

Luiss Lab of European Economics
LLEE Working Document no. 64

**Heterogeneous Firms and Country-Specific
Productivity in a Core-Periphery Model with
Footloose Workers**

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April 2008

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Abstract

This paper presents a simple "core-periphery" model with footloose workers in which it is assumed that the productivity of firms depends on varieties produced and on the inter-country cost of trading ideas. Firms' heterogeneity leads to a "selection effect" that dampens the "Home Market Effect". On other hand, in the presence of costs of trading ideas the symmetric spatial equilibrium is a possible configuration if and only if there are no transportation costs. Finally, in the presence of cross-country efficiency gaps the degree of substitution between varieties matters in driving re-location decisions. If the degree of substitution among varieties is sufficiently small the most productive country becomes more attractive for all firms.

JEL code: F12, L11.

Keywords: New Economic Geography, Firm Heterogeneity, Cross-Country Efficiency Gaps, Home Market Effect.

1 Introduction

The fundamental contribution of the “New Economic Geography” (NEG) literature is to explicitly model "the self-reinforcing character of spatial concentration" (Fujita et al. 1999, p. 4). One of the most convenient assumptions in NEG models is that of identical firms. However, recent empirical research has substantiated the existence of large and persistent differences in terms of size (Cabral and Mata, 2003), as well as in terms of productivity and trade behaviour (Bernard et al., 2003; Helpman et al., 2004).

On the other hand, a look at almost any industry reveals geographic concentration. The possible explanations of why firms tend to cluster together are different and numerous. The first theories of the location of industry explain that this persistence of geographic concentration may arise because organizations benefit economically by locating in efficient positions. Several factors can explain industrial agglomeration: in some cases organizations benefit by minimizing the transportation cost for inputs (Weber, (1909) 1928); alternatively industries may locate near consumers to better serve these constituents (Smithies, 1941). Recently economic theory has paid attention on role of increasing returns to scale for explaining the uneven geographical distribution of economic activity. In particular Krugman (1991a) shows that the interaction of labour migration across regions with increasing returns and trade costs creates a tendency for firms and workers to cluster together as regions integrate. Finally recent studies investigate on role of economic knowledge in generating geographic concentration. Baldwin and Forsild (2000) find that knowledge spillovers are a powerful centripetal force and that integration policies that lower the cost of trading ideas encourage dispersion of economic activity. Our work provides a theoretical support to this empirical literature.

In this paper we develop a simple two-country NEG model in which we remove the standard and unrealistic assumption of identical firms. In particular, we extend the so called ‘core-periphery’ (CP) model (Krugman 1991a) with footloose workers to include heterogeneity among firms’ marginal production costs. As in Melitz (2003) we first assume that the productivity only depends on varieties produced and that the

two regions have the same firm-level productivity distribution function; we then hypothesize that there are inter-country costs of trading ideas and that the variance and the mean of the distribution functions are country-specific. Specifically, we assume that in the North workers are more productive than in the South. The mean and the variance of the marginal production costs are assumed to be lower in the North than those observed in the southern region. Since there exist significant efficiency gaps even across the most advanced industrial nations of the world, it seems to be an important issue to consider the case in which countries differ in their efficiency levels. Existing theoretical literature has provided a mixed picture on this issue. Montagna (2002) demonstrates that exposure to free trade will induce more low productivity firms to enter into the more efficient country; on the other hand, Jean (2002) shows that trade opening will improve the industry productivity in the more efficient country as well as in the less efficient one.

The model shows that the standard assumption of identical firms is not innocuous and that matters in driving location decisions. When productivity only depends on varieties produced, heterogeneity acts as a dispersion force in that re-location to the largest region is attractive only for the more productive firms. Different firm-level productivity dampens the “Home Market Effect” introducing a sort of “selection effect”, implying that the standard CP model overestimates agglomeration economies. In the presence of firms heterogeneity, in fact, full agglomeration occurs only when international trade is frictionless.

When we introduce costs of trading ideas, we still observe that heterogeneity is a dispersion force and tends to dampen the "Home Market Effect". Also in this case, in fact, a sort of "selection effect" is at work: the most efficient firms have a stronger preference for location in the larger market than inefficient firms. In this case, however, the "Home Market Effect" depends on the share of industrial workers located in the largest region; in particular, the effect is stronger than in the case in which regions have not efficiency gaps only if the share of industrial workers located in the bigger country is sufficiently large. Consistently with the NEG literature, the degree of substitution between varieties is also shown to be another crucial variable in driving location decisions. Specifically, if the degree of substitution between varieties

is sufficiently small the more productive country becomes more attractive for all firms. By contrast, for a larger degree of substitution between varieties penetrating more competitive market is more difficult because competition among firm is fiercer. It follows that competition among firms is always a dispersion force.

Finally, the cost of trading ideas is a powerful centripetal force and encourages partial agglomeration; in fact, in this case the symmetric equilibrium is a possible configuration if and only if trade is free, while full agglomeration never takes place.

