to entice and lock in more customers to make up for the loss of business and secure market share. This means that loans, after falling initially, return to their previous steady-state value faster than in the case of no lending relationships and then overshoot it. It's worth noting that bank loans continue to stay above their prior steady-state level for a protracted period. This results in faster recovery in investment and capital which then is reflected also in quicker return to steady-state value by aggregate consumption and output. Employment and wages also revert to their stedy-state faster before overshooting it. This result echoes of "financial accelerator" effects of Bernanke, Gertler, and Gilchrist (1999) and shows that presence of lending relationships can act as an amplifier of credit shocks. In so doing, this work also highlights the important role of borrower-lender relationships and the potential of shocks to credit standards to affect macroeconomic activity. I further show that higher intensity and persistence of credit relationships amplify shocks to credit standards. The reason is that higher intensity and persistence of lending relationships allow banks to charge larger premium after

[3]To be precise, credit standard in this model refers to the process that describes the evolution of probability of loan repayment and it links bank's choice of LTV ratios and credit risk, indicating that as banks raise LTV ratios they obtain higher share of the loans market but at the same time, they expose themselves to greater credit risk.

a shock to credit standards which is reflected in higher spike in spread after a negative shock. This higher spread then leads to both a greater fall in macroeconomic variables and a quicker reversion to their previous steady-state level. The macreconomic aggregates then overshoot their prior steady-state level and stay elevated for an extended period which generates higher macroeconomic amplification and volatility.

This paper is connected to the literature on intersection of macroeconomics and banking. Specifically, it examines how changes in credit conditions affect macroeconomic activity. Over the last several decades, a number of papers have examined systematic fluctuation in credit standards – collateral requirements or spread between deposit and lending rates – over the business cycle. Examples of such work include Rajan (1994), Ruckes (2004) and Dell'Ariccia and Marquez (2006), among others[4]. Empirically, a number of studies have documented lending relationships (Petersen and Rajan, 1994, 1995; Ongena and Smith, 2000; Kosekova, Maddaloni, Papoutsi, and Schivardi, 2023). In this paper, I have modelled lending relationships using deep habits in loan demand from individual banks á la Aliaga-Díaz and Olivero (2010) who incorporated deep habits as in Ravn, Schmitt-Grohé, and Uribe (2006) into banking. Justification for deep habits model comes from information asymmetry between lenders and borrowers (Sharpe, 1990; Kim, Kliger, and Vale, 2003). It is also supported by the fact that propensity of borrowers to switch from their lenders is negatively related with duration of relationship (Chakravarty, Feinberg, and Rhee, 2004) and perceived reliability and resposiveness of banks. Competition on lending rates has been studied by Gerali, Neri, Sessa, and Signoretti (2010) while Ravn (2016) is the first to study competition on collateral requirements in a DSGE setup.

Collateral requirements are pervasive and Avery, Bostic, and Samolyk (1998), for example, have docuemnted that 80% of US small business loans are collaterized. Non-price competition in banking due to agency problems with price competition is widespread (Stiglitz and Weiss, 1981). In terms of modelling, Bestor (1985) presents a model in which banks compete both on lending rates and collateral requirements. The fact that banks accumulate privileged information about their borrowers and their assets and that borrowers with long banking relationships are less likely to pledge collateral has been documented by Berger and Udell (1995). In Ravn (2016), credit standards act as an additonal financial accelerator (Bernanke, Gertler, and Gilchrist, 1999). This generates amplification of technology shocks, an effect usually not produced by financial

[4]These papers offer theoretical explanations for changes in credit standards but do not incorporate it into DSGE models and do not allow a quantitative study of macroeconomic implications of changes in credit standards.

frictions (Kocherlakota, 2000; Liu, Wang, and Zha, 2013). Jensen, Ravn, and Santoro (2018), however, demonstrate that in a model with two types of credit constrained agents strategic complementarities between their repective collateral constraints can create quantitatively relevant amplification of technology shocks. Aliaga-Díaz and Olivero (2010) show deep habits in banking may generate countercyclical spreads between lending and deposit rates as observed in data. Aksoy, Basso, and Coto-Martinez (2013) report a small effect of lending relationships on amplification of output fluctuations. Airaudo and Olivero (2019) embed deep habits in banking with cost channel of monetary policy. Melina and Villa (2014) show that with deep habits in banking, countercyclical movements in interest rates lead to increased government spending multiplier. Melina and Villa (2018) present a DSGE model with banking relationships to study response of monetary policy to credit exuberance. Dell'Ariccia and Marquez (2006) show that during booms, banks lower their collateral requirements to attract more borrrowers. Ruckes (2004) argues that banks have to offer more attractive borrowing terms during booms to their customers since there is increased competition amongst banks for them during an economic upturn. Rajan (1994) makes the assumption that banks have short term concerns relative to their other rivals which induces them to lower their credit standards during a boom, increase competition and generate higher profits. Other notable contributions include papers by Berlin and Butler (2002) and Hainz, Weill, and Godlewski (2013).

Banking in DSGE models has been studied by Marvin and McCallum (2007) while collaterized borrowing has been examined by Iacoviello (2005) and Kiyotaki and Moore (1997). Gerali, Neri, Sessa, and Signoretti (2010) is an example of a study that looks at banking sector with monopolistic competition of Dixit-Stiglitz form. Further, Andrés and Arce (2012) and Andrés, Arce, and Thomas (2013) incorporate Salop form of spatial competition to study effects of a shock to credit availability and monetary policy, respectively.

The remainder of this paper is organized as follows. Section 2 presents the model and Section 3 discusses model solution and parameterization. Section 4 presents results and Section 5 concludes.

2 Model

The model contains two agents – households and entrepreneurs. Entrepreneurs are collateral-constrained and their borrowing is subject to a credit limit. These features resemble setups used

in Iacoviello (2005), Liu, Wang, and Zha (2013), Justiniano, Primiceri, and Tambalotti (2015) and Ravn (2016). I closely follow Ravn (2016) and extend it to include a process for probability of loan repayment which is subject to an exogenous negative shock. In this model, entrepreneurs borrow from banks whose only source of funding is deposits from households. The loan demand by entrepreneurs features external habit à la Aliaga-Díaz and Olivero (2010). Entrepreneurs in this model are more impatient than households, that is, $\beta^E < \beta^P$. Households consume, supply labor and hold deposits with banks. They receive interest on their deposits and share of profits of banks. Entrepreneurs, in turn, own the firms in the economy and run it. They consume non-durables and hire labor from households which they combine with capital and land to produce output. In the following, I explain each agent's optimization problem in greater detail.

2.1 Households

Households have the utility function of the following form:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \left(\beta^P\right)^t \left\{ \log\left(C_{i,t}^P - \gamma^P C_{i,t-1}^P\right) - \frac{N_{i,t}^{\nu}}{\nu} + \varsigma \log H_{i,t}^P \right\} \tag{1}$$

where $C_{i,t}^P$, $N_{i,t}$ and $H_{i,t}^P$ denote consumption, labor and housing respectively of the households, $\beta^P \in (0,1)$ is a discount factor, γ^P measures the degree of habit formation in consumption, ν is Frisch elasticity of labor supply and ς is a weight on housing. The superscript P denotes patient households. The household faces the following budget constraint

$$C_{i,t}^P + Q_t^H \left(H_{i,t}^P - H_{i,t-1}^P\right) + \int_0^1 D_{ik,t}\mathrm{d}k \leq W_t N_{i,t} + \int_0^1 \Pi_{ik,t}\mathrm{d}k + R_{t-1}^D \int_0^1 D_{ik,t-1}\mathrm{d}k \tag{2}$$

Here, Q_t^H is the price of one unit of housing in terms of consumption goods, W_t^P is the real wage and R_{t-1}^D is the gross risk-free interest rate on the stock of deposits $D_{ik,t-1}$ of household i in bank k at the end of period $t-1$. I assume housing does not depreciate. Profits obtained by household i from bank k are denoted by $\Pi_{ik,t}$. After imposing symmetric equilibrium, FOCs of

the households can be written as

$$\frac{1}{C_t^P - \gamma^P C_{t-1}^P} - \beta^P \mathbb{E}_t \frac{\gamma^P}{C_{t+1}^P - \gamma^P C_t^P} = \lambda_t^P \tag{3}$$

$$\beta^P \mathbb{E}_t \lambda_{t+1}^P = \frac{\lambda_t^P}{R_t^D} \tag{4}$$

$$\frac{\varsigma_t}{H_t^P} + \beta^P \mathbb{E}_t \left(\lambda_{t+1}^P Q_{t+1}^H\right) = \lambda_t^P Q_t^H \tag{5}$$

$$N_t^{\nu-1} = \lambda_t^P W_t \tag{6}$$

where λ_t^P is the Lagrange multiplier associated with household's budget constraint (2). One can combine household's first-order conditions with respect to consumption (3) and bank deposits (4) to obtain their Euler equation. Equation (5) describes household's Euler equation for housing and links today's housing price to the utility it provides plus the expected capital gain. Equation (6) describes household's consumption-lesiure tradeoff. First order conditions of the problem are derived in the Appendix A.1.

2.2 Entrepreneurs

Entrepreneur j maximizes the utility obtained from consuming the non-durable consumption goods

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \left(\beta^E\right)^t \log\left(C_{j,t}^E - \gamma^E C_{j,t-1}^E\right) \tag{7}$$

where β^E and γ^E are as defined above. Entrepreneurs face a collateral constraint à la Kiyotaki and Moore (1997) that limits the borrowing of each entrepreneur from each bank to a fraction of their assets

$$l_{jk,t} \leq \frac{1}{R_{k,t}^L} \theta_{k,t} a_{j,t} \tag{8}$$

Here, $l_{jk,t}$ denotes entrepreneur j's loan from bank k, expected value of entrepreneur's assets is $a_{j,t}$ and $R_{k,t}^L$ is the bank-specific lending rate. All entrepreneurs borrowing from bank k are subject to a loan-to-value (LTV) requirement $\theta_{k,t}$. In turn, $a_{j,t}$ is given by

$$a_{j,t} = \mathbb{E}_t \left(Q_{t+1}^H H_{j,t}^E + Q_{t+1}^K K_{j,t}\right) \tag{9}$$

In the above equation, Q_t^K denotes the value of installed capital in units of consumption goods, $K_{j,t}$ stock of capital and $H_{j,t}^E$ stock of housing.

Following Aliaga-Díaz and Olivero (2010) and Ravn (2016), entrepreneurs have deep habits in banking relationships. Let $x_{j,t}$ denote entrepreneur j's effective or habit-adjusted borrowing. Given the continuum of banks in the economy who compete under monopolistic competition, this can be written as

$$x_{j,t} = \left[\int_0^1 \left(l_{jk,t} - \gamma^L s_{k,t-1}\right)^{\frac{\xi-1}{\xi}} \mathrm{d}k\right]^{\frac{\xi}{\xi-1}} \tag{10}$$

where stock of habits $s_{k,t-1}$ evolves according to

$$s_{k,t-1} = \rho_s s_{k,t-2} + (1-\rho_s)\, l_{k,t-1} \tag{11}$$

Here, $\gamma^L \in (0,1)$ denotes the degree of habit formation in demand for loans and $\rho_s \in (0,1)$ measures the persistence of this habit. The parameter ξ denotes of the elasticity of substitution between loans from different banks and is thus a measure of the market power of each individual bank.

Given his total need for financing $x_{j,t}$, each entrepreneur chooses $l_{jk,t}$ to solve the following problem

$$\min_{l_{jk,t}} \int_0^1 \Upsilon_{k,t} l_{jk,t} \mathrm{d}k \tag{12}$$

subject to collateral constraint (8) and his effective borrowing (10). Here, $\Upsilon_{k,t} \equiv R^L_{k,t} + \frac{\eta}{\theta_{k,t}}$ where the first term denotes the interest expenditure and the second term refers to value of pledged collateral. This captures the dual problem the entrepreneur is trying to solve. On one hand, the entrepreneur is trying to minimize his interest expenditure while on the other hand, he is trying to minimize the amount of collateral he has to pledge. The parameter $\eta > 0$ denotes the relative weight to desire for minimizing collateral. Entrepreneurs may want to minimize the amount of collateral they have to post for a number of reasons. For example, they may want to minimize the loss of their asset in case of their liquidation in bankruptcy. They may also want to avoid the legal and paperwork costs involved in collateral valuation and ensuring that it's free from any encumbrances. As a result, entrepreneurs prefer to borrow from banks that offer cheaper interest rates or impose less collateral requirements.

Entrepreneur j's optimal demand for loans from bank k is

$$l_{jk,t} = \left(\frac{\Upsilon_{k,t}}{\Upsilon_t}\right)^{-\xi} x_t + \gamma^L s_{k,t-1} \tag{13}$$

where $\theta_t = \left(\int_0^1 \theta_{k,t}^{1-\xi} \mathrm{d}k\right)^{\frac{1}{1-\xi}}$ represents the aggregate LTV ratio in the economy and $R_t^L \equiv \left[\int_0^1 (R_{k,t}^L)^{1-\xi} \mathrm{d}k\right]^{\frac{1}{1-\xi}}$ is the aggregate lending rate.

