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**National Oligopolies and Economic  
Geography**

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**JEL code: R12, R13, F12.**

**Keywords: Spatial Oligopoly, Antitrust Policy, Welfare.**

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# National Oligopolies and Economic Geography

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## Abstract

We replace monopolistic competition with oligopoly in a model of “new economic geography”. In an international oligopoly agglomeration is more likely to occur than in monopolistic competition. When oligopolies have a national dimension there are many possible types of bifurcation diagrams but, unlike in monopolistic competition, the symmetric equilibrium is always stable for low trade costs. The antitrust policy, though identical in both countries, affects the geographical distribution of firms. In turn, migration attenuates the effectiveness of the antitrust policy in eliminating collusive behavior. For high and intermediate trade costs a toughening of the antitrust policy is likely to result in more agglomeration and, in some cases, may reduce world welfare.

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# 1 Introduction

Monopolistic competition is the market structure typically assumed in models of the new economic geography. In this paper we replace the assumption of monopolistic competition with that of oligopoly. We show that in oligopoly we have the same stable spatial configurations than in monopolistic competition, but agglomeration is more likely to emerge in the former than in the latter. If the oligopoly takes national dimensions then we have a rich variety of stable spatial configurations which does not emerge in monopolistic competition. Further, in the presence of national oligopolies the symmetric equilibrium is stable for low enough trade costs. This result contrasts sharply with the result obtained in models of economic geography that assume monopolistic competition where, for low enough trade costs, the symmetric equilibrium is unstable.

We continue the analysis by considering the possibility of collusive behavior. We find that the stable spatial configurations that emerge in collusive-oligopoly are equally rich but different from the configurations that emerge in non-collusive oligopoly.

We then investigate the effect of an antitrust policy on the geographical distribution of firms. Antitrust policies are designed to fight collusive behavior but have consequences on the equilibrium spatial configurations. We trace a relationship between the intensity of the antitrust policy and the degree of spatial agglomeration. We find that for high trade costs, a toughening of the antitrust policy has a non-linear effect on the degree of agglomeration: first increasing it, then reducing it, and then increasing it again. For intermediate trade costs, a toughening of the antitrust policy increases the degree of agglomeration except for very low level of toughness where the policy is neutral on agglomeration. Only for low enough trade costs a toughening of the antitrust policy has no impact on agglomeration. In addition we show that the link between antitrust policy and agglomeration goes in both direction: agglomeration reduces the effectiveness of the antitrust policy in eliminating collusive behavior.

Lastly, we measure the impact of the antitrust policy on welfare. The antitrust policy influences welfare in two ways: the direct effect on prices and the indirect effect through the location firms. We show that a toughening of the antitrust policy will have negative effects on welfare when the negative effect resulting from an increase in agglomeration dominates the positive effect resulting from a reduction of prices.

## 2 Review of the Literature and Model Choice

The literature on spatial oligopoly has a long tradition and it is beyond the scope of this paper to provide an exhaustive review. One of the earliest paper often cited is that of Hotelling (1929), he finds that competition for market areas leads to the agglomeration of firms. In Hotelling's model, two firms selling a single homogenous product at the same fixed price choose location on a line where customers are evenly distributed. Each firm gains by locating near to its competitor because, in so doing, it gains customers located between itself and the competitor without losing customers located further away from the competitor than from itself. The result, known as the Principle of Minimum (spatial) Differentiation, is that firms will agglomerate in the

center of the line. The agglomeration force is determined by the desire to increase the market share. Models of this type neglect an important aspect, namely, that firms want to be away from competitors in order to better exploit their market power. Taking this aspect into account, d'Aspremont et al. (1979) have shown that price competition leads to dispersion of firms. In their model the presence of trade costs makes goods spatially differentiated. In this situation, when a firm moves closer to its competitor the latter will react by lowering its price thus impacting negatively on the approaching firm's profits. This price competition effect pushes firms apart.

All the location theory is built upon the tension between agglomeration and dispersion forces, the former typically generated by the desire to be close to large markets in order to attract customers, and the latter generated typically by the desire to avoid price competition (or market crowding) effects. In most of this literature there is no interaction between firms location and size of the market. This interaction is brought in by Krugman (1991) who develops a model in monopolistic competition where the agglomeration and dispersion forces take a dynamic aspect. Firms want to locate in large markets because they want to attract customers (like in spatial competition), but (unlike in spatial competition) markets become larger precisely where firms decide to locate. Likewise, firms want to quit large markets in order to stay away from competitions, but markets become less competitive precisely as firms quit them. Thus, the space feature in terms of market size becomes endogenous. This model has generated a vast literature known as the new economic geography (see Fujita, Krugman and Venables, 1999; Fujita and Thisse, 2002; and Baldwin et al., 2003 for extensive treatments on the new economic geography). Most models in this literature assume monopolistic competition. Our model assumes oligopoly (like most of the spatial competition literature), but the size of the market depends on firms location (like in the new economic geography literature). The majority of new economic geography models uses Dixit and Stiglitz (1977) preferences. This structure, however, does not lend itself easily to the treatment of oligopoly. We prefer to use the linear demand structure proposed in Ottaviano et al. (2002) which extends naturally to the treatment of an oligopolistic market. The model in Ottaviano et al. (2002) will also serve as basic structure of our oligopoly variant.

### 3 The Model

The world economy is composed by two regions, Home,  $H$ , and Foreign,  $F$ , and produces two goods. One good is homogenous and is produced under perfect competition and constant returns to scale; we shall refer to this good as the “agricultural” good. The other good is horizontally differentiated and is produced by use of an increasing returns to scale technology in an oligopolistic market. In the oligopolistic market there are  $N$  firms each producing a variety different from any other variety. We shall refer to this good as “manufactures”. There are two factors of production labeled as “workers” ( $L$ ) and “farmers” ( $A$ ), but each industry employs only one factor: the manufacturing sector uses only workers and the agricultural sector uses only farmers. Farmers are geographically immobile, while workers are mobile between regions. For the sake of

symmetry we shall assume that farmers are evenly distributed across the two regions. We assume that the homogenous good can be traded at no cost between regions. Its price is therefore identical in the two regions and we choose it as the numéraire. Each variety of the manufacturing good can only be traded across regions at the cost of  $\tau > 0$  units of the numéraire good for each unit of manufacture shipped. The cost  $\tau$  accounts for all the obstacles to trade and measures the degree of market integration.

### 3.1 Demand

Individual preferences take the form of a quasi-linear utility function with a quadratic sub-utility that is assumed to be symmetric in all varieties and identical across individuals:

$$U(q) = q_0 + \alpha \sum_{i=1}^N q_i - \frac{\beta - \gamma}{2} \sum_{i=1}^N q_i^2 - \frac{\gamma}{2} \left( \sum_{i=1}^N q_i \right)^2, \quad (1)$$

where  $q_i$  is the quantity of variety  $i$  and  $q_0$  is the quantity of the numéraire good.<sup>1</sup> All parameters are assumed to be positive. In particular  $\beta > \gamma > 0$ , implies that consumers love variety. These assumptions ensure that the utility function is strictly concave. The parameter  $\gamma$  measures the degree of substitution between varieties, so that goods are substitutes, independent, or complements according to whether  $\gamma \gtrless 0$ . The larger  $\gamma$  the closer substitutes goods are. If  $\beta$  were allowed to be equal to  $\gamma$ , then goods would be perfect substitutes and the utility function would degenerate into a standard quadratic utility defined over a homogenous product. The parameter  $\alpha$  indicates the intensity of consumers' preferences for differentiated goods.

Each individual is endowed with  $\bar{q}_0 > 0$  units of the numéraire sufficient for positive equilibrium consumption and one unit of labor of type  $L$  or  $A$ . Her budget constraint is defined as follows:

$$\sum_{i=1}^N p_i q_i + q_0 = m + \bar{q}_0, \quad (2)$$

where  $p_i$  is the price of variety  $i$ ,  $m$  is labor income and the price of the agricultural good is normalized to one.

The inverse demand function for each variety is obtained by maximizing the utility function (1) subject to the budget constraint (2). Given the strict concavity of the utility function the following first order conditions are necessary and sufficient for a maximum:

$$\frac{\partial U(q)}{\partial q_i} = -p_i + \alpha - (\beta - \gamma)q_i - \gamma \sum_{j=1}^N q_j = 0, \quad i = 1..N. \quad (3)$$

Solving the budget constraint (2) for the numéraire  $q_0$ , substituting it into the utility function and solving the first order conditions for  $p_i$  yields the inverse demand function:

$$p_i = \alpha - (\beta - \gamma)q_i - \gamma \sum_{j=1}^N q_j. \quad (4)$$

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<sup>1</sup>As is well known the use of a quasi-linear utility function leads to ignore income effects.

Solving the inverse demand function of each variety for  $q_i$  and summing up all the  $N$  inverse demand functions, after some algebraic manipulations, we obtain the demand function:

$$q_i = a - (b + cN) p_i + c \sum_{j=1}^N p_j, \quad (5)$$

where  $a \equiv \frac{\alpha}{\beta + \gamma(N-1)}$ ,  $b \equiv \frac{1}{\beta + \gamma(N-1)}$  and  $c \equiv \frac{\gamma}{(\beta - \gamma)[\beta + \gamma(N-1)]}$ . The parameters in the demand function have a simple economic interpretation:  $c$  is the cross-price effect,  $cN$  is the sum of cross-price effects, and  $(b + cN)$  is the own-price effect.

### 3.2 Supply

We assume that the constant returns to scale technology in agriculture requires one unit of  $A$  to produce one unit of output. The assumption that the agricultural good can be freely traded between regions implies that in equilibrium the wage of farmers is the same in both regions. The technology and the choice of the numéraire ensure that the common wage is equal to one.

We assume that the production of manufactures requires  $\phi$  units of  $L$  for any quantity of output. This implies the assumption that variable costs are zero. Given the fixed input of workers, the market clearing conditions for  $L$  imply that the number of varieties produced in each region,  $n_h$  and  $n_F$ , is proportional to the number of workers present in each region at any time:

$$n_h = \frac{\lambda \bar{L}}{\phi}, \quad (6)$$

$$n_F = \frac{(1 - \lambda) \bar{L}}{\phi}, \quad (7)$$

where  $\lambda$  is the share of workers located in region  $H$  and  $\bar{L}$  is the stock of labor in the world economy. The total number of varieties produced in the economy is:  $N = \frac{\bar{L}}{\phi}$ .

We assume that the equilibrium wage is determined as the result of a bidding process among firms demanding labor. Then, competition among firms makes that the entire profit of the firm is absorbed by labor costs.

Henceforth, whenever possible, we will focus on region  $H$ . The analogous equations for  $F$  can be derived by symmetry.

From (5), the demand functions faced by firm  $i$  resident in  $H$  in its domestic and foreign market are, respectively:

$$q_{i,HH} = a - (b + cN) p_{i,HH} + c P_H, \quad (8)$$

and

$$q_{i,HF} = a - (b + cN) p_{i,HF} + c P_F, \quad (9)$$

where  $P_H$  and  $P_F$  are the price indices in region  $H$  and region  $F$  defined as

$$P_H = \sum_{j=1}^{n_H} p_{j,HH} + \sum_{k=1}^{n_F} p_{k,FH}, \quad (10)$$

$$P_F = \sum_{j=1}^{n_H} p_{j,HF} + \sum_{k=1}^{n_F} p_{k,FF}. \quad (11)$$

Profits in each market for firm  $i$  resident in  $H$  are:

$$\pi_{i,HH} = p_{i,HH} q_{i,HH}(p_{i,HH}) \left( \frac{A}{2} + \lambda L \right), \quad (12)$$

$$\pi_{i,HF} = (p_{i,HF} - \tau) q_{i,HF}(p_{i,HF}) \left[ \frac{A}{2} + (1 - \lambda)L \right], \quad (13)$$

with  $q_{i,HH}(p_{i,HH})$ ,  $q_{i,HF}(p_{i,HF})$  given by (8) and (9). Total profit is, therefore:

$$\Pi_{i,H} = \pi_{i,HH} + \pi_{i,HF} - \phi w_H, \quad (14)$$

where  $w_H$  is the manufacturing wage prevailing in region  $H$ .