The remainder of the paper is organized as follows. Section 2 presents the basic CP footloose workers model in which we include heterogeneity in firms' marginal production costs depending on variety produced ; Section 3 analyzes the case in which there are costs of trading ideas and the variance and the mean of the distribution functions are country specific; finally Section 4 concludes.

2 The model

In the economy there are two regions, North and South, that we assume to be symmetric in terms of tastes, openness to trade and, initially, in terms of their factor's supplies as in the standard CP model.

We extend the CP in two ways: we add heterogeneity among firms' marginal production costs and assume that workers face quadratic adjustment costs when switching from one region to another.

In each region there are two sectors: a perfectly competitive agriculture sector and a manufacturing sector that is characterized by monopolistic competition *à la* Dixit-Stiglitz (1977). Each sector employs a single specific factor: the agriculture sector A produces a homogeneous good under constant returns and uses only labor supplied by agricultural workers, L_A . More specifically it takes a_A unit of farmers to produce one unit of the A sector good. On the other hand, the manufacturing sector M employs labor of industrial workers to produce output subject to increasing returns. In particular, the production of each variety requires a fixed input involving F units of industrial worker labor L and a variable input a units of L per unit of

output produced. All firms share the same fixed cost F but have different productivity levels indexed by a that only depend on the type of varieties produced.

Both factors are assumed to be exogenously given. Farmers are evenly distributed across regions and are spatially immobile, while industrial workers are mobile between regions but they face immigration costs.

Trade in sector A is frictionless: the agricultural good can be traded freely inter and intra regions without incurring in any transportation cost. In sector M international trade is inhibited by iceberg trade costs. Specifically, it is costless to ship industrial goods to local consumers, but to sell one unit in the other region an industrial firm must ship $\tau > 1$ units¹. As usual, τ captures all the costs of selling to distant markets, not just transport costs, and $\tau - 1$ is the tariff-equivalent of these costs.

In the M sector there is a discrete number of firms N and each variety of the differentiated good is produced by a single firm. Each firm is assumed to have a certain market power and can perfectly price discriminate across markets.

2.1 Demand side

The preferences of the representative consumer are given by a two-tier utility function. The upper tier determines the consumer's division of expenditure between the homogeneous good and all differentiated industrial goods. The second tier dictates the consumer's preferences over the various differentiated industrial varieties.

The specific function form is Cobb-Douglas for the first tier and CES for the second. In symbols we have:

$$U = C_M^\mu C_A^{1-\mu}, \quad (1)$$

$$C_M = \left(\int_{i \in N} c_i^{1-\frac{1}{\sigma}} di \right)^{1/(1-\frac{1}{\sigma})}, \quad (2)$$

where C_M and C_A denote consumption of the composite of all differentiated varieties of industrial goods and consumption of the homogeneous good, respectively; N represents the mass of available industrial goods of which n are produced in North

¹The idea is that $\tau - 1$ units of the good "melts" in transit.

country and n^* are produced in South country. These goods are substitutes implying $0 < \frac{\sigma-1}{\sigma} < 1$ and where $\sigma > 1$ is the elasticity of substitution between any two industrial varieties; finally, μ is the expenditure share on differentiated goods.

The aggregate price of this economy is:

$$P = \left(\int_{i \in N} p_i^{1-\sigma} di \right)^{\frac{1}{1-\sigma}}. \quad (3)$$

This aggregate can be used to derive the optimal consumption and expenditure decisions for individual varieties using:

$$PC_M = E = \int_{i \in N} p_i c_i di = \mu I, \quad (4)$$

where E is the expenditure on M good and I is the expenditure on differentiated goods and on homogeneous good.

A well-known feature of the preferences in (1) is that utility maximization yields a constant expenditure allocation between industrial goods and the agricultural good. Thus the demand functions for M and A are:

$$C_M = \mu \frac{I}{P}, \quad (5)$$

$$C_A = (1 - \mu) \frac{I}{p_A}, \quad (6)$$

where p_A is the price of agricultural good.

Utility optimization also yields a standard CES demand function for each industrial varieties, namely:

$$c_i = \frac{p_i^{-\sigma}}{\int_{i \in N} p_i^{1-\sigma} di} \mu I. \quad (7)$$

2.2 Supply side

The A sector is characterized by perfect competition, no increasing returns and no trade cost. Perfect competition forces marginal cost pricing, that is:

$$p_A = w_A a_A, \quad (8)$$

$$p_A^* = w_A^* a_A, \quad (9)$$

where w_A (w_A^*) is the wage prevailing in North (South) region.

Costless trade in A equalizes northern to southern prices, and this, in turn, indirectly equalizes wage rates for agricultural labor in both regions. Henceforth, we take A as numéraire and choose units of A such $a_A = 1$. This simplifies the expression for the price index and expenditure since it implies that $p_A = w_A = p_A^* = w_A^* = 1$.