Production function of each entrepreneur is

$$Y_{j,t} = A_t \left(N_{j,t}\right)^{1-\alpha} \left\{ \left(H_{j,t-1}^E\right)^{\phi} \left(K_{j,t-1}\right)^{1-\phi} \right\}^{\alpha} \tag{14}$$

where $Y_{j,t}$ is output, $N_{i,t}$ is labor input and $\alpha, \phi \in (0,1)$ are factor shares. TFP A_t follows the process

$$\log A_t = (1-\rho_A) \log A + \rho_A \log A_{t-1} + \sigma_A \epsilon_{A,t} \tag{15}$$

with iid innovation $\epsilon_{A,t}$ following a normal process with standard deviation σ_A where $A > 0$ and $\rho_A \in (0,1)$. The evolution of capital obeys the following law of motion

$$K_{j,t} = (1-\delta) K_{j,t-1} + \left[1 - \frac{\Omega}{2}\left(\frac{I_{j,t}}{I_{j,t-1}} - 1\right)^2\right] I_{j,t} \tag{16}$$

where $I_{j,t}$ is firm j's investment level, $\delta \in (0,1)$ the rate of depreciation of capital stock and $\Omega > 0$ is a cost adjustment parameter. The entrepreneur faces the following budget constraint

$$C_{j,t}^E + \int_0^1 R_{k,t-1}^L l_{jk,t-1} \mathrm{d}k \leq Y_{j,t} - W_t N_{j,t} - I_{j,t} - Q_t^H \left(H_{j,t}^E - H_{j,t-1}^E\right) + x_{j,t} + \Phi_t + \Psi_t \tag{17}$$

where $\Phi_t \equiv \gamma^L \int_0^1 \frac{\theta_{k,t}}{\theta_t} s_{k,t-1} \mathrm{d}k$ and $\Psi_t \equiv \int_0^1 \left(1 - p_{k,t-1}\right)\left(R_{t-1}^L L_{k,t-1} - \tau \theta_{t-1} a_{t-1}\right) \mathrm{d}k$ are two transfers made to entrepreneurs to ensure all markets clear. The proof is identical to Ravn (2016) and has been relegated to Appendix E. The term Φ_t is standard in models featuring deep habits (see, for instance, Ravn, Schmitt-Grohé, and Uribe, 2006) and arises because of wedge between actual and effective entreprenerial borrowing. The second transfer Ψ_t is specific to this model and as explained by Ravn (2016), represents the wedge between actual and effective repayment of loans to each bank. These transfers are exogenous to individual entrepreneurs and therefore do not

affect their behavior. After imposing symmetric equilibrium, the FOCs of the entrepreneurs are

$$\lambda_t^E = \frac{1}{C_t^E - \gamma^E C_{t-1}^E} - \beta^E \mathbb{E}_t \frac{\gamma^E}{C_{t+1}^E - \gamma^E C_t^E} \tag{18}$$

$$\lambda_t^E = \beta^E \mathbb{E}_t \lambda_{t+1}^E R_t^L + \mu_t^E R_t^L \tag{19}$$

$$W_t = (1-\alpha)\frac{Y_t}{N_t} \tag{20}$$

$$\lambda_t^E Q_t^H = \beta^E \mathbb{E}_t \left[\lambda_{t+1}^E \left(Q_{t+1}^H + \alpha\phi \frac{Y_{t+1}}{H_t^E}\right)\right] + \mu_t^E \theta_t \mathbb{E}_t Q_{t+1}^H \tag{21}$$

$$\kappa_t^E = \alpha(1-\phi)\beta^E \mathbb{E}_t \left(\frac{\lambda_{t+1}^E Y_{t+1}}{K_t}\right) + \beta^E (1-\delta)\mathbb{E}_t \kappa_{t+1}^E + \mu_t^E \theta_t \mathbb{E}_t Q_{t+1}^K \tag{22}$$

$$\lambda_t^E = \kappa_t^E \left[1 - \frac{\Omega}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 - \Omega \frac{I_t}{I_{t-1}}\left(\frac{I_t}{I_{t-1}} - 1\right)\right] + \beta^E \Omega \mathbb{E}_t \left[\kappa_{t+1}^E \left(\frac{I_{t+1}}{I_t}\right)^2 \left(\frac{I_{t+1}}{I_t} - 1\right)\right] \tag{23}$$

where μ_t^E, κ_t and λ_t^E are Lagrange multipliers associated with entrepreneur's collateral constraint (8), law of motion of capital (16) and entrepreneur's budget constraint (17). Entrepreneur's first order conditions with respect to consumption (18) and loans (19) may be combined to derive his Euler equation for consumption. Equation (20) describes entrepreneur's optimal demand for labor. Entrepreneur's Euler equation for land is described by (21) which relates its price today to its expected resale value tomorrow plus the payoff obtained by holding it for a period as given by its marginal productivity and its ability to serve as a collateral. Likewise, (22) is entrepreneur's Euler equation for capital and it links price of capital today to its price tomorrow and the expected payoff from keeping it for a period as given by its marginal productivity and its ability to serve as a collateral. Finally, entrepreneur's Euler equation for the investment is given by (23). All the derivations of first order conditions are contained in Appendix A.2.

2.3 Banking Sector

Banks in this model accept deposits from households and make loans to entrepreneurs. Banks take the interest rate on deposits R_t^D as given. Each individual bank k chooses its lending rate $R_{k,t}^L$, its LTV ratio $\theta_{k,t}$ and the total amount of lending $L_{k,t}$. By increasing its LTV ratio, a bank can obtain higher share of loans market as entrepreneurs prefer to borrow from banks allowing higher LTV ratio but at the same time, it exposes the bank to greater credit risk. Several studies such as Quercia and Stegman (1992), Edelberg (2004) and Jimenez, Salas, and Saurina (2006), among others, provide empirical support for this assumption. All these studies broadly find that

loans with less collateral turn out to be riskier ex post. The link between lower credit standards and higher credit risk can thus be given as

$$p_{k,t}\zeta_t = \Xi + \varpi\left(\theta_{k,t} - \overline{\theta}\right) \tag{24}$$

where $p_{k,t}$ is a bank-specific probability that a given loan is repaid and $\varpi < 0$ measures the elasticity of this probability with respect to deviations of the bank's LTV ratio from its steady state level $\overline{\theta}$ which is same for all banks. The steady-state repayment probability is given by $\Xi > 0$. I depart from Ravn (2016) by assuming that this probability is subject to a negative exogenous shock. The idea is to capture effects of shocks to lending standards that affect bank credit and impact other macroeconomic variables. My goal here is to study macroeconomic effects of shocks that emerge from variation in credit standards, above and beyond what endogenous changes in credit standards account for. Here, ζ_t is an exogenous shock that follows the law of motion given by

$$\log\zeta_t = (1-\rho_\zeta)\log\zeta + \rho_\zeta\log\zeta_{t-1} + \sigma_\zeta\epsilon_{\zeta,t} \tag{25}$$

where $\epsilon_{\zeta,t}$ is iid innovation which follows a normal distribution with standard deviation σ_ζ and where $\zeta > 0$ and $\rho_\zeta \in (0,1)$.

Each bank faces a standard trade-off when choosing its lending rate $R^L_{k,t}$. By raising its lending rate, each bank k can increase the profitability of its loans but it reduces the demand for its loans as entrepreneurs turn to other banks for loans. Given the presence of deep habits in lending, it means that demand for loans from bank k will be lower in future periods too. Profits of the bank k can be written as

$$\begin{aligned}\Pi_{k,t} &= \left[\Xi + \varpi\left(\theta_{k,t-1} - \overline{\theta}\right)\right]R^L_{k,t-1}L_{k,t-1} + \left[1 - \Xi - \varpi\left(\theta_{k,t-1} - \overline{\theta}\right)\right]\frac{L_{k,t-1}}{\int_0^1 L_{k,t-1}\mathrm{d}k}\tau\theta_{t-1}a_{t-1} \\ &+ \int_0^1 D_{ik,t}\mathrm{d}i - L_{k,t} - R^D_{t-1}\int_0^1 D_{ik,t-1}\mathrm{d}i \\ &= p_t R^L_{k,t-1}L_{k,t-1} + (1-p_t)\frac{L_{k,t-1}}{\int_0^1 L_{k,t-1}\mathrm{d}k}\tau\theta_{t-1}a_{t-1} + \int_0^1 D_{ik,t}\mathrm{d}i - L_{k,t} - R^D_{t-1}\int_0^1 D_{ik,t-1}\mathrm{d}i\end{aligned} \tag{26}$$

With probability $p_{k,t-1}$, the bank receives its loan back with interest. With complementary probability $(1 - p_{k,t-1})$, the loan is not reapid in which case bank k receives a share of the liquidation value of the borrower's total collaterized assets with its share given by its total lending relative to total lending of all other firms. The parameter $\tau \in (0,1)$ reflects that value

of assets is lower in liquidation.

The balance sheet of bank k is

$$L_{k,t} = \int_0^1 D_{ik,t} \mathrm{d}i \tag{27}$$

where $L_{k,t}$ denotes total loans made by bank k to all entrepreneurs, that is, $L_{k,t} \equiv \int_0^1 l_{jk,t} \mathrm{d}j$. Each bank takes the demand for its loans as given

$$L_{k,t} = \int_0^1 l_{jk,t} \mathrm{d}j = \int_0^1 \left[\left(\frac{\Upsilon_{k,t}}{\Upsilon_t} \right)^{-\xi} x_t + \gamma^L s_{k,t-1} \right] \mathrm{d}j \tag{28}$$

Each bank chooses $L_{k,t}$, $\theta_{k,t}$ and $R^L_{k,t}$ to maximize its profits subject to (27) and (28). Considering a symmetric equilibrium in which all banks optimally choose the same LTV ratio and the same lending rate, the FOCs for banks' optimization problem are:

$$\varrho^E_t = \mathbb{E}_t q_{t,t+1} \left[p_{k,t} R^L_{k,t} + (1 - p_{k,t}) \frac{\tau \theta_t a_t}{\int_0^1 L_{k,t} \mathrm{d}k} - R^D_t + \gamma^L (1 - \rho_s) \mathbb{E}_t \varrho^E_{t+1} \right] \tag{29}$$

$$\xi \varrho^E_t x_t \frac{\frac{\eta}{\theta_t}}{R^L_t \theta_t + \eta} = -\varpi \mathbb{E}_t q_{t,t+1} \left(R^L_t L_t - \tau \theta_t a_t \right) \tag{30}$$

$$\xi \varrho^E_t x_t \frac{\theta_t}{\theta_t R^L_t + \eta} = \mathbb{E}_t q_{t,t+1} p_{k,t} L_{k,t} \tag{31}$$

where ϱ^E_t is the Lagrange multiplier on demand for bank's loans (28) and can be interpreted as shadow value to the bank of lending an extra dollar. Banks are owned by households and consequently their stochastic discount factor is given by $q_{t,t+1} = \beta^P \mathbb{E}_t \frac{\lambda^P_{t+1}}{\lambda^P_t}$. The optimality condition (29) states that shadow value of lending an extra dollar is given by repayment minus cost of borrowing that extra dollar from the households. The term $\gamma^L (1 - \rho_s) s_{k,t-1}$ on the right-hand side reflects the fact that if a given bank lends an extra dollar in this period, the borrower of that dollar will develop will develop a habit for loans from that bank and as a result, will borrow more from it also in the susbesequent period. The size of this effect depends on degree γ^L and duration ρ_s of deep habits. In absence of deep habits (when γ^L is zero), the latter term disappears. Equation (30) states that by marginally increasing θ_t, each banks increases its market share as firms prefer to borrow from banks allowing a higher LTV ratio. The left hand side shows the increase in profits which is related to elasticity of substitution of loans from different banks ξ and ϱ^E_t which indicates that bank considers the increase in demand for loans in subsequent periods as given by (29). The right hand side shows the cost of a marginal

increase in θ_t which is given by an increase in credit risk created by lower credit standards. Equation (31) equates the profit gain from a marginal increase in bank's lending rate to the marginal cost. Bank's marginal cost is on the left-hand side and indicates a loss in its market share as it increases its lending rate. The marginal benefit of a higher lending rate appears on the right-hand side and shows the discounted gain made by repayment of loans made at higher lending rates. Derivation of all first order conditions have been consigned to Appendix A.3.

2.4 Aggregation and Market Clearing

Aggregate resource constraint of the economy is

$$C_t^P + C_t^E + I_t = Y_t \tag{32}$$

The clearing condition for the housing market is

$$H_t^P + H_t^E = H \tag{33}$$

where H is the total fixed supply of housing.

3 Equilibrium and Model Solution

The model is solved around its deterministic steady state using standard perturbation techniques in Dynare (Adjemian, Bastani, Juillard, Karamé, Mihoubi, Mutschler, Pfeifer, Ratto, Rion, and Villemot, 2022). A period in the model is a quarter. Appendices B, C and D contain the list of equilibrium conditions, the list of steady-state conditions and the system of loglinear equations, respectively. The model is calibrated using parameter values standard in literature and those in Ravn (2016). The degree of habit formation in consumption is chosen to be 0.6 which is a common estimate in the literature (Smets and Wouters, 2007). The Frisch elasticity of labor supply ν is 1.01 while the weight on housing ς is set to 0.1 (Iacoviello, 2005).

The labor income share takes a standard value of 0.3 which yields a steady-state capital-output ratio of 1.15, consistent with US data (Liu, Wang, and Zha, 2013). The input share of land in production is close to the value estimated by Liu, Wang, and Zha (2013) and in line with the value used in Iacoviello (2005). The investment adjustment cost parameter is set to 1.85

Table 1: Parameter values

	Value	Description	Source/Target
β^P	0.995	Discount factor, households	Iacoviello (2005)
β^E	0.95	Discount factor, entrepreneurs	Iacoviello (2005)
γ^i, $i = \{P, E\}$	0.6	Habits in consumption, households, entrepreneurs	Smets and Wouters (2007)
ν	1.01	Frisch elasticity of labor supply	Iacoviello (2005)
ς	0.1	Weight on housing	Iacoviello (2005)
α	0.3	Non-labor share of production	Iacoviello (2005)
ϕ	0.1	Land share of non-labor input	Ravn (2016)
Ω	1.85	Investment adjustment cost parameter	Ravn (2016)
δ	0.0285	Capital depreciation rate	Ravn (2016)
τ	0.9432	Recovery rate of assets in liquidation	Ravn (2016)
Ξ	0.98	Steady state of repayment probability	Ravn (2016)
γ^L	0.72	Deep habit formation	Aliaga-Díaz and Olivero (2010)
ρ_s	0.93	Persistence of stock of deep habits	See Text
ξ	230	Elasticity of substitution between banks	Ravn (2016)
ϖ	-1.5	Elasticity of credit risk	Ravn (2016)
η	0.05	Weight of collateral minimization desire	Ravn (2016)
ρ_A	0.95	Persistence of technology shock	Smets and Wouters (2007)
ρ_ζ	0.95	Persistence of shock to lending standard	See Text
ζ	1	Steady-state value of shock to lending standard	Normalization
σ_A	0.0014	Standard deviation of technology shock	Standard
σ_ζ	0.01	Standard deviation of shock to lending standard	See Text

Ravn (2016). The available estimates for investment adjustment cost paramter range from close to 0 (Liu, Wang, and Zha, 2013) to above 26 (Christiano, Motto, and Rostagno, 2010). The rate of depreciation of capital is chosen to obtain a steady-state ratio of non-residential investment to output of slightly above 0.13 as consistent with US data (Beaudry and Lahiri, 2014). Following Liu, Wang, and Zha (2013), the recovery rate of assets in liquidation is calibrated to obtain an LTV ratio of 0.75 in steady state. Ravn (2016) reports that the delinquency rate on commercial and industrial business loans in the US has fluctuated around an average close to 2% since mid 1990's. Using this, steady-state value of loan repayment probability Ξ is set to 0.98.