Since demand is linear there is freight absorption. International trade will take place only if the price net of trade cost is positive (for any variety). Trade costs, therefore, must be sufficiently small in order for international trade to take place. We shall refer to  $\tau_{trade}$  as the level of trade costs for which the price of any variety net of trade costs is positive for any distribution of workers between countries. By definition, there is international trade if  $\tau < \tau_{trade}$ . As we shall see, the value of  $\tau_{trade}$  depends on the market structure.

### 3.3 Instantaneous Equilibrium and Dynamic Adjustment

The distribution of farmers is exogenous and constant over time. Workers, instead, take migration decisions by comparing the indirect utility in  $H$ ,  $V_H$ , with the indirect utility in  $F$ ,  $V_F$ . The indirect utility is given by the sum of consumers surplus plus manufacturing wage plus endowment of the numéraire. The indirect utility difference,  $\Delta V$ , is therefore the sum of the differences in consumer surplus and wages since the identical endowment of the numéraire cancels out. The indirect utility difference depends on  $\lambda$  via the effect that  $\lambda$  has on prices and on price indices. Denoting manufacturing wages  $w_H$ ,  $w_F$  and consumers surpluses  $S_H(\lambda)$ ,  $S_F(\lambda)$  we have:

$$\Delta V(\lambda) \equiv V_H(\lambda) - V_F(\lambda) = [S_H(\lambda) - S_F(\lambda)] + [w_H(\lambda) - w_F(\lambda)]. \quad (15)$$

Labor and goods markets clear instantaneously. Equilibrium in these markets yields prices and wages from which the indirect utility difference in (15) is computed. Migration is determined by the indirect utility difference according to

$$\dot{\lambda} \equiv \frac{d\lambda}{dt} = \begin{cases} \Delta V(\lambda), \\ \min \{0, \Delta V(\lambda)\} & \text{if } \lambda = 1, \\ \max \{0, \Delta V(\lambda)\} & \text{if } \lambda = 0, \end{cases} \quad (16)$$

where  $t$  is time. The dynamic system is in a state of rest when  $\dot{\lambda} = 0$ . Henceforth, we shall use the following terminology. We shall refer to any value of  $\lambda$  as to a “spatial configuration”. We shall often make reference to three spatial configurations

to which we give the names of “symmetric spatial configuration” and “core-periphery configurations”. The “symmetric spatial configuration” corresponds to  $\lambda = 1/2$  and the two core-periphery configurations correspond to  $\lambda = 0$  and to  $\lambda = 1$ . A spatial configuration  $\lambda^*$  is a “spatial equilibrium” if  $\Delta V(\lambda^*) = 0$ . We see from (16) that in a spatial equilibrium the system is indeed in a state of rest since  $\dot{\lambda} = \Delta V(\lambda^*) = 0$ . We see from equation (16) that the state of rest also occurs when  $\lambda = 1$  and  $\Delta V(1) > 0$  and when  $\lambda = 0$  and  $\Delta V(0) < 0$ . In these two cases the indirect utility difference is different from zero but all workers have already migrated, therefore no further migration may occur.

A spatial equilibrium is locally stable if  $\left. \frac{d\Delta V(\lambda)}{d\lambda} \right|_{\lambda=\lambda^*} < 0$  and locally unstable if  $\left. \frac{d\Delta V(\lambda)}{d\lambda} \right|_{\lambda=\lambda^*} > 0$ . The stability of spatial equilibria depends on parameter values. In particular, we shall refer to *tau-break* as to the value(s) of trade cost such that  $\left. \frac{d\Delta V(\lambda)}{d\lambda} \right|_{\lambda=\frac{1}{2}} = 0$  and we shall denote this value (or these values)  $\tau_b$ . Thus,  $\tau_b$  is a value of trade costs at which the symmetric equilibrium switches from being stable to being unstable or viceversa. If a core-periphery configuration is not a spatial equilibrium then it is stable if the system is at rest. We shall refer to *tau-sustain* as to the value(s) of trade costs such that  $\Delta V(0) = \Delta V(1) = 0$  and we shall denote this value (or these values)  $\tau_s$ . Thus,  $\tau_s$  is a value of trade costs at which the core-periphery configurations switch from being stable to being unstable or viceversa. In the presence of oligopoly there may be more than one *tau-break* and more than one *tau-sustain*. In one case the critical values  $\tau_s$  and  $\tau_b$  coincide, in this case we denote both of them  $\tau_c$ . We refer to the “economic geography” as to the ensemble of stable spatial configurations and to their position in  $(0, 1)$  for given values of parameters.

### 3.4 International and National Oligopoly

In monopolistic competition each firm neglects the impact that its decision has on market aggregates. Thus, for instance, when the monopolistic-competitive firm decides its own price it neglects the impact that its decision has on the price index or, alternatively, when it decides the quantity of output it neglects the impact that its decision has on total market output. Conversely, in oligopoly each firm takes into account the effect that its action has on market aggregates. The extent of this awareness, however, may be limited by geographical distance. Because of insufficient information on distant markets a firm may consciously decide to neglect the impact that its action has on that market. We take this geographical aspect into account by considering the two cases of international and national oligopolies.

In international oligopoly each firm is aware of the impact that its own decision has on world aggregates. Thus, for instance, when the firm decides its own price it takes into account the effect that its decision has on the price indices in both countries.

In national oligopolies, instead, each firm takes into account only the effect that its decision has on national market aggregates and neglects the impact that its decision has on world aggregates. Thus, for instance, when the firm decides its price it takes into account the effect that its decision has on the price index of its country and neglects

the impact that it has on the price index of the foreign country. The resulting market structure is that firms in each country behave as oligopolist in their home market and as monopolistic-competitive firms in their distant market.

In the following section we turn to the study of the economic geography in the two cases of an international oligopoly and two national oligopolies.

## 4 Oligopoly and Economic Geography

### 4.1 International Oligopoly

Each firm maximizes its profit given by (14) with respect to  $p_{i,hh}$  and  $p_{i,hf}$  taking into account the effect that this decision has on the price index of both countries. Profit maximization yields the reaction functions. Solving the system of reaction functions gives the four equilibrium prices:

$$p_{i,HH}^I = \frac{ac + 2(2b + cN)(b + cN - c)}{(2b + 2cN - c)(2b + cN - c)} p_{HH}^{MC}, \quad \forall i = 1..n_H, \quad (17)$$

$$p_{i,FF}^I = \frac{ac + 2(2b + cN)(b + cN - c)}{(2b + 2cN - c)(2b + cN - c)} p_{FF}^{MC}, \quad \forall i = 1..n_F, \quad (18)$$

$$p_{i,HF}^I = p_{i,FF}^* + \frac{b + cN - c}{2b + 2cN - c} \tau, \quad \forall i = 1..n_H, \quad (19)$$

$$p_{i, FH}^I = p_{i,HH}^* + \frac{b + cN - c}{2b + 2cN - c} \tau, \quad \forall i = 1..n_F, \quad (20)$$

where the superscript  $I$  means equilibrium prices in (non-collusive) international oligopoly and the superscript  $MC$  indicates the equilibrium prices in monopolistic competition:  $p_{HH}^{MC} = \frac{2a+c(1-\lambda)N\tau}{2(2b+cN)}$  and  $p_{FF}^{MC} = \frac{2a+c\lambda N\tau}{2(2b+cN)}$ . It is worth mentioning a few fairly intuitive results. First, equilibrium prices are identical for all firms in the same country since firms face identical demand and have identical costs. Second, equilibrium oligopoly-prices are higher than equilibrium monopolistic-competitive prices,  $p_{HH}^I > p_{HH}^{MC}$  and  $p_{FF}^I > p_{FF}^{MC}$  (the inequality can be easily demonstrated); this is intuitive since oligopolistic firms enjoy a higher degree of market power than monopolistic-competitive firms. Third, equilibrium prices in oligopoly are more sensitive to changes in trade costs than equilibrium prices in monopolistic competition,  $\frac{dp_{HH}^I}{d\tau} > \frac{dp_{HH}^{MC}}{d\tau}$  and  $\frac{dp_{FF}^I}{d\tau} > \frac{dp_{FF}^{MC}}{d\tau}$  (the inequalities can be easily proven); this result implies a stronger procompetitive effect of market integration in oligopoly than in monopolistic competition. Fourth, prices are lower in the largest market because competition among firms is stronger ( $\lambda$  enters expressions (17)-(18) through monopolistic-competitive prices). Fifth, using equations (17)-(20) into the price indices shows that the largest region will have the lowest price index since prices are lower in the largest region and the number of varieties on which consumers pay transport costs is smaller in the largest region. Lastly, for  $\tau = 0$  equilibrium prices do not depend upon the geographical distribution of firms,  $p_{HH} = p_{FH} = p_{FF} = p_{HF}$ . Computing *tau-trade* we have  $\tau_{trade}^I = a \frac{2(cN+b)-c}{(cN-c+2b)(b+cN)} < \frac{a}{b}$ .

Coming to the spatial configurations we observe that the indirect utility difference  $\Delta V(\lambda)$  is a quadratic function of  $\tau$  and a linear function of  $\lambda$  of the form:

$$\Delta V(\lambda) = C(\tau_c - \tau)\tau(\lambda - 1/2), \quad (21)$$

where  $C$  is a positive coefficient which depends on parameters and  $\tau_c$  has been defined above.<sup>2</sup> For trade costs higher than  $\tau_c$  the symmetric equilibrium is stable and the core-periphery configurations are unstable, for trade costs below  $\tau_c$  the symmetric equilibrium is unstable and the core-periphery configurations are stable. Agglomeration of manufacturing activity occurs for sufficiently low trade costs. These results are qualitatively the same as in Ottaviano et al. (2002). The only important difference is in the value of  $\tau_c$ . In Appendix A we demonstrate the following proposition.

**Proposition 1** *The value of  $\tau_c$  in oligopoly is larger than the value of  $\tau_c$  in monopolistic competition.*

This result has the interesting implication that the likelihood of stable core-periphery configuration (complete agglomeration) is higher in international oligopoly than in monopolistic competition. The reason is due to the fact that in the case of oligopoly the procompetitive effect of trade liberalization is larger than in monopolistic competition. Thus, the indirect utility of the largest region becomes the largest in correspondence of a lower level of market integration than in the case of monopolistic competition.

## 4.2 National Oligopolies

In this section we consider the case in which each firm takes account of the impact that its action has on national aggregate variables only. Each firm in  $H$  maximizes (14) with respect  $p_{i,HH}$ , taking into account the effect on  $P_H$ , and maximizes it with respect to  $p_{i,HF}$ , neglecting the effect on  $P_F$ . Profit maximization gives the reaction functions. Solving the system of reaction functions we have the four equilibrium prices:

$$p_{i,HH}^{NC} = \frac{2(2b + cN)(b + cN)}{2(2b + cN)(b + cN) - c(2b + cN + cN\lambda)} p_{HH}^{MC}, \quad \forall i = 1..n_H, \quad (22)$$

$$p_{i,FF}^{NC} = \frac{2(2b + cN)(b + cN)}{2(2b + cN)(b + cN) - c[2b + cN + cN(1 - \lambda)]} p_{FF}^{MC}, \quad \forall i = 1..n_F, \quad (23)$$

$$p_{i,HF}^{NC} = \frac{a + cN(1 - \lambda)p_{FF}^{NC} + (b + cN)\tau}{2b + cN + cN(1 - \lambda)}, \quad \forall i = 1..n_H, \quad (24)$$

$$p_{i,FH}^{NC} = \frac{a + cN\lambda p_{HH}^{NC} + (b + cN)\tau}{2b + cN + cN\lambda}, \quad \forall i = 1..n_F, \quad (25)$$

where the superscript  $NC$  indicates equilibrium prices in non-collusive national oligopolies. As is intuitive,  $p_{HH}^{NC} > p_{HH}^{MC}$  and  $p_{FF}^{NC} > p_{FF}^{MC}$ . Further,  $\frac{dp_{HH}^{NC}}{d\tau} > \frac{dp_{HH}^{MC}}{d\tau}$  and  $\frac{dp_{FF}^{NC}}{d\tau} > \frac{dp_{FF}^{MC}}{d\tau}$  implying a stronger procompetitive effect of market integration in national oligopolies than in monopolistic competition. We observe that equilibrium national oligopoly

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<sup>2</sup>See Appendix A for the derivation of (21).

prices may be different between countries even in the absence of transport costs. Indeed they are different for  $\tau = 0$  whenever  $\lambda \neq 1/2$ . This result marks an interesting difference with the cases of international oligopoly and of monopolistic competition where, instead, in free trade prices are identical everywhere and for any  $\lambda$ . Mathematically, this result is due to the fact that with national oligopolies equilibrium prices are function of  $\lambda$  and  $\tau$  separately. Intuitively, the result is due to the national extent of the oligopolistic markets which makes the oligopoly equilibrium price in each market dependent on the size of that market.

We now come to the stability analysis whose results we illustrate by use of bifurcation diagrams. There are quite a few different possible bifurcation diagrams. This multiplicity of bifurcation diagrams is due to the fact that both the slope of the phase line and the indirect utility difference at core-periphery configurations are quadratic in  $\tau$ . Consequently, there may be two *tau-break* and two *tau-sustain*.

We begin by the stability of the symmetric equilibrium. The slope of the phase line evaluated at the symmetric equilibrium is

$$\left. \frac{d\Delta V(\lambda)}{d\lambda} \right|_{\lambda=\frac{1}{2}} = -B_0^{NC} + B_1^{NC}\tau - B_2^{NC}\tau^2, \quad (26)$$

where  $B_0^{NC}$ ,  $B_1^{NC}$ , and  $B_2^{NC}$  are positive.<sup>3</sup> The polynomial in (26) is a concave parabola. Since it is quadratic in  $\tau$  there may be two distinct positive *tau-break*. When this is the case we denote the smallest and the largest of these two values  $\tau_{b1}^{NC}$  and  $\tau_{b2}^{NC}$ , respectively.

Evaluating the indirect utility difference at  $\lambda = 1$  or  $\lambda = 0$  gives us the information about the stability of the core-periphery configurations. Computing, for instance,  $\Delta V(1)$  we have:

$$\Delta V(1) = -S_0^{NC} + S_1^{NC}\tau - S_2^{NC}\tau^2, \quad (27)$$

where  $S_0^{NC}$ ,  $S_1^{NC}$ , and  $S_2^{NC}$  are positive (see Appendix B). The polynomial in (27) is a concave parabola. Since it is quadratic in  $\tau$  there may be two distinct and positive *tau-sustain*. When this is the case we denote the smallest and the largest of these two values  $\tau_{s1}^{NC}$  and  $\tau_{s2}^{NC}$ , respectively.

Since each of the polynomials in (26)-(27) may have real and distinct roots, or real and coincident roots, or complex roots we have a total of sixteen possible bifurcation diagrams. We begin with the seven bifurcation diagrams generated by real and distinct roots or by complex roots. We first give the list of the diagrams and then we discuss them. The seven diagrams are the following:

1. Both polynomials (26)-(27) have real and distinct roots. There are four possible sub-cases.
  - (a)  $0 < \tau_{b1} < \tau_{s1} < \tau_{s2} < \tau_{b2}$ . The bifurcation diagram is shown in Figure 1a.
  - (b)  $0 < \tau_{s1} < \tau_{b1} < \tau_{b2} < \tau_{s2}$ . The bifurcation diagram is shown in Figure 1b.
  - (c)  $0 < \tau_{b1} < \tau_{s1} < \tau_{b2} < \tau_{s2}$ . The bifurcation diagram is shown in Figure 1c.

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<sup>3</sup>See Appendix B in relation to equations (26) and (27)

- (d)  $0 < \tau_{s1} < \tau_{b1} < \tau_{s2} < \tau_{b2}$ . The bifurcation diagram is shown in Figure 1d.
2. Polynomial (27) has real roots and polynomial (26) has complex roots. The bifurcation diagram is shown in Figure 2.
  3. Polynomial (26) has real roots and polynomial (27) has complex roots. The bifurcation diagram is shown in Figure 3.
  4. Both polynomials have complex roots. The bifurcation diagram is shown in Figure 4.

Looking at the bifurcation diagrams we see that for sufficiently low and sufficiently high trade costs the only stable spatial configuration is the symmetric equilibrium. For intermediate trade costs, instead, there is a rich taxonomy of stable spatial configurations.

For reason of space we do not show the nine bifurcation diagrams that would result if either equation (26) or (27) have coincident real roots. These nine bifurcation diagrams can be easily constructed. These diagrams give the type of results that we have drawn from the seven diagrams listed above: namely, that for low and high trade costs the symmetric equilibrium is stable and that for intermediate trade costs there may be multiple stable spatial configurations.<sup>4</sup>

The economic geography resulting from national oligopoly is very different from that emerging in international oligopoly or in monopolistic competition.

First, in the case of national oligopolies the symmetric configuration and the core-periphery configurations may all be stable, whereas in the case of non-collusive international oligopoly (or in monopolistic competition *à la* Ottaviano et al., 2002) when the symmetric configuration is stable the core-periphery configurations are unstable and viceversa. The simultaneous stability of the symmetric and core-periphery spatial configurations occurs in Figure 1b for  $\tau \in (\tau_{s1}, \tau_{b1})$ , and for  $\tau \in (\tau_{b2}, \tau_{s2})$ , in Figure 1c for  $\tau \in (\tau_{b2}, \tau_{s2})$ , and in Figure 1d for  $\tau \in (\tau_{s1}, \tau_{b1})$ .

Second, in the case of national oligopolies there may be multiple stable spatial equilibria (multiple stable partial agglomerations). This result occurs neither international oligopoly nor in most models of monopolistic competition. Multiple stable spatial equilibria are shown in Figure 1a for  $\tau \in (\tau_{b1}, \tau_{s1})$ , and for  $\tau \in (\tau_{s2}, \tau_{b2})$ , in Figure 1c for  $\tau \in (\tau_{b1}, \tau_{s1})$ , in Figure 1d for  $\tau \in (\tau_{s2}, \tau_{b2})$  and in Figure 3 for  $\tau \in (\tau_{b1}, \tau_{b2})$ .

Third, as shown by all bifurcation diagrams, in the case of national oligopolies the symmetric spatial configuration is stable for low transport costs. This result marks an important difference with respect to the case of international oligopoly and monopo-

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<sup>4</sup>To see this note that if the concave parabola in (26) has coincident real roots, then it must be negative for any value of  $\tau$  except for  $\tau$  equal to the roots. Therefore the symmetric equilibrium is locally stable. Likewise, if the concave parabola in (26) has coincident real roots, then it must be negative for any value of  $\tau$  except for  $\tau$  equal to the roots. Therefore, the core-periphery configurations are unstable.

listic competition, where the symmetric configuration is unstable for low trade costs.<sup>5</sup> In order to illustrate this result in detail we evaluate the two components of the indirect utility difference in expression (15) at one of the core-periphery configuration, for instance for  $\lambda = 1$ . When all firms are in  $H$  we have that both the consumer surplus and the wage are lower in  $H$  than in  $F$ :

$$S_H^{NC}(1) < S_F^{NC}(1), \quad (28)$$

$$w_H^{NC}(1) < w_F^{NC}(1). \quad (29)$$

Therefore

$$\Delta V(1) < 0, \quad (30)$$

and the core-periphery configuration is unstable.<sup>6</sup> We give the intuition for each of the inequalities (28) and (29). The inequality in (28) is due the fact that for sufficiently low transportation costs when all firms are located in  $H$  we have that  $p_{HF}^{NC} < p_{HH}^{NC}$ . The reason is due to the national dimension of the oligopoly: recall that firms set oligopoly prices in their national market and set monopolistically-competitive prices in their distant market. When all firms are in  $H$  then the equilibrium price in  $H$  is oligopolistic, while the equilibrium price in  $F$  is monopolistic-competitive. The oligopoly price is higher than the monopolistic competitive price. The latter, however, includes trade costs since all firms are in  $H$ . But if trade costs are sufficiently low then we have  $p_{HF}^{NC} < p_{HH}^{NC}$  and, therefore, we have a higher consumers' surplus in  $F$  than in  $H$ .

The inequality in (29) is explained by the balance of the size effect on gross profits. The largest local profit is made on the largest market; that is, for  $\lambda = 1$ ,  $\pi_{HH}^{NC} > \pi_{FF}^{NC}$ . Because of this effect, wages would tend to be higher in  $H$  and agglomeration in  $H$  would tend to be stable. But the size effect is present in profits on distant market too. The largest profit on the distant market is made on the largest distant market; that is, for  $\lambda = 1$ ,  $\pi_{HF}^{NC} < \pi_{FH}^{NC}$ . Because of this effect, wages would tend to be higher in  $F$  and agglomeration in  $H$  would tend to be unstable. Total profits are the sum of local and distant profits. For low trade cost, the size effect on distant markets profits dominates that on local markets profits. Thus, we have inequality (29).

Last, we have to comment about *tau-trade*. Computing *tau-trade* for national oligopolies gives  $\tau_{trade}^{NC} = a \frac{2(cN+b)-c}{(cN-c+2b)(b+cN)} < \frac{a}{b}$ . Depending on parameter values  $\tau_{trade}^{NC}$  can be larger or smaller than any of the *tau-break* and *tau-sustain*. The bifurcation diagrams should therefore be interpreted taking into account that the maximum meaningful value of trade cost is  $\tau_{trade}^{NC}$  (not shown in the diagrams). The spatial configurations to the right of  $\tau_{trade}^{NC}$  will, of course, never occur. A similar truncation of the phase diagram occurs in Ottaviano et al. (2002) where, in monopolistic competition, the single *tau-break* coincides with the single *tau-sustain* and where *tau-trade* may be smaller than these.

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<sup>5</sup>In the new economic geography literature there are a few models where core-periphery configurations are unstable for low trade costs and/or there are multiple spatial equilibria, but this is due to the presence of congestion forces or diminishing returns added to the models and not to the market structure *per se*. See, for instance, Helpman (1998), Puga (1999), Brühlhart and Trionfetti (2004).

<sup>6</sup>See Appendix B for the proof.

The results of this section may be summarized as follows.