Following Aliaga-Díaz and Olivero (2010), the deep habit parameter in banking γ^L is set to 0.72. I take this value as baseline and later vary it to capture in a transparent fashion the impact of intensity of credit relationships on shocks to lending standards. Petersen and Rajan (1995) report the duration of bank-firm relationship as 11 years. The persistence of stock of habits ρ_s is then selected to match it. Follwoing Ravn (2016), this is done by setting the persistence parameter ρ_s so that if the stock of habits $s_{k,t}$ were to increase exogeneously, only 5% of this increase would persist after 44 quarters. This implies a value of $\rho_s = 0.93$ which is close to

the value of 0.85 used by Ravn, Schmitt-Grohé, and Uribe (2006) and Aliaga-Díaz and Olivero (2010). I later conduct an exercise in which I vary this parameter to analyze how it affects the impact of shocks to lending standards. Elasticity of substitution between loans from different banks is set to 230 (Ravn, 2016). Aliaga-Díaz and Olivero (2010) use an elasticity of substitution of 190 whereas Melina and Villa (2018) use a value of 427.

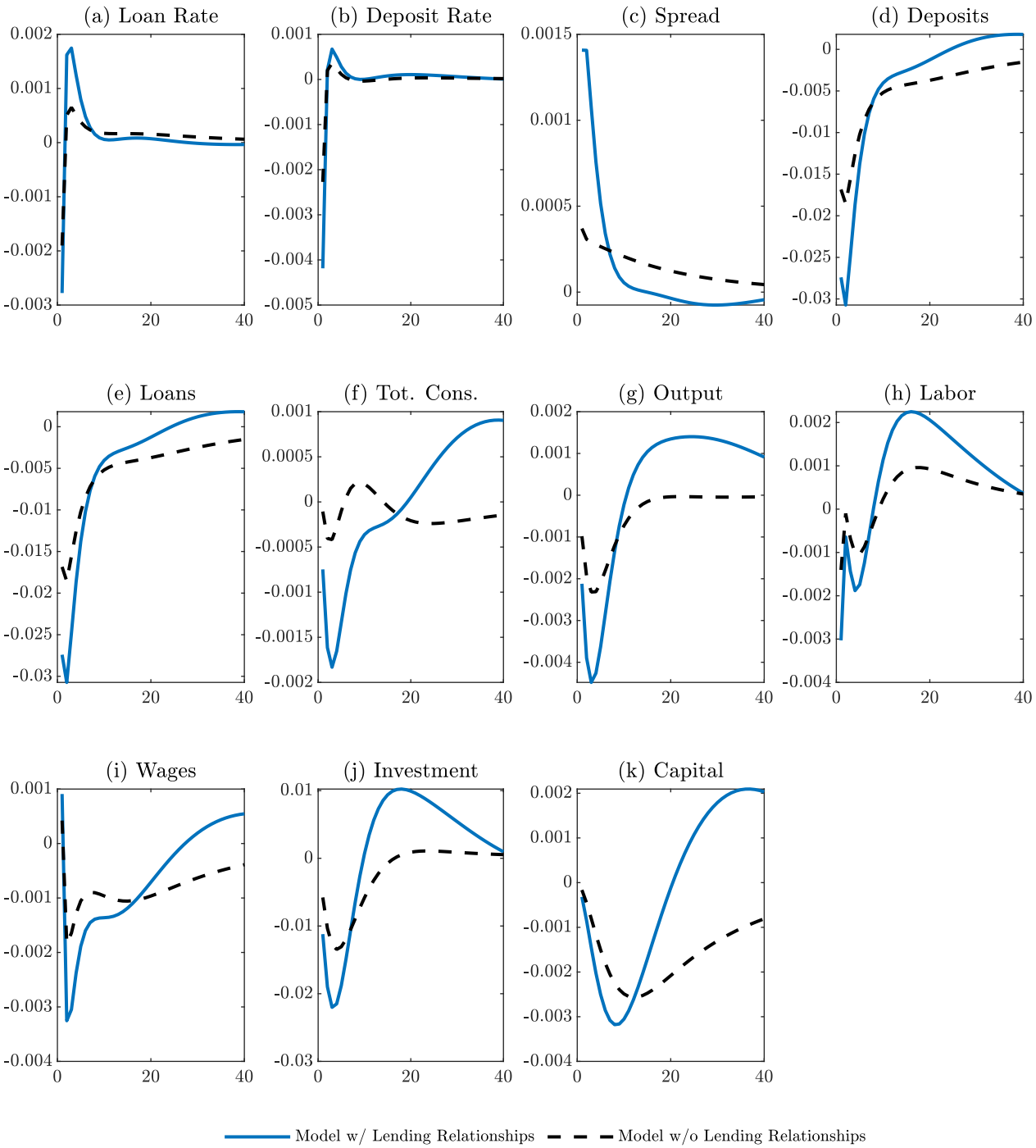
The parameter ϖ measures the elasticity of credit risk with respect to changes in LTV ratio. Using data from US mortgage loans originated between 1995 and 2008, Lam, Dunsky, and Kelly (2013) conclude that foreclosure and delinquency rates tend to rise around one for one with the delinquency ratio while Von Furstenberg (1969) reports a higher elasticity 'in excess of unity'. The value of this elasticity is therefore chosen to be 1.5 which is the same value used in Ravn (2016). Estimates of the value for η which denotes entrepreneur's desire to minimize collateral pledges relative to cost minimization motive, are scarce. Booth and Booth (2006) find that firms' collateral minimization concern is of limited importance and they tend to choose the least costly form of borrowing. They point out that firms' willingness to accept higher lending rates in order ro reduce collateral requirements is rather small and therefore the value of η is set at 0.05 – a small value. Following Smets and Wouters (2007), persistence of technology shock σ_A is set to 0.95 and its standard deviation to 0.0014 which is standard in the literature. Estimates for parameters governing shocks to lending standards are hard to come by. I normalize the steady-state value of shock to the lending standards ζ to 1. Following typical values in the literature for calibration of other shocks, its persistence ρ_ζ is set to 0.95 and its standard deviation is set to 0.01.

4 Results

This section illustrates the effects of a shock to the lending standards and explains its effects on macroeconomic activity. After a shock, spread spikes and bank credit falls, see Figure 1. Note that spread rises more than thrice when credit relationships are present versus the case when lending relationships are absent (that is, when $\gamma^L = \rho_s = 0$) and this contributes to a much greater fall in bank credit in the case when lending relationships are present. Drop in bank loans reduces the amount of funding available to entrepreneurs who as a result cut down on investment. Capital, labor and output fall on impact while wages drop rapidly after a brief rise at impact. This is then reflected in fall in consumption and output. All these variables fall

more when credit relationships are present.

FIGURE 1: EFFECTS OF A SHOCK TO THE LENDING STANDARDS



Note: Impact of a shock to the lending standards. Numbers on the horizontal axis represent quarters since the shock. Numbers on the vertical axis show percentage deviation from the steady state.

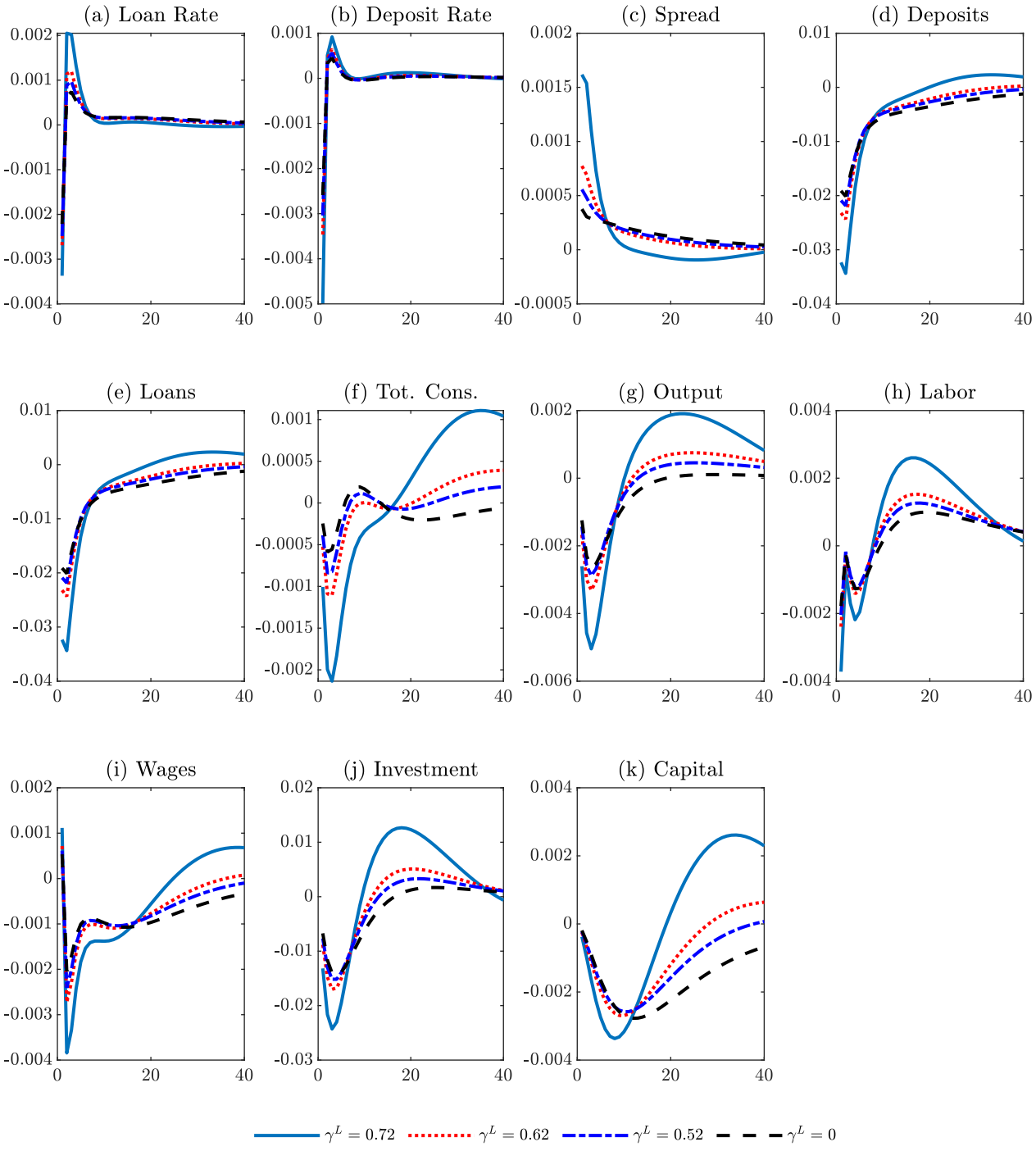
After intial surge in spread, it falls quickly to its previous steady-state value before overshooting it. It then stays below its steady-state level for extended period (once it overshoots its preivous steady-state level, it doesn't return to its steady state value even until 40 quarters). After banks raise their spread after an adverse shock to credit standards and bank credit falls as a

result, they rapidly lower their spread to regain their market share and win customers back. This leads spread to overshoot its previous steady-state level and stay below it for protracted period. This first results in a greater fall in bank credit followed by rapid reversion to its steady state before it overshoots it and stays persistently above it for extended period (it stays above its steady state even after 40 quarters). This first leads to greater fall in other macroeconomic variables such as investment and capital and then larger increases. This is accompanied by larger fall and then bigger increases in labor and wages. Consequently, aggregate output and consumption first drop more before increasing and overshooting their previous steady-state level. This illustrates how presence of lending relationships acts as an amplifier of macroeconomic volatility. This result is reminiscent of "financial accelerator" effect of Bernanke, Gertler, and Gilchrist (1999) and sheds light on ability of borrower-lender relationships to act as propagator and amplifier of financial shocks.

To better understand the forces at work and their interplay, consider what happens when a negative shock to probability of loan repayment arrives. First consider the case when there is no bank-firm lending relationship. Since banks want to protect their profits on the loans they make, the begin charging higher spread which jumps as a result. A spike in spreads makes it more expensive for entrepreneurs to take out loans who respond by reducing their borrowing. Notice that deposits also fall by equal magnitude since in this model loans and deposits are mirror images of each other. Fall in bank credit is then followed by a drop in investment, capital, output, labor and consumption. Eventually, as spreads gradually near their previous steady state value, bank credit approaches its pre-shock level which is accompanied by a wider macroeconomic recovery. Notice that macroeconomic variables display little to no amplification. In the aftermath of a shock to lending standards, they fall and then eventually return to their equilibrium value.

Now consider the case when bank-firm lending relationships are present. After a shock to lending standards materializes, banks raise spreads more than threefold. This indicates banks' market power in loans market. This jump in spreads makes bank credit more expensive than the case in which credit relationships between lenders and borrowers are absent. Consequently, bank loans fall by almost twice the magnitude than in the case of no lending relationships. Greater fall in bank credit presages the larger drops in investment, output, consumption and labor. After a while, banks bagin to rapidly lower their spread. This reflects their desire to win back their customers and regain their market share. Banks realize that because of presence of

FIGURE 2: SHOCK TO THE LENDING STANDARDS AND INTENSITY OF LENDING RELATIONSHIPS



Note: Impact of a shock to the lending standards. Numbers on the horizontal axis represent quarters since the shock. Numbers on the vertical axis show percentage deviation from the steady state.

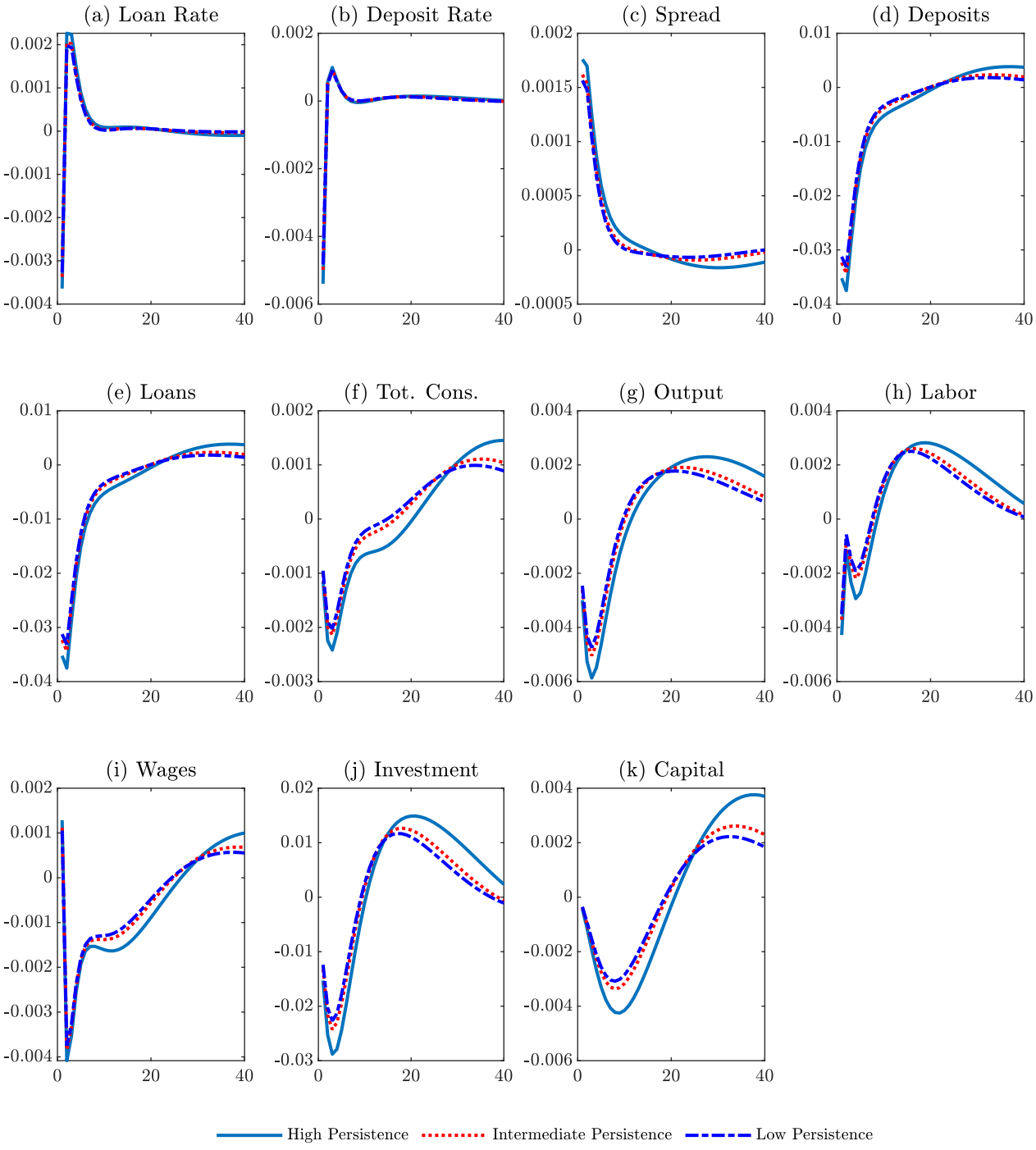
lending relationships, these customers will be with them in future periods too, yielding them even higher profits. As a consequence, banks lower their spread below its prior steady-state value and it remains there for a prolonged period. Note that spread does not return to its equilibrium value even after a decade. Also notable is how spread quickly reaches its equilibrium value before overshooting it when credit relationships are present while it does not return to

its prior steady state even after a decade in absence of bank-firm lending relationships. This explains why bank credit does not fully recover in case of no lending relationships. An extended period with low spread makes bank loans cheaper and attractive for entrepreneurs who begin borrowing until it reaches its previous steady-state level and then overshoots it. Surge in bank credit leads to an investment boom which results in large increases in capital, output, labor, wages and consumption all of which overshoot their prior equilibrium value and stay elevated for an extended period. In fact, these variables do not return to their steady state even after a decade. This mechanism illustrates how presence of lending relationships amplifes a shock to credit standards and why these effects are muted when credit relationships are absent.

I now perform an exercise in which I vary the intensity and persistence of lending relationships to see their impact on effects of shocks to credit standards, The purpose of this exercise is to capture in a transparent fashion how intensity and persistence of credit relationships affect impact and transmission of shocks to lending standards. Figure 2 shows effects of a shock to lending standards at different degrees of habits in lending. I consider the benchmark case of $\gamma^L = 0.72$ and then gradually reduce it to 0.62, 0.52 and 0. The last number corresponds to the case of no credit relationships. At higher intensity of lending relationships, a shock to the lending standards has greater impact and its effects decrease as degree of habits in lending decreases. Higher degrees of habits in lending allow banks to seek greater rents or premium after a negative shock to the lending standard which is reflected in bigger jump in spread at higher degrees of habits in lending. This larger spikes in spread at higher degree of habits in lending then leads to larger macroeconomic fluctuations and higher economic volatility.

I now vary the persistence of lending relationships and Figure 3 displays how it affects the impact of a shock to the lending standards. I consider three cases – when stock of habits left after a decade is 10%, 5% and 2.5%, respectively. I label these three cases 'high persistence', 'intermediate persistence' and 'low persistence', respectively. Like higher degree of intensity of lending relationships, higher persistence of credit relationships allow banks to charge larger rents or premiums after a negative shock to the credit standard which is manifested in larger spike in spread at greater persistence of lending relationships. Additionally, it's notable that spread stays below its prior steady-state level for extended period which contributes to first greater fall and then larger increase in bank credit. This bigger drop followed by faster recovery and larger increase after overshooting previous stady state, leads to higher macroeconomic amplification and volatilty. This casts lights on the fact that after a negative shock to the lending standards,

FIGURE 3: SHOCK TO THE LENDING STANDARDS AND PERSISTENCE OF LENDING RELATIONSHIPS



Note: Impact of a shock to the lending standards. Numbers on the horizontal axis represent quarters since the shock. Numbers on the vertical axis show percentage deviation from the steady state.

greater persistence of lending relationships, like greater degrees of habits in lending, lead to larger macroeconomic amplification and volatility.

The findings from these exercises can be summed up as the following. A negative shock to the bank lending standard leads banks to raise their spread which results in larger drop in bank credit. Banks raise their spread several times more when they have lending relationships with

borrowers than when they do not. This reflects their ability to seek rents and their effort to protect their profits after a negative shock. After bank credit has fallen, banks rapidly lower their spread to win back customers and gain market share which then leads to bank credit rising fast, overshooting its previous steady-state level and staying above it for extended period. Notice that spread stays below its prior steady-state value during this protracted period. This greater fall, faster recovery and bigger increase in bank credit feeds into other macroeconomic variables such as investment, capital, labour, output and consumption which then leads to significant macroeconomic amplification and volatility. These effects are largely absent in a model without lending relationships. Macroeconomic aggregates in that setup react less to a shock to the lending standard and mostly revert to their previous steady-state value after a shock. They show no amplification and display a muted response. This highlights the important role of credit relationships and shows how shocks to the credit standards can have much higher impact and how they can cause macroeconomic amplification and volatility when presence of borrower-lender credit relationships are taken into account.

5 Conclusion

This paper contributes to the macro-banking and macro-finance literature by building a model in which banks compete on both interest rates and collateral requirements and have lending relationships with their borrowers. A shock to credit standards in this economy leads to a spike in spreads, fall in bank credit and a drop in overall economic activity. The model shows significant macroeconomic volatility and amplification in the aftermath of a shock to credit standards which is missing in a model without lending relationships. The paper shows that taking bank-firm lending relationships into account is important for study of financial shocks and illustrates how shocks originating in financial sector can affect larger real economy. It also shows that effects of shocks to lending standards are increasing in intensity and persistence of credit relationships. These results suggest that features of banking sector, and presence of borrower-lender relationships in particular, should be considered when examining effects of various shocks that might affect lending standards since they can have ripple effects on wider macroeconomy.

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Appendix (For Online Publication)

Shocks to the Lending Standards and the Macroeconomy

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A Derivation of FOCs

A.1 Households

The Lagrangian of patient households is

$$\mathscr{L}_t = \mathbb{E}_t \left\{ \sum_{t=0}^{\infty} \left(\beta^P\right)^t \left[\begin{array}{c} \log\left(C_{i,t}^P - \gamma^P C_{i,t-1}^P\right) - \frac{N_{i,t}^{\nu}}{\nu} + \varsigma \log H_{i,t}^P \\ -\lambda_{i,t}^P \left[\begin{array}{c} C_{i,t}^P + Q_t^H \left(H_{i,t}^P - H_{i,t-1}^P\right) + \int_0^1 D_{ik,t} \mathrm{d}k \\ -W_t N_{i,t} - \int_0^1 \Pi_{ik,t} \mathrm{d}k - R_{t-1}^D \int_0^1 D_{ik,t-1} \mathrm{d}k \end{array} \right] \end{array} \right] \right\} \tag{A.1}$$

The problem yields the following first order conditions (here, I ignore all the i's denoting individual patient households):

$$\frac{\partial \mathscr{L}}{\partial C_t^P} : \frac{1}{C_t^P - \gamma^P C_{t-1}^P} - \beta^P \mathbb{E}_P \frac{\gamma^P}{C_{t+1}^P - \gamma^P C_t^P} = \lambda_t^P \tag{A.2}$$

$$\frac{\partial \mathscr{L}}{\partial D_t} : \beta^P \mathbb{E}_t \lambda_{t+1}^P = \frac{\lambda_t^P}{R_t^D} \tag{A.3}$$

$$\frac{\partial \mathscr{L}}{\partial H_t^P} : \frac{\varsigma}{H_t^P} + \beta^P \mathbb{E}_t \left(\lambda_{t+1}^P Q_{t+1}^H\right) = \lambda_t^P Q_t^H \tag{A.4}$$

$$\frac{\partial \mathscr{L}}{\partial N_t} : N_t^{\nu-1} = \lambda_t^P W_t \tag{A.5}$$

A.2 Entrepreneurs

Entrepreneur's optimization problem features two parts. The first part consists of choosing how much to borrow from each individual bank, $l_{jk,t}$ to minimize his total interest rate expenditure and amount of collateral he has to pledge. Denoting by parameter $\eta \in (0,1)$ the relative weight attached to collateral minimization motive, the problem can be framed as

$$\min_{l_{jk,t}} \left[\int_0^1 R_{k,t}^L l_{jk,t} \mathrm{d}k + \eta \int_0^1 \frac{l_{jk,t}}{\theta_{k,t}} \mathrm{d}k \right] - \chi_t \left[x_{j,t} - \left(\int_0^1 \left(l_{jk,t} - \gamma^L s_{k,t-1}\right)^{\frac{\xi-1}{\xi}} \mathrm{d}k \right)^{\frac{\xi}{\xi-1}} \right] \tag{A.6}$$

This can be rewritten as

$$\min_{l_{jk,t}} \left[\int_0^1 \Upsilon_{k,t} l_{jk,t} \mathrm{d}k \right] - \chi_t \left[x_{j,t} - \left(\int_0^1 \left(l_{jk,t} - \gamma^L s_{k,t-1}\right)^{\frac{\xi-1}{\xi}} \mathrm{d}k \right)^{\frac{\xi}{\xi-1}} \right] \tag{A.7}$$

The first order condition for this problem is

$$R_{k,t}^L + \eta \frac{1}{\theta_{k,t}} = -\frac{\xi}{\xi - 1} \chi_t \left(\int_0^1 \left(l_{jk,t} - \gamma^L s_{k,t-1} \right)^{\frac{\xi-1}{\xi}} \mathrm{d}k \right)^{\frac{1}{\xi-1}} \frac{\xi - 1}{\xi} \left(l_{jk,t} - \gamma^L s_{k,t-1} \right)^{-\frac{1}{\xi}} \tag{A.8}$$