**Proposition 2** *In national oligopolies there are sixteen possible types of bifurcation diagrams, there may be multiple stable spatial configurations, and there exists a sufficiently low level of trade costs for which the symmetric equilibrium is stable.*

Perhaps the most interesting implication of these results is that a sufficiently high degree of market integration eliminates industrial agglomeration when oligopolies have a national dimension.

## 5 Collusion, Antitrust Policy, and Welfare

The possibility of collusion among producers is always present in oligopolistic markets and antitrust laws have been a constant threat to collusive behavior.<sup>7</sup> The objective of antitrust policies is to fight collusive behavior and is usually unrelated to international trade or to economic geography. Yet, there is an interaction between antitrust policies and location that we want to illustrate in this section. We begin by showing the effect of collusion on economic geography. We then illustrate by use of a numerical example the effect of the antitrust policy on the economic geography and welfare. In the latter we do not aspire to provide results of general validity since the number of possible cases and the analytical intractability of the model make it impossible to provide a complete taxonomy. We simply want to highlight the consequences of antitrust policies on economic geography and welfare.

### 5.1 Equilibrium Spatial Configurations in the Presence of Collusive National Oligopolies

We limit our attention to collusive national oligopolies since this is the most interesting case. The case of collusive international oligopoly may be studied analogously.<sup>8</sup> The collusive behavior is defined as follows: firms located in the same country maximize jointly the sum of profits on domestic markets:

$$\underset{p_{HH}}{\text{Max}} [n_H \pi_{HH} (p_{HH})], \quad (31)$$

$$\underset{p_{FF}}{\text{Max}} [n_F \pi_{FF} (p_{FF})], \quad (32)$$

and each firm maximizes individually its profits on the distant market:

$$\underset{p_{i,HF}}{\text{Max}} [\pi_{HF} (p_{i,HF})], \quad (33)$$

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<sup>7</sup>An early example of antitrust law is the *Lex Julia de Annona*, enacted during the Roman Republic around 50 BC.

<sup>8</sup>We know from the industrial organization literature that collusion is sustainable in repeated games when the discount rate is sufficiently low. We assume that this is the case without explicitly modelling the sustainability of collusive behavior.

$$\underset{p_{i,FH}}{\text{Max}} [\pi_{FH} (p_{i,FH})]. \quad (34)$$

By solving the four maximization problems we obtain the best-response functions. Solving the system of best-response functions we obtain the equilibrium prices:

$$p_{HH}^C = \frac{2(2b+cN)(b+cN)}{2(2b+cN)(b+cN) - c\lambda N(2b+cN+cN\lambda)} p_{HH}^{MC}, \quad (35)$$

$$p_{FF}^C = \frac{2(2b+cN)(b+cN)}{2(2b+cN)(b+cN) - cN(1-\lambda)[2b+cN+cN(1-\lambda)]} p_{FF}^{MC}, \quad (36)$$

$$p_{HF}^C = \frac{a+cN(1-\lambda)p_{FF}^C + (b+cN)\tau}{2b+cN+cN(1-\lambda)}, \quad (37)$$

$$p_{FH}^C = \frac{a+cN\lambda p_{HH}^C + (b+cN)\tau}{2b+cN+cN\lambda}, \quad (38)$$

where the superscript  $C$  indicates equilibrium prices in collusive national oligopolies. Like in the previous section, and very intuitively, oligopoly prices are higher than monopolistic competitive prices, they are more sensitive to changes in trade costs, and are different in different markets even if trade cost is zero.

The stability analysis of collusive national oligopoly gives analogous results to that of non-collusive national oligopolies (see Appendix C). There may be two *tau-break* (denoted  $\tau_{b1}^C$  and  $\tau_{b2}^C$  with  $0 < \tau_{b1}^C < \tau_{b2}^C$ ) and two *tau-sustain* (denoted  $\tau_{s1}^C$  and  $\tau_{s2}^C$  with  $0 < \tau_{s1}^C < \tau_{s2}^C$ ). There are sixteen possible types of bifurcation diagrams which are qualitatively the same as those of the non-collusive national oligopolies. Naturally, the values of *tau-break* and *tau-sustain* of non-collusive national oligopolies need not to coincide with those of collusive oligopoly. Computing *tau-trade* we have  $\tau_{trade}^C = \frac{a(2b+cN+c\lambda^\circ N)}{c^2 N^2 \lambda^\circ (1-\lambda^\circ) + 2b(b+cN)} \leq \frac{a}{b}$  where  $\lambda^\circ \equiv \max \left[ 0, \frac{\sqrt{2(b+cN)} - (2b+cN)}{cN} \right]$  which can be larger or smaller of any of the *tau-break* and *tau-sustain*.

**Proposition 3** *In collusive national oligopolies there are the same types of bifurcation diagrams as in non-collusive national oligopolies, but the values of tau-break and tau-sustain need not to coincide.*

This is as far as we can go with analytical results. In the following two sections we resort to numerical simulations to show the possible effects of the antitrust policy on economic geography and welfare.

## 5.2 The Effect of the Antitrust Policy on the Economic Geography

The antitrust policy, if severe enough, will eliminate collusive behavior. This is hardly surprising. What is more interesting is that the antitrust policy has an indirect effect on the economic geography. We illustrate this effect in the context of three numeric examples of high, intermediate, and low trade costs. We shall see that the antitrust policy may have a different impact on the economic geography depending on the

degree of economic integration. In this section we do not intend to draw general results. The intricacy of the algebra and the large number of parameters make a general demonstration or a complete taxonomy beyond the scope of the paper. Rather, we want to highlight the economic mechanism through which the antitrust policy influences the economic geography.

We have chosen parameter values (reported in Appendix E) such that the collusive and non-collusive bifurcation diagrams are very different: the bifurcation diagram for non-collusive oligopoly is of the type represented in Figure 1a, whereas the bifurcation diagram for collusive-oligopoly is of the type represented in Figure 3. Therefore, an antitrust policy that eliminates collusive behavior is subject to impact the economic geography in an important way.

The antitrust policy we consider is extremely simple. We assume that each government fines firms who are found guilty of collusive behavior. Governments apply a fine equal to a fraction  $\psi$  of local profits. Enforcement of anti-collusive regulations is not perfect and each collusive firm faces a probability  $\delta$  of being fined. Thus, the expected fine for a collusive firm is a fraction  $\eta = \psi\delta$  of local profits. Firms will collude if

$$(1 - \eta)\pi_{ii}^C - \pi_{ii}^{NC} > 0, \quad i = H, F. \quad (39)$$

Firms will not collude if the sign of the inequality is reversed. Firms are indifferent between colluding and not colluding when  $(1 - \eta)\pi_{ii}^C - \pi_{ii}^{NC} = 0$ , in such case we assume that the status quo prevails. Thus, if firms were not colluding before becoming indifferent they will continue not to collude when they become indifferent; if firms were colluding before becoming indifferent they will continue to collude when they become indifferent. Obviously, if  $\eta = 0$  firms will collude since collusive profits,  $\pi_{ii}^C$ , are larger than non-collusive profits,  $\pi_{ii}^{NC}$ .

It can be shown that for every spatial configuration there exists a unique expected fine rate  $\eta$  for which firms are indifferent between colluding or not. In Appendix D we demonstrate that the locus in the space  $(\lambda, \eta)$  in which  $(1 - \eta)\pi_{ii}^C - \pi_{ii}^{NC} = 0$  does not depend upon the level of economic integration  $\tau$ . Further, it can be easily shown that for firms located in  $H$  the critical level of  $\eta$  is always increasing in  $\lambda$ , while for the representative firm in  $F$  the critical level of  $\eta$  is always decreasing in  $\lambda$ . The economic intuition for this is straightforward. The higher the number of colluding firms located in the same region, the higher the gains from colluding and, therefore, the higher the fraction of profits that colluding firms can afford paying as a fine. Figure 5 shows the combinations of  $\eta$  and  $\lambda$  for which firms operating in  $H$  and  $F$  collude or not. In the dark-gray area firms in both countries collude, thus we have two collusive national oligopolies. In the light-gray area firms collude in  $H$  or in  $F$  but not in both; note that it is firms in the largest market who collude. In the white areas firms in neither country collude. We have marked two threshold values of  $\eta$ :  $\eta = 0.1247$ , and  $\eta = 0.6694$  for future reference.

We can visualize the effect of the antitrust policy on the economic geography by use of phase diagrams. Each phase diagram plots the phase line given by the indirect utility difference as function of  $\lambda$ . Since the competition regime may change we have to pay attention to the construction of the phase line. We have seen in Figure 5 that,

for a given  $\eta$ , by moving along the abscissa we cross different competition regimes.<sup>9</sup> In turn, to each competition regime there corresponds a phase line. Therefore, each phase line shown in the diagrams pulls together the sections of the phase lines corresponding to the competition regime applicable to any value of  $\lambda$ . Naturally, the line composed by these sections need not to be continuous. The resulting phase line will present discontinuities in correspondence of values of  $\lambda$  where the competition regime is subject to change.

We shall illustrate three examples: high, intermediate, and low trade costs. We begin with the example of high trade costs where we set  $\tau = 1.73$ . For this value of trade costs there is trade in all competition regimes (see Appendix E).

Each panel in Figure 6 represents a phase diagram for six different values of  $\eta$ . In each diagram, we plot the phase line represented by the indirect utility difference  $\Delta V(\lambda)$ .

Figure 6a plots the indirect utility difference in the absence of antitrust policy (i.e.  $\eta = 0$ ). For  $\eta = 0$  firms in both countries collude for any value of  $\lambda$  as we have already seen. There are three equilibria: the symmetric equilibrium which is unstable and two stable lateral equilibria.

Figure 6b plots the indirect utility difference for  $\eta = 0.04$ . The phase line exhibits two stable spatial equilibria on either side of the symmetric equilibrium. For  $\eta = 0.04$  all firms collude when  $\lambda \in (0.3217, 0.6783)$ .<sup>10</sup> The stable spatial configurations are within the zone of collusion. We conclude that for  $\eta = 0.04$  there will be partial agglomeration with collusive national oligopolies. As mentioned above, we observe the discontinuities of the phase line for  $\lambda = 0.3217$  and  $\lambda = 0.6783$  where the competition regime is subject to change.

Figure 6c plots the indirect utility difference for  $\eta = 0.08$ . For  $\eta = 0.08$  firms collude for  $0.4193 < \lambda < 0.5807$ .<sup>11</sup> In correspondence of these values of lambda the phase line exhibits a discontinuity. The consequence of these discontinuities is that there is no real value of  $\lambda$  such that  $\Delta V(\lambda) = 0$ . In other words, there is no spatial equilibrium. The absence of a spatial equilibrium bears no consequences on the determination of the stable spatial configuration. It is apparent that for any initial value of  $\lambda \in (\frac{1}{2}, 1]$  the law of motion (16) will drive  $\lambda$  towards 0.5807. Conversely, for any value of  $\lambda \in [0, \frac{1}{2})$  the law of motion (16) will drive  $\lambda$  towards 0.4193. Said it differently,  $\lambda = 0.5807$  is the limit point for any phase trajectory originating in the set  $(\frac{1}{2}, 1]$  and  $\lambda = 0.4193$  is the limit point for any phase trajectory originating in the set  $[0, \frac{1}{2})$ . Or, equivalently, the sets  $(\frac{1}{2}, 1]$  and  $[0, \frac{1}{2})$  are the basins of attraction of 0.5807 and 0.4193, respectively. We conclude that for  $\eta = 0.08$  there is partial agglomeration and firms in the largest country are indifferent between colluding and not colluding while firms in the smallest country do not collude. We further note that the stable spatial configurations in Figure

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<sup>9</sup>For instance, with  $\eta = 0.2$ , moving along the abscissa we cross from the area of collusion in  $F$  to the area of no-collusion, to the area of collusion in  $H$ .