The first order condition can be rewritten as

$$\begin{aligned}
R_{k,t}^L + \eta \frac{1}{\theta_{k,t}} &= -\chi_t \left(\int_0^1 \left(l_{jk,t} - \gamma^L s_{k,t-1} \right)^{\frac{\xi-1}{\xi}} \mathrm{d}k \right)^{\frac{1}{\xi-1}} \left(l_{jk,t} - \gamma^L s_{k,t-1} \right)^{-\frac{1}{\xi}} \\
\left(R_{k,t}^L + \eta \frac{1}{\theta_{k,t}} \right) \left(l_{jk,t} - \gamma^L s_{k,t-1} \right) &= -\chi_t \left(\int_0^1 \left(l_{jk,t} - \gamma^L s_{k,t-1} \right)^{\frac{\xi-1}{\xi}} \mathrm{d}k \right)^{\frac{1}{\xi-1}} \left(l_{jk,t} - \gamma^L s_{k,t-1} \right)^{\frac{\xi-1}{\xi}} \\
\int_0^1 \left(R_{k,t}^L + \eta \frac{1}{\theta_{k,t}} \right) \left(l_{jk,t} - \gamma^L s_{k,t-1} \right) \mathrm{d}k &= -\chi_t \left(\int_0^1 \left(l_{jk,t} - \gamma^L s_{k,t-1} \right)^{\frac{\xi-1}{\xi}} \mathrm{d}k \right)^{\frac{1}{\xi-1}} \int_0^1 \left(l_{jk,t} - \gamma^L s_{k,t-1} \right)^{\frac{\xi-1}{\xi}} \mathrm{d}k
\end{aligned}$$

$$\int_0^1 R_{k,t}^L \left(l_{jk,t} - \gamma^L s_{k,t-1} \right) \mathrm{d}k + \eta \int_0^1 \frac{1}{\theta_{k,t}} \left(l_{jk,t} - \gamma^L s_{k,t-1} \right) \mathrm{d}k = -\chi_t \left(\int_0^1 \left(l_{jk,t} - \gamma^L s_{k,t-1} \right)^{\frac{\xi-1}{\xi}} \mathrm{d}k \right)^{\frac{\xi}{\xi-1}} \tag{A.9}$$

$$x_{j,t} = -\frac{1}{\chi_t} \left[\int_0^1 R_{k,t}^L \left(l_{jk,t} - \gamma^L s_{k,t-1} \right) \mathrm{d}k + \eta \int_0^1 \frac{1}{\theta_{k,t}} \left(l_{jk,t} - \gamma^L s_{k,t-1} \right) \mathrm{d}k \right] \quad \ddagger \tag{A.10}$$

At the optimum, the following conditions must hold

$$\begin{aligned}
\frac{1}{\theta_t} x_{j,t} &= \int_0^1 \frac{1}{\theta_{k,t}} \left(l_{jk,t} - \gamma^L s_{k,t-1} \right) \mathrm{d}k \\
R_t^L x_{j,t} &= \int_0^1 R_{k,t}^L \left(l_{jk,t} - \gamma^L s_{k,t-1} \right) \mathrm{d}k
\end{aligned}$$

‡ can be rewritten as

$$\begin{aligned}
x_{j,t} &= -\frac{1}{\chi_t} \left[R_t^L x_{j,t} + \eta \frac{1}{\theta_t} x_{j,t} \right] \\
-\chi_t &= R_t^L + \eta \frac{1}{\theta_t}
\end{aligned}$$

Inserting this in first order condition

$$
\begin{aligned}
R_{k,t}^{L}+\eta\frac{1}{\theta_{k,t}} &= -\frac{\xi}{\xi-1}\chi_t\left(\int_0^1\left(l_{jk,t}-\gamma^L s_{k,t-1}\right)^{\frac{\xi-1}{\xi}}\mathrm{d}k\right)^{\frac{1}{\xi-1}}\frac{\xi-1}{\xi}\left(l_{j,t}-\gamma^L s_{k,t-1}\right)^{-\frac{1}{\xi}}\\
R_{k,t}^{L}+\eta\frac{1}{\theta_{k,t}} &= \left(R_t^L+\eta\frac{1}{\theta_t}\right)\left(\int_0^1\left(l_{jk,t}-\gamma^L s_{k,t-1}\right)\mathrm{d}k\right)^{\frac{1}{\xi-1}}\left(l_{jk,t}-\gamma^L s_{k,t-1}\right)^{-\frac{1}{\xi}}\\
R_{k,t}^{L}+\eta\frac{1}{\theta_{k,t}} &= \left(R_{k,t}^L+\eta\frac{1}{\theta_t}\right)x_t^{\frac{1}{\xi}}\left(l_{jk,t}-\gamma^L s_{k,t-1}\right)^{-\frac{1}{\xi}}\\
\left(l_{jk,t}-\gamma^L s_{k,t-1}\right)^{\frac{1}{\xi}} &= (x_t)^{\frac{1}{\xi}}\frac{R_t^L+\eta\frac{1}{\theta_t}}{R_{K,t}^L+\eta\frac{1}{\theta_{k,t}}}\\
l_{jk,t} &= \left(\frac{R_t^L+\eta\frac{1}{\theta_t}}{R_{k,t}^L+\eta\frac{1}{\theta_{k,t}}}\right)^{\xi}x_t+\gamma^L s_{k,t-1}\\
l_{jk,t} &= \left(\frac{\Upsilon_t}{\Upsilon_{k,t}}\right)^{\xi}x_t+\gamma^L s_{k,t-1}\\
l_{jk,t} &= \left(\frac{\Upsilon_{k,t}}{\Upsilon_t}\right)^{-\xi}x_t+\gamma^L s_{k,t-1}
\end{aligned}
$$

When η is high, the entrepreneur attaches higher importance to collateral minimization motive. As a result, relative LTV ratios become more important for determination of demand for loans from each bank. Indeed, as η tends to zero, the entrepreneur cares only about minimizing his interest expenditure

$$
\lim_{\eta\to 0}\left(\frac{\Upsilon_{k,t}}{\Upsilon}\right)^{-\xi}=\lim_{\eta\to 0}\left(\frac{R_{k,t}^L+\eta\frac{1}{\theta_{k,t}}}{R_t^L+\eta\frac{1}{\theta_t}}\right)^{-\xi}=\left(\frac{R_{k,t}^L}{R_t^L}\right)^{-\xi}
$$

which is a standard demand function under monopolistic competition which states that demand for loans from bank k is a negative function of bank k's lending rate $R_{k,t}^L$ relative to aggregate lending rate R_t^L. Now, the second part of entrepreneur's optimization problem can be written as

$$
\mathscr{L}_t=\mathbb{E}_t\left\{\sum_{t=0}^{\infty}\left(\beta^E\right)^t\left[\begin{array}{c}
\log\left(C_{j,t}^E-\gamma^E C_{j,t-1}^E\right)\\
-\lambda_{j,t}^E\left[\begin{array}{c}C_{j,t}^E+R_{k,t-1}^L\int_0^1 l_{jk,t-1}\mathrm{d}k-Y_{j,t}+W_tN_{j,t}+I_{j,t}\\+Q_t^H\left(H_{j,t}^E-H_{j,t-1}^E\right)-x_{j,t}-\Phi_t-\Psi_t\end{array}\right]\\
-\mu_{j,t}^E\left[R_{k,t}^L\int_0^1 l_{jk,t}\mathrm{d}k-\int_0^1\theta_{k,t}\mathrm{d}k\mathbb{E}_t\left(Q_{t+1}^H H_{j,t}^E+Q_{t+1}^K K_{j,t}\right)\right]\\
-\kappa_{j,t}^E\left[K_{j,t}-(1-\delta)K_{j,t-1}-\left\{1-\frac{\Omega}{2}\left(\frac{I_{j,t}}{I_{j,t-1}}-1\right)^2\right\}I_{j,t}\right]\\
-\epsilon_{j,t}^E\left[x_{j,t}-\left\{\int_0^1\left(l_{jk,t}-\gamma^L s_{k,t-1}\right)^{\frac{\xi-1}{\xi}}\mathrm{d}k\right\}^{\frac{\xi}{\xi-1}}\right]
\end{array}\right]\right\} \tag{A.11}
$$

where $Y_{j,t}=A_t\left(N_{j,t}\right)^{1-\alpha}\left\{\left(H_{j,t-1}^E\right)^{\phi}\left(K_{j,t-1}\right)^{1-\phi}\right\}^{\alpha}$ may be inserted for $Y_{j,t}$ in the budget constraint.

Solving entrepreneur's optmization problem, the first order conditions are (I ignore all j's here):

$$\frac{\partial \mathscr{L}}{\partial C_t^E} : \frac{1}{C_t^E - \gamma^E C_{t-1}^E} - \beta^E \mathbb{E}_t \frac{\gamma^E}{C_{t+1}^E - \gamma^E C_t^E} = \lambda_t^E \tag{A.12}$$

$$\frac{\partial \mathscr{L}}{\partial x_t} : \lambda_t^E = \epsilon_t^E \tag{A.13}$$

$$\frac{\partial \mathscr{L}}{\partial l_t} : \epsilon_t^E = \beta^E \mathbb{E}_t \lambda_{t+1}^E R_t^L + \mu_t^E R_t^L \tag{A.14}$$

$$\frac{\partial \mathscr{L}}{\partial N_t} : W_t = (1-\alpha)\frac{Y_t}{N_t} \tag{A.15}$$

$$\frac{\partial \mathscr{L}}{\partial H_t^E} : \lambda_t^E Q_t^H = \beta^E \mathbb{E}_t \left\{ \lambda_{t+1}^E \left(Q_{t+1}^H + \alpha\phi \frac{Y_{t+1}}{H_t^E} \right) \right\} + \mu_t^E \theta_t \mathbb{E}_t Q_{t+1}^H \tag{A.16}$$

$$\frac{\partial \mathscr{L}}{\partial K_t} : \kappa_t^E = \alpha(1-\phi)\beta^E \mathbb{E}_t \left(\frac{\lambda_{t+1}^E Y_{t+1}}{K_t} \right) + \beta^E (1-\delta) \mathbb{E}_t \kappa_{t+1}^E + \mu_t^E \theta_t \mathbb{E}_t Q_{t+1}^K \tag{A.17}$$

$$\frac{\partial \mathscr{L}}{\partial I_t} : \lambda_t^E = \kappa_t^E \left\{ 1 - \frac{\Omega}{2}\left(\frac{I_t}{I_{t-1}} - 1 \right)^2 - \Omega \frac{I_t}{I_{t-1}} \left(\frac{I_t}{I_{t-1}} - 1 \right) \right\} + \beta^E \Omega \mathbb{E}_t \left\{ \kappa_{t+1}^E \left(\frac{I_{t+1}}{I_t} \right)^2 \left(\frac{I_{t+1}}{I_t} - 1 \right) \right\} \tag{A.18}$$

Using $\lambda_t^E = \epsilon_t^E$ from (A.13), (A.14) becomes

$$\beta^E \mathbb{E}_t \left(\lambda_{t+1}^E R_t^L \right) + \mu_t^E R_t^L = \lambda_t^E \tag{A.19}$$

A.3 Banks

The problem of banks is to choose their lending rate, LTV ratio and the total amount of lending. The bank considers deep habits in loan demand as well as adverse selection which is given by

$$p_{k,t}\zeta_t = \Xi + \varpi \left(\theta_{k,t} - \overline{\theta} \right) \tag{A.20}$$

The bank solves the following problem

$$\begin{aligned} \max_{L_{k,t}, \theta_{k,t}, R_{k,t}^L} \Pi_t = & \left[\Xi + \varpi \left(\theta_{k,t} - \overline{\theta} \right) \right] R_{k,t-1}^L L_{k,t-1} + \left[1 - \Xi + \varpi \left(\theta_{k,t} - \overline{\theta} \right) \right] \frac{L_{k,t-1}}{\int_0^1 L_{k,t-1} \mathrm{d}k} \tau \theta_{t-1} a_{t-1} \\ & - R_{t-1}^D L_{k,t-1} + \varrho_t^E \left(\int_0^1 \left[\left(\frac{R_t^L + \eta \frac{1}{\theta_t}}{R_{k,t}^L + \eta \frac{1}{\theta_{k,t}}} \right)^{\xi} x_t + \gamma^L s_{k,t-1} \right] \mathrm{d}j - L_{k,t} \right) \end{aligned}$$

The first order condition for $L_{k,t}$ is

$$\begin{aligned} & \mathbb{E}_t q_{t,t+1} p_{k,t} R_{k,t}^L + \mathbb{E}_t q_{t,t+1} (1 - p_{k,t}) \frac{\tau \theta_t a_t}{\int_0^1 L_{k,t} \mathrm{d}k} - \mathbb{E}_t q_{t,t+1} R_t^D + \gamma^L (1 - \rho_s) \mathbb{E}_t \left(q_{t,t+1} \varrho_{t+1}^E \right) - \varrho_t^E = 0 \\ & \varrho_t^E = \mathbb{E}_t q_{t,t+1} \left[p_{k,t} R_{k,t}^L + (1 - p_{k,t}) \frac{\tau \theta_t a_t}{\int_0^1 L_{k,t} \mathrm{d}k} - R_t^D + \gamma^L (1 - \rho_s) \mathbb{E}_t \varrho_{t+1}^E \right] \end{aligned} \tag{A.21}$$

The first order condition for $\theta_{k,t}$ is

$$\varpi \mathbb{E}_t q_{t,t+1} R^L_{k,t} L_{k,t} - \varpi \mathbb{E}_t q_{t,t+1} \frac{L_{k,t}}{\int_0^1 L_{k,t} \mathrm{d}k} \tau \theta_t a_t + \xi \varrho^E_t \left(\frac{R^L_t + \eta^{\frac{1}{\theta_t}}}{R^L_{k,t} + \eta^{\frac{1}{\theta_{k,t}}}} \right)^{\xi - 1} x_t \left(\frac{\eta^{\frac{1}{\theta^2_{k,t}\left(R^L_t + \eta^{\frac{1}{\theta_t}}\right)}}}{R^L_{k,t} + \eta^{\frac{1}{\theta_{k,t}}}} \right)^2 = 0$$

$$\Rightarrow \xi \varrho^E_t x_t \left(\frac{R^L_t + \eta^{\frac{1}{\theta_t}}}{R^L_{k,t} + \eta^{\frac{1}{\theta_{k,t}}}} \right)^{\xi - 1} \frac{\eta^{\frac{1}{\theta^2_{k,t}}} \left(R^L_t + \eta^{\frac{1}{\theta_t}} \right)}{\left(R^L_{k,t} + \eta^{\frac{1}{\theta_{k,t}}} \right)^2} = -\varpi \mathbb{E}_t q_{t,t+1} \left[R^L_{k,t} L_{k,t} - \frac{L_{k,t}}{\int_0^1 L_{k,t} \mathrm{d}k} \tau \theta_t a_t \right] \tag{A.22}$$