<sup>10</sup>These threshold values are computed from expressions (D-3) and (D-4) in Appendix D. They can also be visualized in Figure 5 by noticing that the first tickmark on the ordinates starting from the bottom corresponds to  $\eta = 0.04$ . Then, drawing an imaginary horizontal line from  $\eta = 0.04$  we cross the competition zones at  $\lambda = 0.3217$  and  $\lambda = 0.6783$ .

<sup>11</sup>Again, these threshold values are computed from expressions (D-3) and (D-4) in Appendix D and can be visualized in Figure 5.

6c are closer to each other than those in Figure 6b; firms are less dispersed.

Figure 6d plots the indirect utility difference for  $\eta = 0.1247$ . From expressions (D-3) and (D-4) in Appendix D we see that for  $\eta = 0.1247$  firms are indifferent between colluding and not colluding when  $\lambda = 0.5$ , for  $\lambda < 0.5$  firms in  $F$  collude and firms in  $H$  do not, for  $\lambda > 0.5$  firms in  $H$  collude and firms in  $F$  do not. There is, therefore, a discontinuity of the phase line at  $\lambda = 0.5$ . This value of  $\eta$  is marked in Figure 5. Like before, this discontinuity bears no consequences on the determination of the stable spatial configuration. It is apparent that the symmetric equilibrium is the only stable spatial configuration towards which the economy will converge. Indeed,  $\lambda = 0.5$  is the limit point of all phase trajectories  $\lambda(t)$ . We conclude that for  $\eta = 0.1247$  there is no agglomeration and firms in the largest country are indifferent between colluding and not colluding, while firms in the smallest country do not collude.

Figure 6e plots the indirect utility difference for  $\eta = 0.4$ . From expressions (D-3) and (D-4) we see that for this value of  $\eta$  firms do not collude for  $0.1954 < \lambda < 0.8046$ , firms collude in  $H$  and not in  $F$  for  $\lambda > 0.8046$ , and firms collude in  $F$  and not in  $H$  for  $\lambda < 0.1954$ . The phase line exhibits therefore two discontinuities in correspondence of  $\lambda = 0.1954$ , and  $\lambda = 0.8046$ . The symmetric equilibrium is unstable, but there are two stable spatial configurations on either side of the symmetric equilibrium represented by the two values of  $\lambda$  in correspondence of the discontinuities. We conclude that for  $\eta = 0.4$  there will be partial agglomeration and firms in the largest country are indifferent between colluding and not colluding, while firms in the smallest country do not collude. We note at this point that if migration had not been possible an expected fine of 0.4 would have induced all firms to not collude. To see this graphically, take the symmetric equilibrium as the exogenous spatial configuration in the absence of migration and refer to Figure 5 where for  $\eta = 0.4$  and for  $\lambda = 0.5$  we are in the white area of no collusion.<sup>12</sup> Therefore, migration attenuates the effectiveness of the antitrust policy in eliminating collusive behavior.

Figure 6f plots the indirect utility difference for  $\eta = 0.8$ . The symmetric spatial configuration is unstable while the core-periphery configurations are stable. We see in Figure 5 that firms never collude when the expected fine rate is this high. We conclude that for  $\eta = 0.8$  there is agglomeration and non-collusion.

The results of Figure 6 can be summarized by the bifurcation diagram depicted in Figure 7. The bifurcation diagram has  $\eta$  on the abscissa reflecting the evolution of stable and unstable spatial configurations as  $\eta$  changes. Continuous lines indicate stable spatial configurations, while the dashed horizontal line represents the only unstable spatial equilibrium (the symmetric equilibrium). The different intensities of gray recall those of Figure 5: in the dark-gray area firms in both countries collude, in the white area firms collude in neither country, in the light-gray area firms are indifferent between colluding and the status quo prevails.<sup>13</sup> For  $\eta = 0$  we have two

<sup>12</sup>The symmetric equilibrium is unstable but in the absence of migration its stability is not an issue. Had we taken any of the stable configurations at  $\eta = 0$  as initial configurations, i.e.,  $\lambda = 0.3934$  or  $\lambda = 0.6066$ , we would have had the same conclusion; namely, that an expected fine of 0.4 would eliminate collusion if migration were not possible.

<sup>13</sup>If collusion prevails it will take place in the largest country only. This is why we use a light-gray that recalls the situation “firms in  $H$  collude” or “firms in  $F$  collude” of Figure 5.

partial agglomeration equilibria in which all firms collude in their local markets. A small increase in  $\eta$  will push these two stable spatial equilibria further apart until  $\eta$  reaches the value  $\eta = 0.0511$  beyond which the two equilibria move closer to each other. Then, again they move apart until they reach the maximum possible distance between them. Coming to the effect of the antitrust policy on collusive behavior we see that within each zone of collusion (dark-gray and light-gray) changes in the severity of the policy are ineffective on firms behavior. In the dark-grey area of Figure 7 the policy is ineffective because it is not strong enough, but in the light-grey area of Figure 7 an increase in severity of the antitrust policy is ineffective because migration neutralizes it.<sup>14</sup>

Four main lessons can be drawn from this analysis. First, that the antitrust policy influences the economic geography. Second, that the effect of the antitrust policy on the economic geography is non-monotonic: starting from no antitrust policy, a weak antitrust policy will increase the degree of agglomeration, a tougher policy will reduce the degree of geographical agglomeration and further increases in toughness will induce an increase in the degree of agglomeration until complete agglomeration is reached. Third, a small increase in the toughness of the anti-trust policy from zero will not eliminate the collusive behavior; a further increase will leave firms indifferent between colluding and not colluding; and further increases will eliminate collusion only when complete agglomeration is reached. Fourth, the antitrust policy would be more effective in eliminating collusion if migration were not possible.

Figure 8 plots a bifurcation diagram analogous to that in Figure 7 but for an intermediate level of transportation costs,  $\tau = 0.5$ . We see that there will be collusion and no agglomeration for  $0 < \eta < 0.1247$ . For values  $0.1247 < \eta < 0.6694$  firms are indifferent between colluding and not colluding and any increase in the toughness of the antitrust policy is accompanied by an increase in the degree of agglomeration until complete agglomeration is reached. Finally, for any  $\eta > 0.6694$  firms will not collude while the degree of agglomeration remains at its maximum level.

Figure 9 plots a bifurcation diagram analogous to that in Figure 7 but for a low level of trade costs,  $\tau = 0.03$ . In this case the symmetric spatial configuration is stable for any value of  $\eta$ , there is collusion for  $\eta < 0.1247$  and non-collusion for higher values of  $\eta$ .

The results of the analysis in this section may be summarized as follows:

**Simulation result 1** *An increase of the fine on collusive behavior has a non monotonic effect on the degree of agglomeration when trade costs are high; first increasing it, then decreasing it, then increasing it again until complete agglomeration is reached. When trade costs are at an intermediate level, a small fine has no impact on agglomeration, but further increases in the fine increase the degree of agglomeration up to complete agglomeration. Only when trade costs are low the fine has no impact on the degree of agglomeration.*

What is interesting of this result is that the antitrust policy, typically designed to

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<sup>14</sup>The coordinates given by any stable spatial configuration and the corresponding values of  $\eta$  give us points on the frontier between the light-grey and white areas in Figure 5

fight collusive behavior, may bear collateral effects on the seemingly unrelated issue of the geographical location of industries.

**Simulation result 2** *Migration attenuates the effectiveness of the antitrust policy in eliminating collusive behavior.*

This result is interesting because it discloses an unexpected link between antitrust polices and policies that enable migration of factors.

### 5.3 The Effect of the Antitrust Policy on Welfare

We want to study the effect of the antitrust policy on economic welfare. For simplicity we assume that fine revenues are redistributed to consumers in a lump sum fashion, so that the fine *per se* is neutral on welfare. We are concerned only with world welfare, i.e., the sum of the indirect utilities of all the individuals in the world economy. We show the results by simulating the same three representative cases of high, intermediate, and low trade costs; respectively,  $\tau = 1.73$ ,  $\tau = 0.5$ , and  $\tau = 0.03$ .

Before we start the detailed analysis it is useful to grasp the intuition for the results. The effect of the antitrust policy on welfare goes through two channels: (1) the effect on oligopolistic prices (collusive or non-collusive) and (2) the effect on the spatial configuration of economic activity. It is intuitive that, other things equal, collusive oligopolies will yield lower welfare than non-collusive oligopolies simply because the deadweight loss due to price distortion is smaller in the latter than in the former. The first channel therefore conveys a positive effect of the antitrust policies on welfare. The second channel carries an effect that may be positive or negative. To see it through we have to distinguish three cases: non-collusive oligopoly, collusive oligopoly in one country only, and collusive oligopoly in both countries. Consider first the case of two non-collusive national oligopolies. In such case the welfare function is a strictly concave symmetric parabola in  $\lambda$  with a unique global maximum at  $\lambda = 1/2$  for high trade costs (the symmetric equilibrium therefore yields higher welfare than any other configuration); conversely, for low trade costs the welfare function is a symmetric convex parabola in  $\lambda$  with unique minimum at  $\lambda = 1/2$  (the symmetric equilibrium yields lower welfare than any other configuration). The logic of this result is that for high trade costs the welfare loss from agglomeration (represented by the numéraire lost in transit) outweighs the benefit from agglomeration (represented by increased local competition and, thereby, lower local deadweight loss). For low trade costs the balance is reversed.

Consider now the case of two collusive national oligopolies. In this case the welfare function is a concave symmetric parabola in  $\lambda$  with unique global maximum at  $\lambda = 1/2$  for any level of trade costs. The logic of this result is simple: in the presence of collusion agglomeration will not generate welfare gains in terms of an increase in the intensity of local competition. Therefore any agglomeration will bring about a reduction in world welfare because of the increase in the quantity of the numéraire lost in transit. The balance between the two channels will determine the welfare effect of the antitrust policy.

This same logic applies to the case of collusive oligopoly in one country and non-collusive oligopoly in the other country, the reason is that collusion takes place in the largest country.

Figures 10, 11, and 12 plot the welfare function against  $\eta$  for the case of high, intermediate, and low transport costs; respectively,  $\tau = 1.73$ ,  $\tau = 0.5$ , and  $\tau = 0.03$ . The different intensities of gray have the same meaning as those used in the bifurcation diagrams.

Consider first the case of high trade costs. This case is depicted in Figure 10. For  $\eta < 0.0511$  welfare is decreasing in  $\eta$ . The reason is that for these low values of  $\eta$  all firms of the economy collude (see Figure 5) and stable equilibria tend to be more dispersed (see Figure 7). For  $0.0511 < \eta < 0.6694$  we have two welfare functions because in all stable spatial configurations corresponding to these values of  $\eta$  firms located in the largest market are indifferent between colluding and not colluding and the prevailing status quo may be of collusion or non-collusion. If the status quo is of non-collusion the corresponding welfare function in the figure is titled “firms do not collude”. If the status quo is of collusion the corresponding welfare functions are titled “all firms collude” or “firms in  $H$  or  $F$  collude” according to which of the cases applies. Thus, if we read Figure 10 from low to high values of  $\eta$  the collusive status quo prevails since for the lowest values of  $\eta$  there is collusion. It follows that the relevant welfare function is always the lowest. Conversely, if we read Figure 10 from high to low values of  $\eta$  the non-collusive status quo prevails since for the highest values of  $\eta$  there is no collusion. It follows that the relevant welfare function is always the highest. Having clarified this matter consider now values of  $\eta \in (0.0511, 0.1247)$ . For these values of  $\eta$  both welfare functions are increasing in  $\eta$  because, as we have learnt in Figure 7, the two stable spatial configurations move closer to each other. Obviously the welfare function “all firms collude” is below the welfare function “firms in  $H$  or in  $F$  collude” since the latter contains a smaller deadweight loss. For values of  $\eta \in (0.1247, 0.6694)$  both welfare functions are decreasing in  $\eta$  because, as we have learnt in Figure 7, the stable spatial configurations get further apart. For values of  $\eta > 0.6694$  there is no collusion and welfare is independent from  $\eta$  since  $\eta$  does not influence the position of the stable spatial configurations.