The first order condition for $R^L_{k,t}$ is

$$\mathbb{E}_t q_{t,t+1} p_{k,t} L_{k,t} + \xi \varrho^E_t \left(\frac{R^L_t + \eta^{\frac{1}{\theta_t}}}{R^L_{k,t} + \eta^{\frac{1}{\theta_t}}} \right)^{\xi - 1} x_t \left(\frac{-\left(R^L_t + \eta^{\frac{1}{\theta_t}} \right)}{\left(R^L_{k,t} + \eta^{\frac{1}{\theta_{k,t}}} \right)^2} \right) = 0$$

$$\Rightarrow \mathbb{E}_t q_{t,t+1} p_{k,t} L_{k,t} = \xi \varrho^E_t x_t \left(\frac{R^L_t + \eta^{\frac{1}{\theta_t}}}{R^L_{k,t} + \eta^{\frac{1}{\theta_t}}} \right)^{\xi - 1} \left(\frac{\left(R^L_t + \eta^{\frac{1}{\theta_t}} \right)}{\left(R^L_{k,t} + \eta^{\frac{1}{\theta_{k,t}}} \right)^2} \right) \tag{A.23}$$

In a symmetric equilibrium all banks have the same LTV ratio $\theta_{k,t} = \theta, \forall k$ and the same lending rate $R^L_{k,t} = R^L_t, \forall k$ and consequently lend the same amount $L_{k,t} = L_t, \forall k$. Bank's first order condition in this case can be rewritten as

$$\varrho^E_t = \mathbb{E}_t q_{t,t+1} \left[p_{k,t} R^L_{k,t} + (1 - p_{k,t}) \frac{\tau \theta_t a_t}{\int_0^1 L_{k,t} \mathrm{d}k} - R^D_t + \gamma^L (1 - \rho_s) \mathbb{E}_t \varrho^E_{t+1} \right] \tag{A.24}$$

$$\xi \varrho^E_t x_t \frac{\frac{\eta}{\theta}}{R^L_t \theta_t + \eta} = -\varpi \mathbb{E}_t q_{t,t+1} \left(R^L_t L_t - \tau \theta_t a_t \right) \tag{A.25}$$

$$\xi \varrho^E_t x_t \frac{\theta_t}{\theta_t R^L_t + \eta} = \mathbb{E}_t q_{t,t+1} p_{k,t} L_{k,t} \tag{A.26}$$

where I have imposed $L_t = l_t$ in a symmetric equilibrium and that the collateral constraint is always binding (holds with equality at all times).

B List of Equations

B.1 Households

$$\frac{1}{C^P_t - \gamma^P C^P_{t-1}} - \beta^P \mathbb{E}_t \frac{\gamma^P}{C^P_{t+1} - \gamma^P C^P_t} = \lambda^P_t \tag{B.1}$$

$$\beta^P \mathbb{E}_t \lambda^P_{t+1} = \frac{\lambda^P_t}{R^D_t} \tag{B.2}$$

$$\frac{\varsigma}{H^P_t} + \beta^P \mathbb{E}_t \left(\lambda^P_{t+1} Q^H_{t+1} \right) = \lambda^P_t Q^H_t \tag{B.3}$$

$$N_t^{\nu-1} = \lambda_t^P W_t \tag{B.4}$$

B.2 Entrepreneurs

$$\frac{1}{C_t^E - \gamma^E C_{t-1}^E} - \beta^E \mathbb{E}_t \frac{\gamma^E}{C_{t+1}^E - \gamma^E C_t^E} = \lambda_t^E \tag{B.5}$$

$$\beta^E \mathbb{E}_t \left(\lambda_{t+1}^E R_t^L\right) + \mu_t^E R_t^L = \lambda_t^E \tag{B.6}$$

$$W_t = (1-\alpha)\frac{Y_t}{N_t} \tag{B.7}$$

$$\lambda_t^E Q_t^H = \beta^E \mathbb{E}_t \left\{\lambda_{t+1}^E \left(Q_{t+1}^H + \alpha\phi \frac{Y_{t+1}}{H_t^E}\right)\right\} + \mu_t^E \theta_t \mathbb{E}_t Q_{t+1}^H \tag{B.8}$$

$$\kappa_t^E = \alpha(1-\phi)\beta^E \mathbb{E}_t \left(\frac{\lambda_{t+1}^E Y_{t+1}}{K_t}\right) + \beta^E (1-\delta)\mathbb{E}_t \kappa_{t+1}^E + \mu_t^E \theta_t \mathbb{E}_t Q_{t+1}^K \tag{B.9}$$

$$\lambda_t^E = \kappa_t^E \left\{1 - \frac{\Omega}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 - \Omega \frac{I_t}{I_{t-1}}\left(\frac{I_t}{I_{t-1}} - 1\right)\right\} + \beta^E \Omega \mathbb{E}_t \left\{\kappa_{t+1}^E \left(\frac{I_{t+1}}{I_t}\right)^2 \left(\frac{I_{t+1}}{I_t} - 1\right)\right\} \tag{B.10}$$

$$s_t = \rho_s s_{t-1} + (1-\rho_s) l_t \tag{B.11}$$

$$x_t = \left(l_t - \gamma^L s_{t-1}\right) \tag{B.12}$$

$$L_t = l_t \tag{B.13}$$

$$C_t^E + R_{t-1}^L l_{t-1} = Y_t - W_t N_t - I_t - Q_t \left(H_t^E - H_{t-1}^E\right) + x_t + \Phi_t + \Psi_t \tag{B.14}$$

$$l_t = \frac{\theta_t a_t}{R_t^L} \tag{B.15}$$

$$a_t = \mathbb{E}_t \left(Q_{t+1}^H H_t^E + Q_{t+1}^K K_t\right) \tag{B.16}$$

$$\kappa_t^E = \lambda_t^E Q_t^K \tag{B.17}$$

B.3 Banks

$$\varrho_t^E = \mathbb{E}_t q_{t,t+1} \left[p_{k,t} R_{k,t}^L + (1-p_{k,t}) \frac{\tau \theta_t a_t}{\int_0^1 L_{k,t} \mathrm{d}k} - R_t^D + \gamma^L (1-\rho_s) \mathbb{E}_t \varrho_{t+1}^E\right] \tag{B.18}$$

$$\xi \varrho_t^E x_t \frac{\frac{\eta}{\theta}}{R_t^L \theta_t + \eta} = -\varpi \mathbb{E}_t q_{t,t+1} \left(R_t^L L_t - \tau \theta_t a_t\right) \tag{B.19}$$

$$\xi \varrho_t^E x_t \frac{\theta_t}{\theta_t R_t^L + \eta} = \mathbb{E}_t q_{t,t+1} p_{k,t} L_{k,t} \tag{B.20}$$

$$\Pi_{k,t} = p_t R_{k,t-1}^L L_{k,t-1} + (1-p_t) \frac{L_{k,t-1}}{\int_0^1 L_{k,t-1} \mathrm{d}k} \tau \theta_{t-1} a_{t-1} + \int_0^1 D_{ik,t} \mathrm{d}i - L_{k,t} - R_{t-1}^D \int_0^1 D_{ik,t-1} \mathrm{d}i \tag{B.21}$$

$$L_t = D_t \tag{B.22}$$

$$q_{t,t+1} = \beta^P \mathbb{E}_t \frac{\lambda^P_{t+1}}{\lambda^P_t} \tag{B.23}$$

$$p_t \zeta_t = \Xi + \varpi \left(\theta_t - \overline{\theta}\right) \tag{B.24}$$

B.4 Market Clearing and Resource Constraints

$$C^P_t + C^E_t + I_t = Y_t \tag{B.25}$$

$$H^P_t + H^E_t = H \tag{B.26}$$

$$Y_t = A_t \left(N_t\right)^{1-\alpha} \left\{ \left(H^E_{t-1}\right)^{\phi} \left(K_{t-1}\right)^{1-\phi} \right\}^{\alpha} \tag{B.27}$$

$$K_t = \left(1-\delta\right) K_{t-1} + \left\{ 1 - \frac{\Omega}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \right\} I_t \tag{B.28}$$

C Steady State Conditions

All $i's$, $j's$ and $k's$ denoting individual household, entrepreneur and bank respectively are ignored. From household's FOC with respect to consumption (B.1) and labor (B.4), I have

$$\frac{1-\beta^P \gamma^P}{\left(1-\gamma^P\right) C^P} = \lambda^P, \tag{C.1}$$

and

$$N^{\nu-1} = \lambda^P W, \tag{C.2}$$

respectively. Household's FOC with respect to deposit (B.2) yields the steady-state gross interest rate

$$R^D = \frac{1}{\beta^P}, \tag{C.3}$$

underscoring that the time preference of most patient agent determines the steady-state rate of interest. From (B.3), I obtain

$$\begin{aligned} \frac{\varsigma}{H^P} + \beta^P \lambda^P Q^H &= \lambda^P Q^H, \\ \Rightarrow Q^H H^P &= \frac{\varsigma}{\lambda^P \left(1-\beta^P\right)}, \\ \Rightarrow H^P &= \frac{\varsigma}{Q^H \lambda^P \left(1-\beta^P\right)}. \end{aligned} \tag{C.4}$$

I next turn to entrepreneurs. Their consumption FOC (B.5) yields

$$\frac{1-\beta^E\gamma^E}{(1-\gamma^E)C^E}=\lambda^E \tag{C.5}$$

Entrepreneur's FOC with respect to loans (B.6) gives

$$\begin{aligned}\beta^E\lambda^E R^L+\mu^E R^L&=\lambda^E\\ \Rightarrow \mu^E&=\frac{\lambda^E(1-\beta^E R^L)}{R^L}\end{aligned} \tag{C.6}$$

The borrowing constraint for entrepreneurs binds if and only if μ^E is positive. This implies that β^E must be less than R^L. In the baseline calibration, β^E is set to 0.95 whereas the steady state value of R^L is 1.0219 which implies that β^E must be less than 0.9786 which is indeed the case.

Entrepreneur's production function is

$$Y=A(N)^{1-\alpha}\left[(H^E)^{\phi}(K)^{1-\phi}\right]^{\alpha} \tag{C.7}$$

Firm's labor choice for households (B.7) yields

$$W=(1-\alpha)\frac{Y}{N} \tag{C.8}$$

From entrepreneur's FOC with respect to housing (B.8), I obtain

$$\begin{aligned}\lambda^E Q^H&=\beta^E\lambda^E\left(Q^H+\alpha\phi\frac{Y}{H^E}\right)+\mu^E\theta Q^H\\ \Rightarrow \frac{Q^H H^E}{Y}&=\frac{\beta^E\alpha\phi R^L}{(1-\beta^E)R^L-\theta(1-\beta^E R^L)}\end{aligned} \tag{C.9}$$

Aggregate law of motion for capital (B.28) gives

$$\begin{aligned}K&=(1-\delta)K+\left[1-\frac{\Omega}{2}\left(\frac{I}{I}-1\right)\right]I\\ \Rightarrow I&=\delta K\end{aligned} \tag{C.10}$$

I have the following steady-state resource constraints

$$Y=C^P+C^E+I \tag{C.11}$$

$$H=H^P+H^E \tag{C.12}$$

$$D = L \tag{C.13}$$

Also, I have the following steady-state version of agents' budget constraints (one of them is redundant because of Walras' Law)

$$C^P = WN - (R^D - 1)D + \Pi \tag{C.14}$$

$$C^E = Y - R^L l - WN - I - x - \Phi - \Psi \tag{C.15}$$

So the steady state is characterized by the vector

$$\left[Y, C^P, C^E, I, H^P, H^E, K, N, L, D, Q^H, Q^K, R^D, R^L, W, \lambda^P, \lambda^E, \mu^E, \varrho^E, p\right]$$

From entrepreneur's optimal choice of capital (B.9), I have

$$\begin{aligned} \kappa_t^E &= \alpha\left(1-\alpha\right)\beta^E \mathbb{E}_t\left(\frac{\lambda_{t+1}^E Y_{t+1}}{K_t}\right) + \beta^E\left(1-\delta\right)\mathbb{E}_t \kappa_{t+1}^E + \mu_t^E \theta_t \mathbb{E}_t Q_{t+1}^K \\ \Rightarrow \frac{\kappa^E}{\lambda^E}\left(1-\left(1-\delta\right)\beta^E\right) &= \alpha\left(1-\phi\right)\beta^E \frac{Y}{K} + \frac{\left(1-\beta^E R^L\right)}{R^L}\theta Q^K \end{aligned} \tag{C.16}$$

Entrepreneur's optimal choice of investment (B.10) yields

$$\begin{aligned} \lambda_t^E(j) = \kappa_t^E(j) &\left[1 - \frac{\Omega}{2}\left(\frac{I_t(j)}{I_t(j-1)} - 1\right)^2 - \Omega \frac{I_t(j)}{I_t(j-1)}\left(\frac{I_t(j)}{I_{t-1}(j)} - 1\right)\right] \\ &+ \beta^E \Omega \mathbb{E}_t\left[\kappa_{t+1}^E(j)\left(\frac{I_{t+1}(j)}{I_t(j)}\right)^2\left(\frac{I_{t+1}(j)}{I_t(j)} - 1\right)\right] \\ \Rightarrow \lambda^E &= \kappa^E \end{aligned} \tag{C.17}$$

Combining this with steady state version of

$$\kappa^E = \lambda^E Q^K \tag{C.18}$$

I obtain $Q^K = 1$ in the steady state. Plugging this into (C.16), I obtain the expression for capital-to-output ratio

$$\begin{aligned} \frac{\kappa^E}{\lambda^E}\left(1-\left(1-\delta\right)\beta^E\right) &= \alpha\left(1-\phi\right)\beta^E \frac{Y}{K} + \frac{\left(1-\beta^E R^L\right)}{R^L}\theta Q^K \\ \Rightarrow \frac{K}{Y} &= \frac{\alpha\left(1-\phi\right)R^L \beta^E}{R^L\left(1-\left(1-\delta\right)\beta^E\right) - \theta\left(1-\beta^E R^L\right)} \end{aligned} \tag{C.19}$$