Consider now the case of intermediate trade costs. This case is depicted in Figure 11. For  $\eta < 0.1247$  all firms collude, the symmetric equilibrium is stable, and there is no effect of  $\eta$  on welfare. For  $0.1247 < \eta < 0.6694$  the stable spatial configurations get further apart as shown in Figure 8. Welfare is increasing in  $\eta$  if firms do not collude. The reason is that for this value of trade costs the benefits from agglomeration in terms of lower deadweight loss are larger than the loss of agglomeration in terms of the numéraire lost in shipping. If, instead, firms collude in  $H$  or in  $F$  the gains from reduced deadweight loss are smaller and welfare declines in  $\eta$ . Again, for  $\eta > 0.6694$  there is no collusion and welfare is independent from  $\eta$  since  $\eta$  does not influence the economic geography.

Lastly, consider the case of very low trade costs. This case is depicted in Figure 12. We have learnt from Figure 9 that when trade costs are this low  $\eta$  does not influence the economic geography. The expected fine  $\eta$  influences only the decision of colluding or not. For  $\eta < 0.1247$  there is collusion and welfare is at a low level constant with

respect to  $\eta$ . For  $0.1247 < \eta < 1$  there is no collusion and welfare is at a high level constant with respect to  $\eta$ .

The results of the numeric analysis in this section may be summarized as follows:

**Simulation result 3** *A toughening of the antitrust policy may be welfare reducing even when it eliminates collusive-behavior. The antitrust policy is more likely to be welfare improving as economic integration progresses.*

What is interesting of this numerical exploration is not that the antitrust policy may increase welfare, after all that is precisely what is expected from these policies. What is interesting is the possibility that the antitrust policy, although successfully eliminates collusion, results in welfare reduction.

## 6 Conclusions

The objective of this paper has been to study the consequences of the market structure on economic geography and welfare. We have replaced monopolistic competition with oligopoly in the two forms of national and international oligopoly. We have shown that the likelihood of complete agglomeration is higher in oligopoly than monopolistic competition. In the presence of national oligopolies (collusive or not) there is a rich variety of stable spatial configurations (which includes multiple stable configurations), but for low enough trade costs the symmetric equilibrium is stable. The stability of the symmetric equilibrium for low trade costs is a result that marks an important difference between oligopoly and monopolistic competition. We then moved to the analysis of the effects of the antitrust policy. We have highlighted the collateral consequences of the antitrust policy on economic geography. We have also found that the possibility of migration attenuates the ability of the antitrust policy to eliminate collusion even when the policy is implemented by all governments with the same severity. Lastly, we have shown that the antitrust policy is welfare reducing when its negative effect conveyed through the economic geography outweighs its positive effect transmitted through prices.

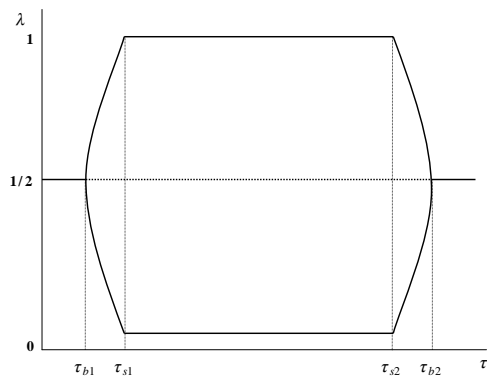
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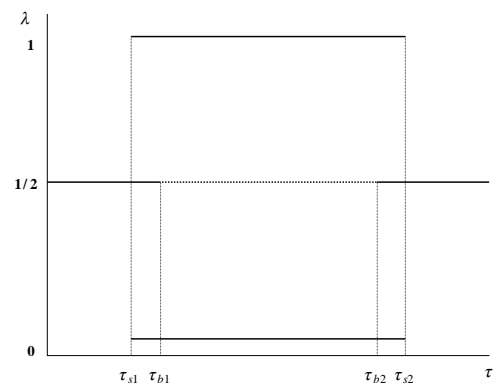
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Figure 1: Bifurcation diagrams with two break points and two sustain points