Next, combining (B.15) and (B.16)

$$l = \frac{\theta}{R^L}\Big[Q^H H^E + Q^K K\Big] \tag{C.20}$$

Dividing by Y, the above expression becomes

$$\frac{l}{Y} = \frac{\theta}{R^L}\left[\frac{Q^H H^E}{Y} + \frac{Q^K K}{Y}\right]$$

Plugging in the values of $\frac{Q^H H^E}{Y}$ and $\frac{Q^K K}{Y}$, I obtain entrepreneur's debt-to-output ratio

$$\frac{l}{Y} = \alpha\theta\beta^E\left[\frac{\phi}{R^L\left(1-\beta^E\right)-\theta\left(1-\beta^E R^L\right)} + \frac{(1-\phi)}{R^L\left(1-(1-\delta)\beta^E\right)-\theta\left(1-\beta^E R^L\right)}\right] \tag{C.21}$$

From entrepreneur's budget constraint (B.14)

$$C^E + R^L l = Y - WN - I + x + \Phi + \Psi \tag{C.22}$$

Rewriting this in ratios to output

$$\begin{aligned}\frac{C^E}{Y} + \frac{R^L l}{Y} &= 1 - \frac{WN}{Y} - \frac{I}{Y} + \frac{x}{Y} + \frac{\Phi}{Y} + \frac{\Psi}{Y} \\ \Rightarrow \frac{C^E}{Y} &= \alpha - \delta\frac{K}{Y} + \left(1-\gamma^L - R^L\right)\frac{l}{Y} + \frac{\Phi}{Y} + \frac{\Psi}{Y}\end{aligned} \tag{C.23}$$

where steady-state expressions for W and x have been used. Now, using the steady-state expressions for Φ and Ψ

$$\begin{aligned}\frac{C^E}{Y} &= \alpha - \delta\frac{K}{Y} + \left(1-\gamma^L - R^L\right)\frac{l}{Y} + \frac{\gamma^L s}{Y} + \frac{(1-p)\left(R^L L - \tau\theta a\right)}{Y} \\ \Rightarrow \frac{C^E}{Y} &= \alpha - \delta\frac{K}{Y} + \Big[1 - pR^L - (1-p)\tau R^L\Big]\frac{l}{Y}\end{aligned} \tag{C.24}$$

Entrepreneur's stock of habits for loans (B.11) leads to

$$\begin{aligned}s_t &= \rho_s s_{t-1} + (1-\rho_s)\, l_t \\ s &= l\end{aligned} \tag{C.25}$$

Entrepreneur's effective demand for loans (B.12) writes

$$\begin{aligned} x_t &= \left(l_t - \gamma^L s_{t-1}\right) \\ \Rightarrow x &= \left(1 - \gamma^L\right) l \end{aligned} \tag{C.26}$$

Total loans of entrepreneurs (B.13) are equal to loans given to all entrepreneurs by all banks

$$L = l \tag{C.27}$$

From bank's balance sheet condition (B.22), deposits must equal total loans to entrepreneurs

$$D = L \tag{C.28}$$

Steady-state version of bank's stochastic discount factor (B.23) reads

$$q = \beta^P \tag{C.29}$$

Now using the previous result and $\frac{\theta a}{L} = R^L$

$$\varrho^E = \beta^P \left[pR^L + (1-p)\,\tau R^L - R^D + \gamma^L (1-\rho_s)\,\varrho^E \right]$$

which can be rewritten as

$$\varrho^E = \beta^P \frac{pR^L + (1-p)\,\tau R^L - R^D}{1 - \beta^P \gamma^L (1-\rho_s)} \tag{C.30}$$

From bank's second FOC

$$\xi \varrho^E \left(\frac{\frac{\eta}{\theta}}{\theta R^L + \eta} \right) x = -\varpi \beta^P \left(R^L L - \tau \theta a \right)$$

After subsituting the expression for x

$$\xi \varrho^E \left(\frac{\frac{\eta}{\theta}}{\theta R^L + \eta} \right) \left(1 - \gamma^L\right) l = -\varpi \beta^P \left(R^L l - \tau R^L l \right)$$

This finally simplifies to

$$\xi \varrho^E \left(\frac{\frac{\eta}{\theta}}{\theta R^L + \eta} \right) \left(1 - \gamma^L\right) = -\varpi \beta^P R^L (1 - \tau) \tag{C.31}$$

The final FOC of banks optimization problem reads

$$\xi \varrho^E \left(\frac{\theta}{\theta R^L + \eta} \right) x = \beta^P pL$$

Rewriting this equation

$$\begin{aligned}
\xi \varrho^E \left(\frac{\theta}{\theta R^L + \eta} \right) \left(1 - \gamma^L\right) &= \beta^P p \\
\Rightarrow \xi \varrho^E \left(1 - \gamma^L\right) \frac{\theta}{\theta R^L + \eta} &= \beta^P p \\
\Rightarrow \xi \varrho^E \left(1 - \gamma^L\right) \theta &= \beta^P p \left(\theta R^L + \eta\right) \\
\Rightarrow \theta \Big[\xi \varrho^E \left(1 - \gamma^L\right) - \beta^P p R^L \Big] &= \beta^P p \eta \\
\Rightarrow \theta &= \frac{\beta^P p \eta}{\xi \varrho^E \left(1 - \gamma^L\right) - \beta^P p R^L}
\end{aligned} \tag{C.32}$$

(C.30), (C.31) and (C.32) form a system of 3 equations in 3 unknowns: ϱ^E, θ and R^L. In order to solve this syetm of equations, I first insert for ϱ^E from (C.30) into (C.31) and (C.32). This gives the following system of equation

$$\begin{aligned}
\xi \left(1 - \gamma^L\right) \frac{p R^L + (1-p)\tau R^L - R^D}{1 - \beta^P \gamma^L (1 - \rho_s)} \frac{\eta}{\theta} &= -\varpi R^L (1 - \tau) \left(\theta R^L + \eta\right) \\
\theta &= \frac{\beta^P p \eta}{\xi \left(1 - \gamma^L\right) \beta^P \dfrac{p R^L + (1-p)\tau R^L - R^D}{1 - \beta^P \gamma^L (1 - \rho_s)} - \beta^P p R^L}
\end{aligned}$$

Plugging the value of θ from the second equation into the first, I obtain the value of R^L after which values of ϱ^E and θ follow directly. This procedure determines the value of R^L exclusively from bank's problem which allows it to be inserted into equations derived from entrepreneur's problem.

Steady state version of aggregate resource constraint (B.25) is

$$\begin{aligned}
C^P + C^E + I &= Y \\
\Rightarrow \frac{C^P}{Y} &= 1 - \frac{C^E}{Y} - \delta \frac{K}{Y}
\end{aligned} \tag{C.33}$$

From steady state value of (B.24)

$$p\zeta = \Xi \tag{C.34}$$

Combining (C.1), (C.2) and (C.8) gives steady-state equilibrium condition for households

$$\begin{aligned} N^{\nu-1} &= \lambda^P W \\ \Rightarrow N^{\nu-1} &= \frac{1-\beta^P\gamma^P}{\left(1-\gamma^P\right)C^P}\left(1-\alpha\right)\frac{Y}{N} \\ \Rightarrow N &= \left[\frac{\left(1-\beta^P\gamma^P\right)\left(1-\alpha\right)}{\left(1-\gamma^P\right)}\left(\frac{C^P}{Y}\right)^{-1}\right]^{\frac{1}{\nu}} \end{aligned} \tag{C.35}$$

From (B.27), steady state output is

$$\begin{aligned} Y &= A\left(N\right)^{1-\alpha}\left[\left(H^E\right)^{\phi}\left(K\right)^{1-\phi}\right]^{\alpha} \\ Y^{1-\alpha} &= A\left(N\right)^{1-\alpha}\left[\left(\frac{H^E}{Y}\right)^{\phi}\left(\frac{K}{Y}\right)^{1-\phi}\right]^{\alpha} \\ Y^{1-\alpha} &= A\left(N\right)^{1-\alpha}\left[\left(\frac{H^E}{Y}\right)^{\phi}\left(\frac{\alpha\left(1-\phi\right)R^L\beta^E}{R^L\left(1-\left(1-\delta\right)\beta^E\right)-\theta\left(1-\beta^E R^L\right)}\right)^{1-\phi}\right]^{\alpha} \end{aligned} \tag{C.36}$$

From Equation (C.4)

$$Q^H = \frac{\varsigma}{H^P\lambda^P\left(1-\beta^P\right)} \tag{C.37}$$

D System of Loglinear Equations

The system of equations log-linearized around their steady state is as below:

D.1 Optimality Conditions of Households

Equations (B.1), (B.2) and (B.4) become

$$\beta^P\gamma^P\mathbb{E}_t\widehat{C}^P_{t+1} - \left(1+\left(\gamma^P\right)^2\beta^P\right)\widehat{C}^P_t + \gamma^P\widehat{C}^P_{t-1} = \left(1-\beta^P\gamma^P\right)\left(1-\gamma^P\right)\widehat{\lambda}^P \tag{D.1}$$

$$\mathbb{E}_t\widehat{\lambda}^P_{t+1} = \widehat{\lambda}^P_t - \widehat{R}^D_t \tag{D.2}$$

$$\left(\nu-1\right)\widehat{N}_t = \widehat{\lambda}^P_t + \widehat{W}_t \tag{D.3}$$

Log-linearization of (B.3) yields

$$\beta^P\mathbb{E}_t\left[\widehat{\lambda}^P_{t+1} + \widehat{Q}^H_{t+1} + \widehat{H}^P_t\right] = \widehat{\lambda}^P_t + \widehat{Q}^H_t + \widehat{H}^P_t \tag{D.4}$$

D.2 Optimality Conditions of Entrepreneurs

From (B.5) and (B.6), I have

$$\beta^E\gamma^E\mathbb{E}_t\widehat{C}^E_{t+1}-\left(1+\left(\gamma^E\right)^2\beta^E\right)\widehat{C}^E_t+\gamma^E\widehat{C}^E_{t-1}=\left(1-\beta^E\gamma^E\right)\left(1-\gamma^E\right)\widehat{\lambda}^E_t \tag{D.5}$$

and

$$\widehat{\lambda}^E_t=\widehat{R}^L_t+\beta^E R^L\mathbb{E}_t\widehat{\lambda}^E_{t+1}+\left(1-\beta^E R^L\right)\widehat{\mu}^E_t \tag{D.6}$$

Equation (B.7) yields

$$\widehat{W}_t=\widehat{Y}_t-\widehat{N}_t \tag{D.7}$$

From (B.8), I derive

$$\begin{aligned}\left(\widehat{\lambda}^E_t+\widehat{Q}^H_t\right)=\beta^E\mathbb{E}_t\left(\widehat{\lambda}^E_{t+1}+\widehat{Q}^H_{t+1}\right)+\left(\frac{1}{R^L}-\beta^E\right)\theta\mathbb{E}_t\left(\widehat{\mu}^E_t+\widehat{\theta}_t+\widehat{Q}^H_{t+1}\right)\\+\left[\left(1-\beta^E\right)-\theta\left(\frac{1}{R^L}-\beta^E\right)\right]\mathbb{E}_t\left[\widehat{\lambda}^E_{t+1}+\widehat{Y}_{t+1}-\widehat{H}^E_t\right]\end{aligned} \tag{D.8}$$

Equation (B.9) becomes

$$\begin{aligned}\widehat{Q}^K_t=&\left[1-\beta^E\left(1-\delta\right)-\theta\left(\frac{1}{R^L}-\beta^E\right)\right]\mathbb{E}_t\left[\widehat{\lambda}^E_{t+1}-\lambda^E_t+\widehat{Y}_{t+1}-K_t\right]\\&+\beta^E\left(1-\delta\right)\mathbb{E}_t\left(\widehat{Q}^K_{t+1}+\widehat{\lambda}^E_{t+1}-\widehat{\lambda}^E_t\right)+\left(1-\beta^E R^L\right)\frac{1}{R^L}\theta\mathbb{E}_t\left[\widehat{\mu}^E_t-\widehat{\lambda}^E_t+\widehat{\theta}_t+\widehat{Q}^K_{t+1}\right]\end{aligned} \tag{D.9}$$

Equation (B.10) is approximated as

$$\widehat{Q}^K_t=\left(1+\beta^E\right)\Omega\widehat{I}_t-\beta^E\Omega\mathbb{E}_t\widehat{I}_{t+1}-\Omega\widehat{I}_{t-1} \tag{D.10}$$

From (B.11) and (B.12), I get

$$\widehat{s}_t=\rho_s\widehat{s}_{t-1}+\left(1-\rho_s\right)\widehat{l}_t \tag{D.11}$$

and

$$\widehat{x}_t=\frac{\widehat{l}_t}{1-\gamma^L}-\frac{\gamma^L\widehat{s}_{t-1}}{1-\gamma^L} \tag{D.12}$$

Entrepreneurs' budget contraint (B.14) becomes

$$\begin{aligned} C^E\widehat{C}_t^E + R^L l\left(\widehat{R}_{t-1}^L + \widehat{l}_{t-1}\right) = Y\widehat{Y}_t - WN\left(\widehat{W}_t + \widehat{N}_t\right) - I\widehat{I}_t - Q^H H^E\left(\widehat{H}_t^E - \widehat{H}_{t-1}^E\right) + x\widehat{x}_t \\ + \gamma^L s\widehat{s}_{t-1} + R^L L\left(\widehat{R}_{t-1}^L + \widehat{L}_{t-1}\right) - \tau a\widehat{a}_{t-1} \\ - pR^L L\left(\widehat{p}_{t-1} + \widehat{R}_{t-1}^L + \widehat{L}_{t-1}\right) + \tau pa\left(\widehat{p}_{t-1} + \widehat{a}_{t-1}\right) \end{aligned} \tag{D.13}$$