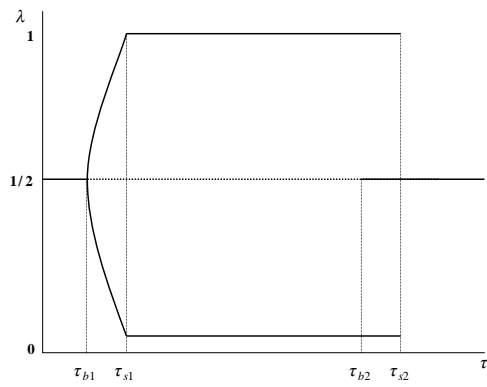
1a



1b



1c



1d

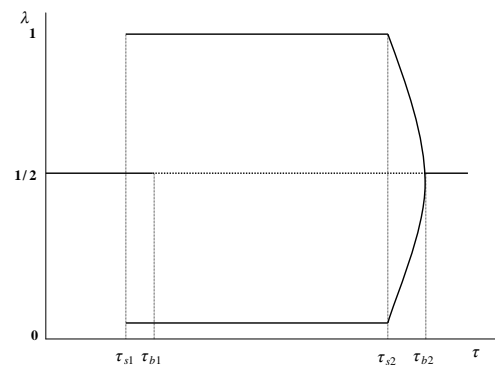


Figure 2: Bifurcation diagram with two sustain points and no break points

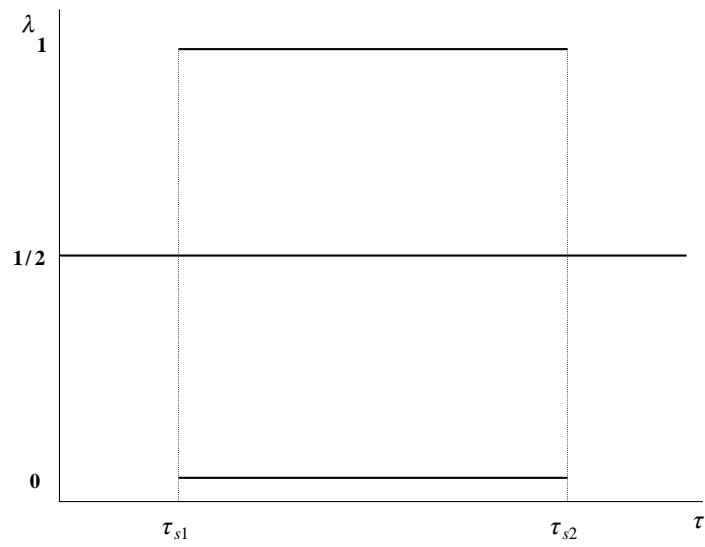


Figure 3: Bifurcation diagram with two break points and no sustain points

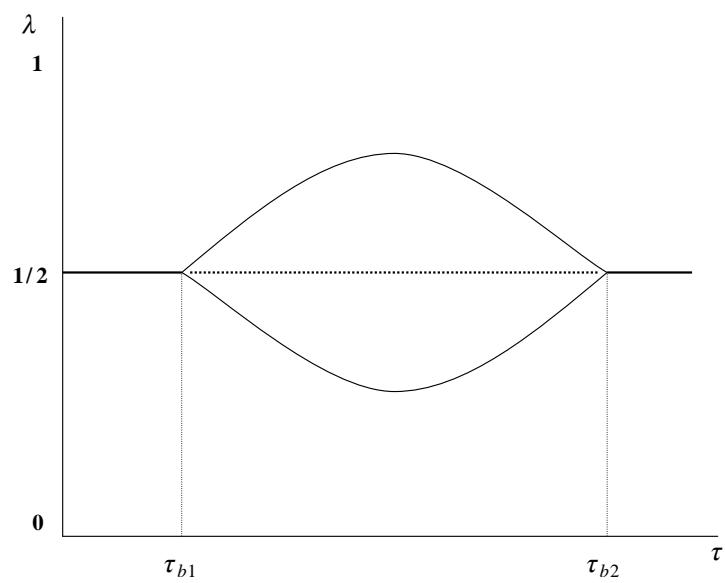


Figure 4: Bifurcation diagram with no critical points

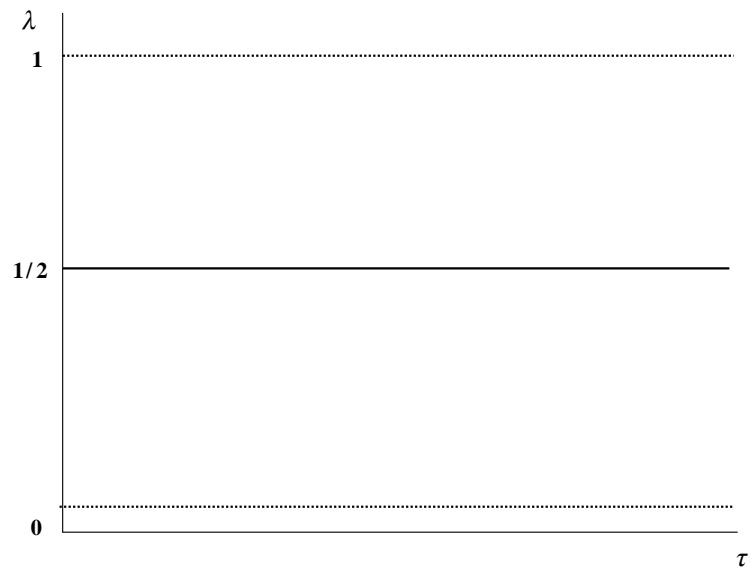


Figure 5: Competition regimes as function of  $\eta$  and  $\lambda$

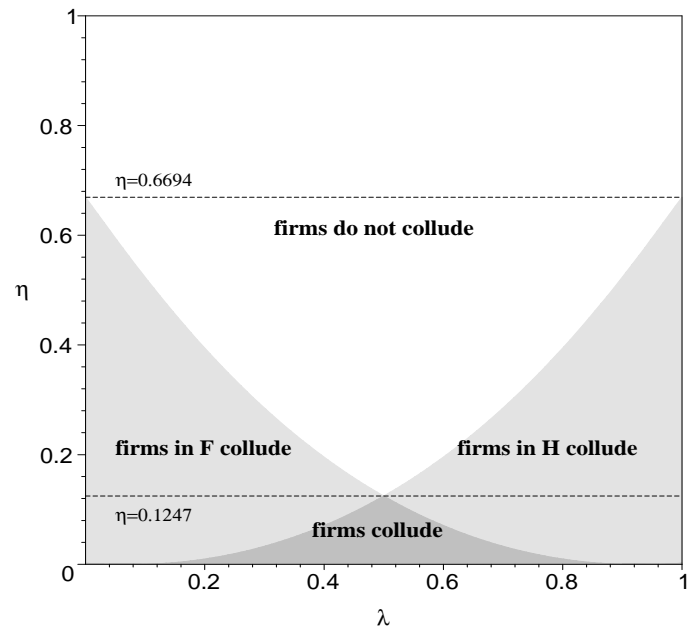


Figure 6: Phase diagrams,  $\tau = 1.73$

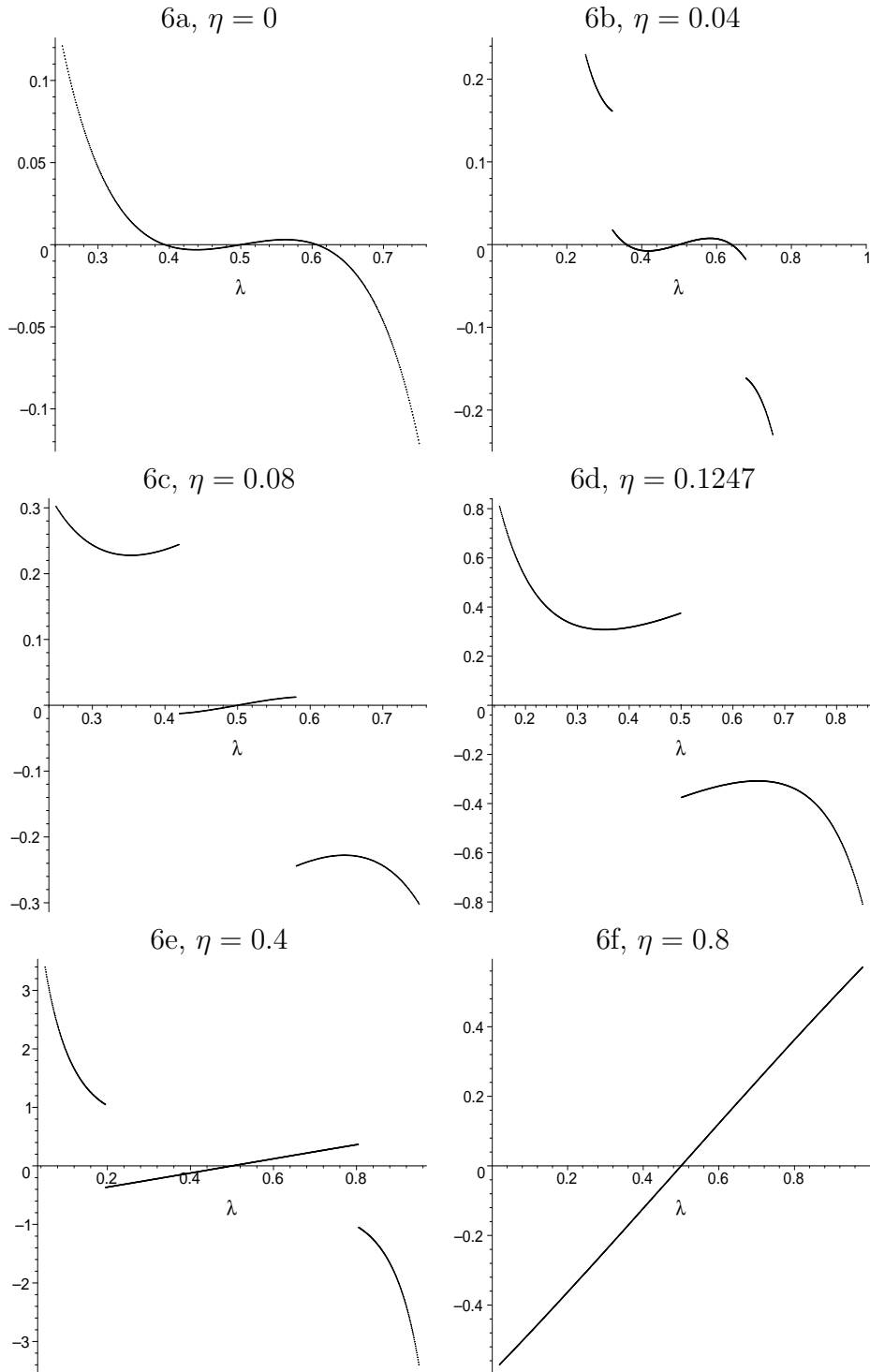


Figure 7: Bifurcation diagram,  $\tau = 1.73$

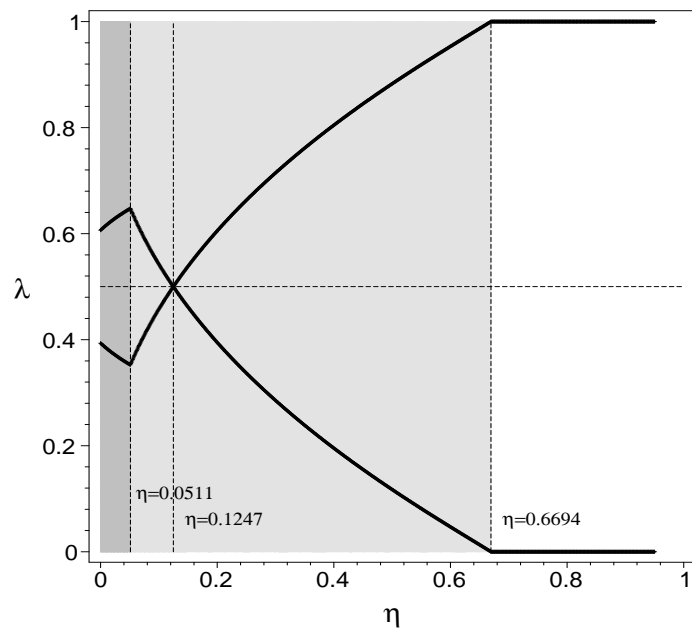


Figure 8: Bifurcation diagram,  $\tau = 0.5$

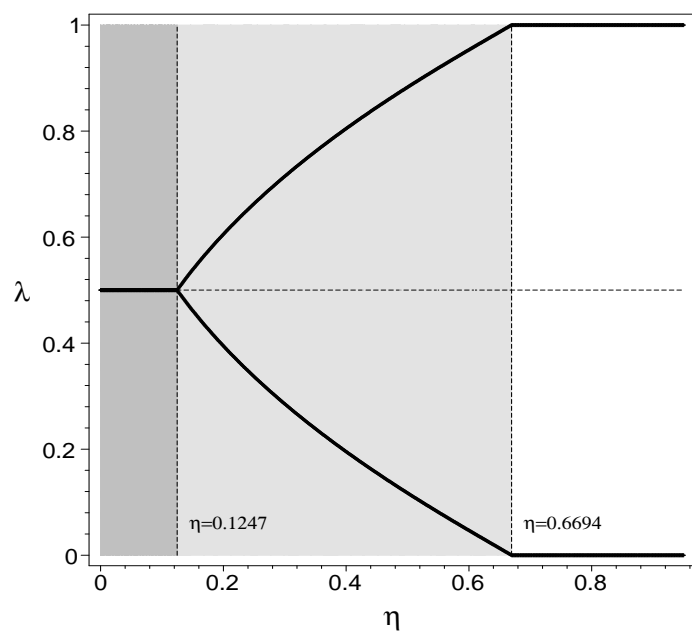


Figure 9: Bifurcation diagram,  $\tau = 0.03$

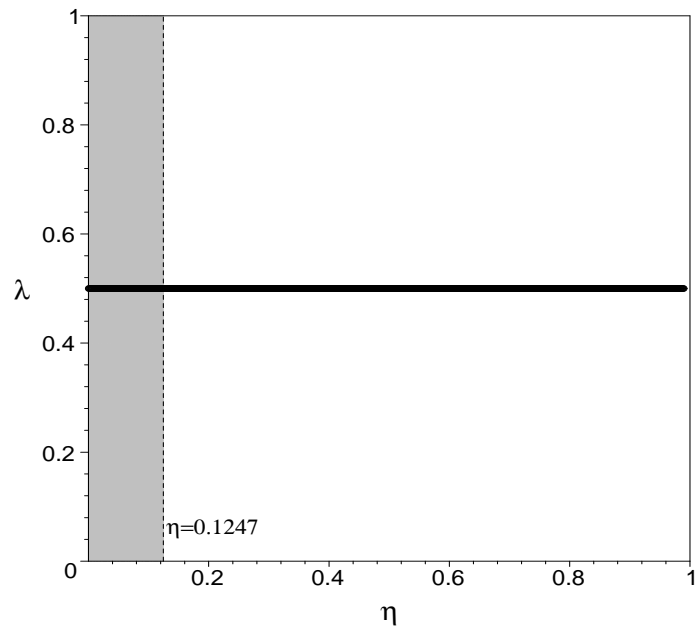


Figure 10: Welfare,  $\tau = 1.73$

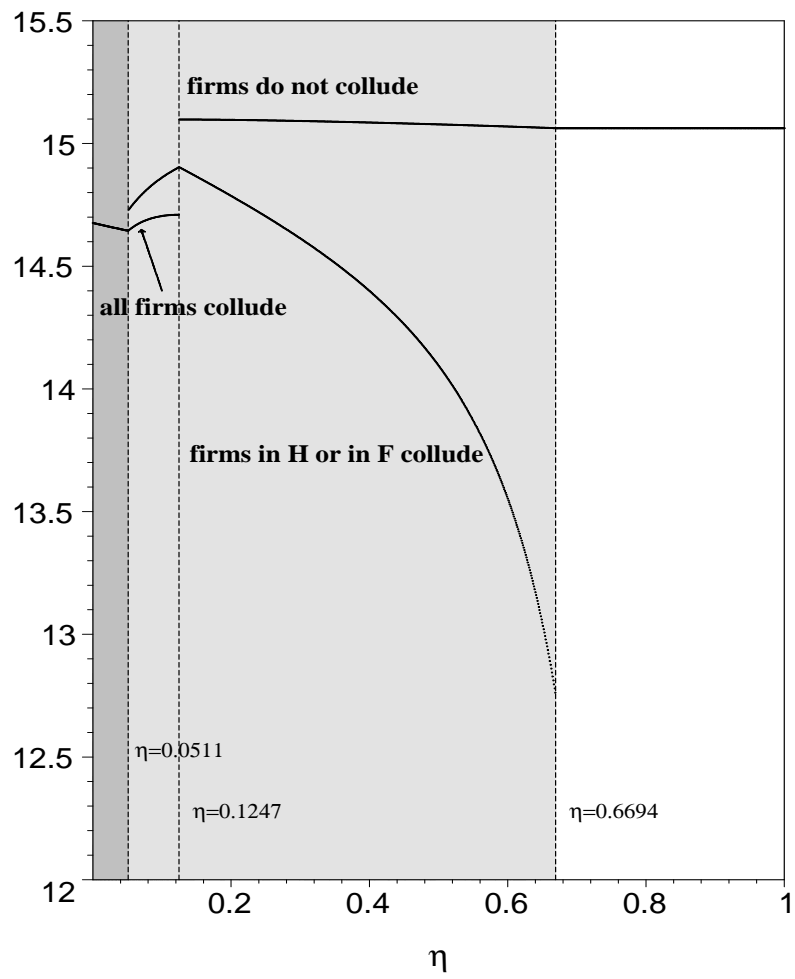


Figure 11: Welfare,  $\tau = 0.5$

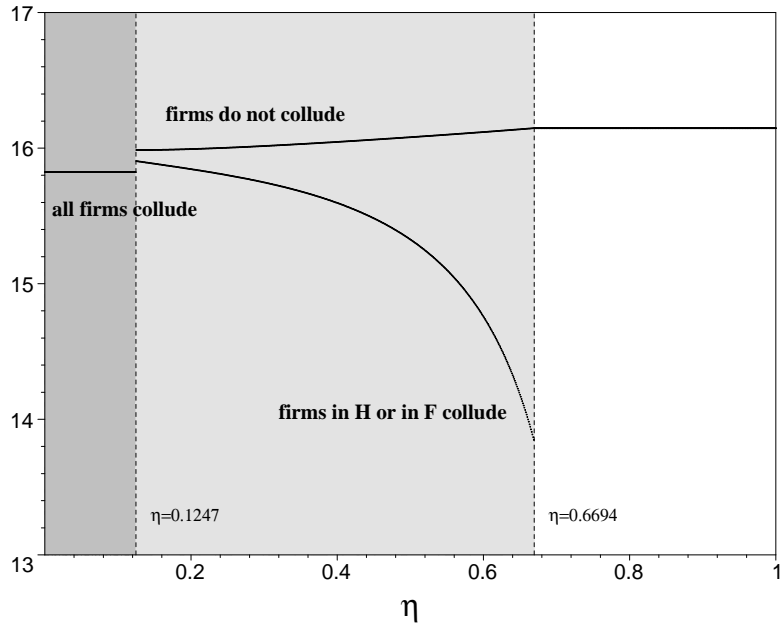
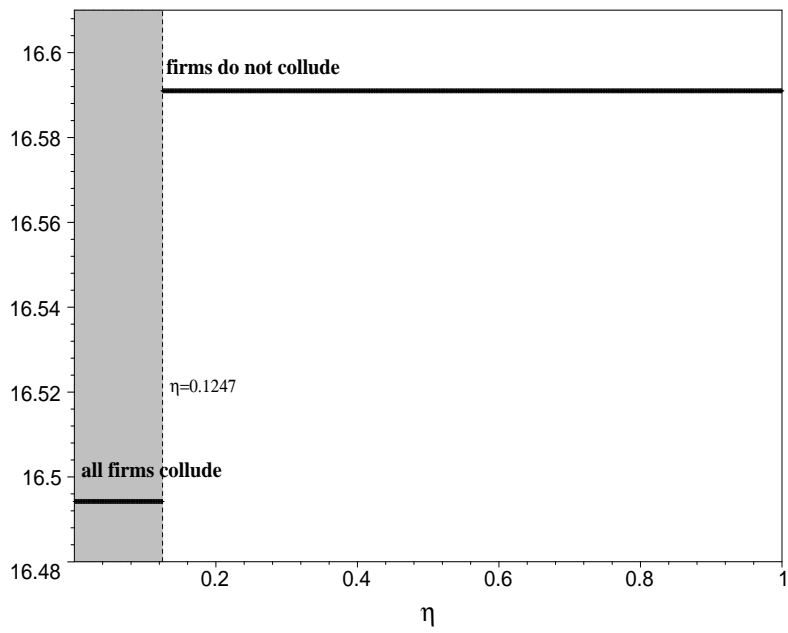


Figure 12: Welfare,  $\tau = 0.03$



## Appendix A

Full derivation of (21) can be obtained from (15) by substituting the equilibrium wages and consumer surplus in regions  $H$  and  $F$  associated with the equilibrium prices (17)-(20). Equilibrium wages,  $w_H$  and  $w_F$ , can be obtained by using the fact that all profits are absorbed by labor costs:

$$w_H = (\pi_{HH} + \pi_{HF}) / \phi, \quad (\text{A-1})$$

$$w_F = (\pi_{FF} + \pi_{FH}) / \phi, \quad (\text{A-2})$$

while  $S_H$  and  $S_F$  are given by the following expressions:

$$S_H = \frac{a^2 N}{2b} - aP_H^* + \frac{(b + cN)N}{2} [\lambda (p_{HH}^*)^2 + (1 - \lambda) (p_{FH}^*)^2] - \frac{c}{2} P_H^{*2}, \quad (\text{A-3})$$

$$S_F = \frac{a^2 N}{2b} - aP_F^* + \frac{(b + cN)N}{2} [(1 - \lambda) (p_{FF}^*)^2 + \lambda (p_{HF}^*)^2] - \frac{c}{2} P_F^{*2}, \quad (\text{A-4})$$

where  $P_H^* = N [\lambda p_{HH}^* + (1 - \lambda) p_{FH}^*]$  and  $P_F^* = N [(1 - \lambda) p_{FF}^* + \lambda p_{HF}^*]$ .

With the help of a symbolic mathematical software it can be shown that (15) can be expressed as:

$$\Delta V(\lambda) = C(\tau_c - \tau)\tau(\lambda - 1/2)$$

with  $C > 0$ ,  $\tau_c > 0$  where  $\tau_c > 4 \frac{a\phi(3b\phi + 2cN\phi)}{2b\phi(3b\phi + 3cN\phi + cA) + c^2 N\phi(A + N\phi)}$ , that is  $\tau_c$  is larger than the corresponding critical level of transportation costs in monopolistic competition. For full derivation details, see MAPLE file **Appendix.A**.<sup>15</sup>

## Appendix B

In the case of national oligopoly the indirect utility gap  $\Delta V(\lambda)$  can be obtained from (15) by substituting the equilibrium wages and consumer surplus in regions  $H$  and  $F$  associated with the equilibrium prices (22)-(25). In the MAPLE file **Appendix.B** we show that the slope of  $\Delta V(\lambda)$  at  $\lambda = 1/2$  is a quadratic function of trade costs  $\tau$  of the form (26). By Descartes' sign rule we know that the polynomial has roots with positive real parts.

In the same file, we derive equation (27), showing that coefficients' signs ensure the existence of roots with positive real parts. We also show that conditions (28) and (29) hold at  $\lambda = 1$  for sufficiently low values of transportation costs by simply demonstrating that consumers' surplus and wage gaps between countries are quadratic functions of  $\tau$  with negative constant terms.

Finally, turning to profits' differences computed at  $\lambda = 1$ , we show that the local profits' gap,  $\pi_{HH}^{NC} - \pi_{FF}^{NC}$ , is a quadratic function of  $\tau$  with positive constant term, while profits difference on distant markets,  $\pi_{HF}^{NC} - \pi_{FH}^{NC}$ , is a quadratic function of trade costs with negative constant term. These results imply that for sufficiently low values of  $\tau$  the following inequalities hold:  $\pi_{HH}^{NC} > \pi_{FF}^{NC}$  and  $\pi_{HF}^{NC} < \pi_{FH}^{NC}$ .

For further details see MAPLE file **Appendix.B**.

<sup>15</sup>All MAPLE files for this paper are downloadable at <http://www.economia.uniroma2.it/dei/professori/annicchiarico/maplefiles.htm>.

## Appendix C

In the MAPLE file `Appendix_C` we study the stability analysis of collusive national oligopoly by showing that using equilibrium prices (35)-(38) we have that:

$$\left. \frac{d\Delta V(\lambda)}{d\lambda} \right|_{\lambda=\frac{1}{2}} = -B_0^C + B_1^C \tau - B_2^C \tau^2, \quad (\text{C-1})$$

$$\Delta V(1) = -S_0^C + S_1^C \tau - S_2^C \tau^2, \quad (\text{C-2})$$

where all parameters are positive.

We also show that for low values of trade costs and full agglomeration of manufacturing activity in country  $H$  (i.e.  $\lambda = 1$ ) we have that the following inequalities hold:

$$S_H^C < S_F^C, \quad (\text{C-3})$$

$$w_H^C < w_F^C, \quad (\text{C-4})$$

$$\pi_{HH}^C > \pi_{FF}^C, \quad (\text{C-5})$$

$$\pi_{HF}^C < \pi_{FH}^C. \quad (\text{C-6})$$

## Appendix D

Consider the locus in the space  $(\lambda, \eta)$  for which firms located in  $H$  are indifferent between colluding or not:

$$(1 - \eta)\pi_{HH}^C - \pi_{HH}^{NC} = 0, \quad (\text{D-1})$$

Let  $\eta_H$  denote the value of the fine  $\eta$  satisfying (D-1). In the MAPLE file `Appendix_D` we show the critical level  $\eta_H$  does not depend upon the level of trade costs  $\tau$ . Further,  $\eta_H$  can be expressed in function of  $\lambda$  as follows:

$$\eta_H = \frac{(\lambda - 1/N)(D_{H,3}\lambda^3 + D_{H,2}\lambda^2 - D_{H,1}\lambda + D_{H,0})}{-\Lambda_H(\lambda)} \quad (\text{D-2})$$

where  $D_{H,3}, D_{H,2}, D_{H,1}, D_{H,0} > 0$  and

$$\Lambda_H(\lambda) = [cN(1 - \lambda) + b] \{(2b + cN)[c - 2(b + cN)] + c^2 N \lambda\}^2 > 0.$$

In the companion file `Appendix_D` we also show that the critical fine  $\eta_H$  is always increasing in  $\lambda$  for  $\lambda \in (1/N, 1)$ .

The representative firm located in country  $F$  will be indifferent between colluding or not in its local market if:

$$(1 - \eta)\pi_{FF}^C - \pi_{FF}^{NC} = 0. \quad (\text{D-3})$$

The critical level of  $\eta$  for which condition (D-3) holds, say  $\eta_F$ , is independent of  $\tau$  and can be written as function of  $\lambda$ :

$$\eta_F = \frac{\{\lambda - [(N - 1)/N]\}(-D_{F,3}\lambda^3 + D_{F,2}\lambda^2 + D_{F,1}\lambda + D_{F,0})}{\Lambda_H(\lambda)}, \quad (\text{D-4})$$

with  $D_{F,3}, D_{F,2}, D_{F,0} > 0, D_{F,1} \geq 0$ ;

$$\Lambda_F(\lambda) = (cN\lambda + b) \{(2b + cN) [c - 2(b + cN)] + c^2N(1 - \lambda)\}^2 > 0.$$

It can be easily shown that  $\eta_F$  is a decreasing in  $\lambda$  for  $\lambda \in (0, 1 - 1/N)$ . Full demonstration can be found in the Maple worksheet [Appendix D](#).

## Appendix E

The top panel of Table 1 reports the parameter values used in sections 5.2 and 5.3. The other panels report the implied critical values of trade costs in non-collusive and collusive oligopoly and the implied critical values of  $\eta$ .

<b>Table 1</b>	
<i>Baseline parameter values</i>	
$\bar{L} = 1$	labor supply in manufacturing
$A = 2.2$	labor force in agriculture
$\alpha = 10$	preference parameter
$\beta = 20$	preference parameter
$\gamma = 10$	preference parameter
$\phi = 0.1$	technology parameter
$N = 10$	number of firms
<i>Implied critical values of <math>\tau</math></i>	
<i>Non-collusive national oligopoly</i>	
$\tau_{trade}^{NC} = 1.7355$	threshold level for trade costs
$\tau_{b1}^{NC} = 0.0394$	first break point
$\tau_{b2}^{NC} = 2.2182$	second break point
$\tau_{s1}^{NC} = 0.3942$	first sustain point
$\tau_{s2}^{NC} = 2.2006$	second sustain point
<i>Collusive national oligopoly</i>	
$\tau_{trade}^C = 3.4634$	threshold level for trade costs
$\tau_{b1}^C = 1.5468$	first break point
$\tau_{b2}^C = 2.0676$	second break point
<i>Implied critical values of <math>\eta</math></i>	
0.1247	critical $\eta$ for which collusion ceases to be sustainable in both countries at $\lambda=1/2$
0.6694	critical $\eta$ for which collusion ceases to be sustainable in $H(F)$ at $\lambda=1(0)$