The borrowing constraint (B.15) yields

$$\widehat{l}_t = \widehat{\theta}_t + \widehat{a}_t - \widehat{R}_t^L \tag{D.14}$$

The definition of entrepreneurs' total assets (B.16) gives

$$\widehat{a}_t = \frac{Q^H H^E}{Q^H H^E + Q^K K}\mathbb{E}_t\left(\widehat{Q}_{t+1}^H + \widehat{H}_t^E\right) + \frac{Q^K K}{Q^H H^E + Q^K K}\mathbb{E}_t\left(\widehat{Q}_{t+1}^K + \widehat{K}_t\right) \tag{D.15}$$

Linearized version of (B.17) is

$$\widehat{\kappa}_t^E = \widehat{\lambda}_t^E + \widehat{Q}_t^K \tag{D.16}$$

D.3 Optimality Conditions of Banks

From (B.18), I obtain

$$\begin{aligned} \frac{\varrho^E}{\beta^P}\widehat{\varrho}_t^E - \varrho^E\gamma^L(1-\rho_s)\mathbb{E}_t\varrho_{t+1}^E = \left[pR^L + (1-p)\tau R^L - R^D + \varrho^E\gamma^L(1-\rho_s)\right]\mathbb{E}_t\widehat{q}_{t,t+1} \\ + pR^L\left(\widehat{p}_t + \widehat{R}_t^L\right) - R^D\widehat{R}_t^D + (1-p)\tau R^L\widehat{R}_t^L - p\tau R^L\widehat{p}_t \end{aligned} \tag{D.17}$$

Equation (B.19) becomes

$$\begin{aligned} \frac{\eta\xi\varrho^E x}{\theta}\left(\widehat{\varrho}_t^E + \widehat{x}_t - \widehat{\theta}_t\right) = -\varpi\beta^P\left(R^L\right)^2 L\theta\left(2\widehat{R}_t^L + \widehat{L}_t + \widehat{\theta}_t + \mathbb{E}_t\widehat{q}_{t,t+1}\right) \\ - \eta\varpi\beta^P R^L L\left(\widehat{R}_t^L + \widehat{L}_t + \mathbb{E}_t\widehat{q}_{t,t+1}\right) \\ + \varpi\tau\beta^P a\theta^2 R^L\left(\widehat{a}_t + 2\widehat{\theta}_t + \widehat{R}_t^L + \mathbb{E}_t\widehat{q}_{t,t+1}\right) \\ + \eta\varpi\tau\beta^P\theta a\left(\widehat{a}_t + \widehat{\theta}_t + \mathbb{E}_t\widehat{q}_{t,t+1}\right) \end{aligned} \tag{D.18}$$

From (B.20), I get

$$\begin{aligned} \xi\varrho^E x\theta\left(\widehat{\varrho}_t^E + \widehat{x}_t + \widehat{\theta}_t\right) = \theta\beta^P R^L pL\left(\widehat{\theta}_t + \widehat{R}_t^L + \widehat{p}_t + \widehat{L}_t + \mathbb{E}_t\widehat{q}_{t,t+1}\right) \\ + \eta\beta^P pL\left(\widehat{p}_t + \widehat{L}_t + \mathbb{E}_t\widehat{q}_{t,t+1}\right) \end{aligned} \tag{D.19}$$

Linearized versions of (B.23) and (B.24) are

$$\widehat{q}_{t,t+1} = \widehat{\lambda}^P_{t+1} - \widehat{\lambda}^P_t \tag{D.20}$$

and

$$p\widehat{p}_t + \zeta\widehat{\zeta}_t = \varpi\theta\widehat{\theta}_t \tag{D.21}$$

Equation (B.22) gives

$$\widehat{D}_t = \widehat{L}_t \tag{D.22}$$

D.4 Market Clearing and Resource Constraints

Equations (B.25) and (B.26) yield

$$\widehat{Y}_t = \frac{C^P}{C}\widehat{C}^P_t + \frac{C^E}{Y}\widehat{C}^E_t + \frac{I}{Y}\widehat{I}_t \tag{D.23}$$

and

$$H^P\widehat{H}^P_t + H^E\widehat{H}^E_t = 0 \tag{D.24}$$

From (B.27) we have

$$\widehat{Y}_t = \widehat{A}_t + (1-\alpha)\,\widehat{N}_t + \alpha\phi\widehat{H}^E_{t-1} + \alpha\,(1-\phi)\,\widehat{K}_{t-1} \tag{D.25}$$

Equation (B.28) gives

$$\widehat{K}_t = (1-\delta)\,\widehat{K}_{t-1} + \delta\widehat{I}_t \tag{D.26}$$

E Market Clearing

The derivation of market clearing conditon is identical to Ravn (2016) and I include it here for the sake of completeness. As mentioned in the main text, two types of transfers Ψ_t and Φ_t to entrepreneurs are needed to ensure all markets clear. This section demonstrates this and shows the derivation of the expression for Ψ_t. Let's start by adding together the budget constraints of households and entrepreneurs. We sum over both households and entrepreneurs, respectively:

$$\begin{aligned}
&\int_0^1 \left(C^P_{i,t} + Q^H_t \left(H^P_{i,t} - H^P_{i,t-1} \right) + \int_0^1 D_{ik,t}\mathrm{d}k \right)\mathrm{d}i + \int_0^1 \left(C^E_{j,t} + R^L_{t-1}\int_0^1 l_{jk,t-1}\mathrm{d}k \right)\mathrm{d}j \\
&= \int_0^1 \left(W_t N_{i,t} + \int_0^1 \Pi_{ik,t}\mathrm{d}k + R^D_{t-1}\int_0^1 D_{ik,t-1}\mathrm{d}k \right)\mathrm{d}i \\
&+ \int_0^1 \left(Y_{j,t} - W_t N_{j,t} - I_{j,t} - Q^H_t \left(H^E_{j,t} - H^E_{j,t-1} \right) + x_{j,t} + \Phi_t + \Psi_t \right)\mathrm{d}j
\end{aligned}$$

After doing the outer integral, I obtain:

$$
\begin{aligned}
&C_t^P + Q_t^H\left(H_t^P - H_{t-1}^P\right) + \int_0^1\int_0^1 D_{ik,t}\mathrm{d}i\mathrm{d}k + C_t^E + R_{t-1}^L\int_0^1\int_0^1 l_{jk,t-1}\mathrm{d}k\mathrm{d}j \\
&= W_tN_t + \int_0^1\int_0^1 \Pi_{ik,t}\mathrm{d}k\mathrm{d}i + R_{t-1}^D\int_0^1\int_0^1 D_{ik,t-1}\mathrm{d}k\mathrm{d}i \\
&+ Y_t - W_tN_t - I_t - Q_t^H\left(H_t^E - H_{t-1}^E\right) + \int_0^1 x_{j,t}\mathrm{d}j + \int_0^1 \Phi_t\mathrm{d}j + \int_0^1 \Psi_t\mathrm{d}j
\end{aligned}
$$

Using housing market clearing condition, rewrite the above expression:

$$
\begin{aligned}
&C_t^P + C_t^E + I_t - Y_t + Q_t\left(\left(H - H_t^E\right) - \left(H - H_{t-1}^E\right)\right) + Q_t^H\left(H_t^E - H_{t-1}^E\right) \\
&+ \int_0^1\int_0^1 D_{ik,t}\mathrm{d}i\mathrm{d}k + R_{t-1}^L\int_0^1\int_0^1 l_{jk,t-1}\mathrm{d}k\mathrm{d}j = \int_0^1\int_0^1 \Pi_{ik,t}\mathrm{d}k\mathrm{d}i \\
&+ R_{t-1}^D\int_0^1\int_0^1 D_{ik,t-1}\mathrm{d}k\mathrm{d}i + \int_0^1 x_{j,t}\mathrm{d}j + \int_0^1 \Phi_t\mathrm{d}j + \int_0^1 \Psi_t\mathrm{d}j
\end{aligned}
$$

After cancelling terms using the resource constraint, I now plug the expressions for $x_{j,t}$, Φ_t and $\Pi_{k,t}$ from the main text:

$$
\begin{aligned}
&\int_0^1\int_0^1 D_{ik,t}\mathrm{d}i\mathrm{d}k = R_{t-1}^D\int_0^1\int_0^1 D_{ik,t-1}\mathrm{d}k\mathrm{d}i - R_{t-1}^L\int_0^1\int_0^1 l_{jk,t-1}\mathrm{d}k\mathrm{d}j \\
&+ \int_0^1\left[\int_0^1\left(l_{jk,t} - \gamma^L s_{k,t-1}\right)^{\frac{\xi-1}{\xi}}\mathrm{d}k\right]^{\frac{\xi}{\xi-1}}\mathrm{d}j + \gamma^L\int_0^1\int_0^1 \frac{\theta_{k,t}}{\theta_t}s_{k,t-1}\mathrm{d}k\mathrm{d}j + \Psi_t\mathrm{d}j \\
&+ \int_0^1\int_0^1\left(p_{k,t-1}R_{t-1}^L L_{k,t-1} + \left(1-p_{k,t-1}\right)\frac{L_{k,t-1}}{\int_0^1 L_{k,t-1}\mathrm{d}k}\tau\theta_{t-1}a_{t-1} - L_{k,t}\right)\mathrm{d}k\mathrm{d}i \\
&+ \int_0^1\int_0^1\left(\int_0^1 D_{ik,t}\mathrm{d}i - R_{t-1}^D\int_0^1 D_{ik,t-1}\mathrm{d}i\right)\mathrm{d}k\mathrm{d}i
\end{aligned}
$$

Letting $\xi \to \infty$ and simplifying:

$$
\begin{aligned}
\int_0^1\int_0^1 D_{ik,t}\mathrm{d}i\mathrm{d}k &= R_{t-1}^D\int_0^1\int_0^1 D_{ik,t-1}\mathrm{d}k\mathrm{d}i - R_{t-1}^L\int_0^1\int_0^1 l_{jk,t-1}\mathrm{d}k\mathrm{d}j + \int_0^1\int_0^1\left(l_{jk,t} - \gamma^L s_{k,t-1}\right)\mathrm{d}k\mathrm{d}j \\
&+ \gamma^L\int_0^1\int_0^1 \frac{\theta_{k,t}}{\theta_t}s_{k,t-1}\mathrm{d}k\mathrm{d}j + \int_0^1 \Psi_t\mathrm{d}j - R_{t-1}^D\int_0^1\int_0^1 D_{ik,t-1}\mathrm{d}i\mathrm{d}k \\
&+ \int_0^1\int_0^1\left(p_{k,t-1}R_{t-1}^L L_{k,t-1} + \left(1-p_{k,t-1}\right)\frac{L_{k,t-1}}{\int_0^1 L_{k,t-1}\mathrm{d}k}\tau\theta_{t-1}a_{t-1}\right)\mathrm{d}k\mathrm{d}i
\end{aligned}
$$

Cancelling terms and further simplifying:

$$
\begin{aligned}
\int_0^1\int_0^1 D_{ik,t}\mathrm{d}i\mathrm{d}k &= -R_{t-1}^L\int_0^1\int_0^1 l_{jk,t-1}\mathrm{d}k\mathrm{d}j + \int_0^1\int_0^1\left(l_{jk,t} - \gamma^L s_{k,t-1} + \gamma^L\frac{\theta_{k,t}}{\theta_t}s_{k,t-1}\right)\mathrm{d}k\mathrm{d}j \\
&+ \int_0^1 \Psi_t\mathrm{d}j + \int_0^1\left(1-p_{k,t-1}\right)\frac{L_{k,t-1}}{\int_0^1 L_{k,t-1}\mathrm{d}k}\tau\theta_{t-1}a_{t-1}\mathrm{d}k + \int_0^1 p_{k,t-1}R_{t-1}^L L_{k,t-1}\mathrm{d}k
\end{aligned}
$$

Cancelling yet more terms and after simplifying more:

$$\int_0^1 \int_0^1 D_{ik,t} \mathrm{d}i \mathrm{d}k = -R_{t-1}^L \int_0^1 L_{k,t-1} \mathrm{d}k + \int_0^1 \int_0^1 l_{jk,t} \mathrm{d}k \mathrm{d}j + \int_0^1 \Psi_t \mathrm{d}j$$
$$+ \int_0^1 \Big(1 - p_{k,t-1}\Big) \tau \theta_{t-1} a_{t-1} \mathrm{d}k + R_{t-1}^L \int_0^1 p_{k,t-1} L_{k,t-1} \mathrm{d}k$$

After moving some terms around:

$$\int_0^1 \left(\int_0^1 D_{ik,t} \mathrm{d}i - L_{k,t} \right) \mathrm{d}k = \int_0^1 \Psi_t \mathrm{d}m + \int_0^1 \Big(1 - p_{k,t-1}\Big) \tau \theta_{t-1} a_{t-1} \mathrm{d}k - \int_0^1 \Big(1 - p_{k,t-1}\Big) R_{t-1}^L L_{k,t-1} \mathrm{d}k$$

Due to bank's balance sheet identity, the LHS becomes zero and I now have

$$\int_0^1 \Big(1 - p_{k,t-1}\Big) R_{t-1}^L L_{k,t-1} \mathrm{d}k - \int_0^1 \Big(1 - p_{k,t-1}\Big) \tau \theta_{t-1} a_{t-1} \mathrm{d}k = \int_0^1 \Psi_t \mathrm{d}j$$

Finally,

$$\int_0^1 \Psi_t \mathrm{d}j = \Psi_t = \int_0^1 \Big(1 - p_{k,t-1}\Big) \Big(R_{t-1}^L L_{k,t-1} - \tau \theta_{t-1} a_{t-1} \Big) \mathrm{d}k$$

where Fubini's theorem has been used to switch the order of integrals where necessary.