Turning to the industrial sector firm, technology is represented by a cost function that exhibits constant marginal costs with a fixed overhead cost. Labor used is thus a linear function of output \bar{x} :

$$l = F + a\bar{x}. \quad (10)$$

All firms share the same fixed cost $F > 0$, but have different productivity levels indexed by $a > 0$. We assume that each unit of labor in each region is associated with a particular level of productive efficiency as measured by the unit labor requirement a .

As in Helpman et al. (2004) the cumulative distribution of the marginal costs, identical in both country, is assumed to be a Pareto:

$$G[a] = \frac{a^\beta}{a_0^\beta}, \quad 1 = a_0 \geq a \geq 0, \quad \beta \geq 1, \quad (11)$$

where a_0 is the scale parameter, denoting the highest marginal cost, and β is a shape parameter. Without loss of generality, we normalize a_0 to unity.

The density function is:

$$f[a] = \frac{dG[a]}{da} = \beta \frac{a^{\beta-1}}{a_0^\beta}. \quad (12)$$

As in Baldwin and Okubo (2005) we assume that relocation is subject to quadratic adjustment costs. The specific assumption is:

$$\chi = \gamma m, \quad (13)$$

where χ denotes the cost of switching region, expressed in units of labor per worker, m is the flow of migrating workers, and γ is a positive parameter.

Each industrial firm is atomistic and thus rationally ignores the impact of its price on the aggregate price. Moreover since varieties are differentiated no direct strategic interaction among firms arises. As a consequence the typical firm acts as if it is a monopolist facing a demand curve with a constant elasticity equal to σ . Given the standard formula for marginal revenue, this implies that profit-maximizing consumer price is a constant mark-up of marginal cost. More specifically, the first-order conditions for a typical industrial firm's sales to its local market and its export market are:

$$p = \frac{\sigma}{\sigma - 1}aw, \quad (14)$$

$$p^* = \frac{\sigma}{\sigma - 1}\tau aw, \quad (15)$$

where p and p^* are the local and export prices of a North-based industrial firm, respectively. The restriction $\sigma > 1$ ensures that p and p^* are always positive and finite. Dixit-Stiglitz monopolistic competition assumptions on market structure imply “mill pricing” is optimal.

Profits made by the representative North-based firm i in both markets are defined as follows:

$$\Pi_i = p_i c_i + p_i^* c_i^* - w [F + a(x_i + x_i^*)], \quad (16)$$

where c_i and c_i^* are the demand functions faced by North-based firm in the two regions and w is the wage prevailing in the North.

The zero-profit condition requires operating profit to be equal to the fixed cost. Using the mill pricing rule and the equilibrium conditions $c_i = x_i$ and $c_i^* = x_i^*/\tau$, equation (16) can be re-written as:

$$\Pi_i = \frac{\sigma}{\sigma - 1}wax_i + \frac{\sigma}{\sigma - 1}w\tau a\frac{1}{\tau}x_i^* - wa\bar{x}_i - Fw = 0, \quad (17)$$

where \bar{x}_i denotes total firm i production, that is $\bar{x}_i = x_i + x_i^*$. It follows that the equilibrium firm size must satisfy:

$$\bar{x}_i = \frac{\sigma - 1}{a}F. \quad (18)$$

2.3 The Home Market Effect

In this section we analyze whether heterogeneity of firm level productivity influences the share of world production moving to the largest region.

We start by considering the equilibrium condition of industrial North market starting from the initial situation where no firms have moved:

$$\bar{x}_i = c_i + \tau c_i^*, \quad (19)$$

where c_i and c_i^* are the demand functions faced by a North firm i in North and in South, respectively, and \bar{x}_i is the total production of variety i .

Using (3) and (7), considering the mill pricing rule, noting that $I = wL$ and $I^* = w^*L^*$ and substituting into (19) we obtain:

$$\bar{x}_i = \frac{p_i^{-\sigma}}{P^{1-\sigma}} \mu I + \tau \frac{p_i^{*-\sigma}}{P^{*1-\sigma}} \mu I^*, \quad (20)$$

$$\begin{aligned} \bar{x}_i = & \frac{p_i^{-\sigma}}{\left(\frac{\sigma}{\sigma-1}w\right)^{1-\sigma} \left[\int_{\in n} (a)^{1-\sigma} di + \tau^{1-\sigma} \int_{\in n^*} (a)^{1-\sigma} di \right]} \mu I + \\ & + \tau \frac{p_i^{*-\sigma}}{\left(\frac{\sigma}{\sigma-1}w\right)^{1-\sigma} \left[\tau^{1-\sigma} \int_{\in n} (a)^{1-\sigma} di + \int_{\in n^*} (a)^{1-\sigma} di \right]} \mu I^*. \end{aligned} \quad (21)$$

To go from this expression defined in terms of an integral over goods to one defined in terms of an integral of marginal costs, we use the density function of a 's in each country, namely n times $f[a]$ for North and n^* times $f[a]$ for South. The support in both cases is the unit interval:

$$\begin{aligned} \bar{x}_i = & \frac{p_i^{-\sigma} \mu I}{\left(\frac{\sigma}{\sigma-1}w\right)^{1-\sigma} \left[n \int_0^1 a^{1-\sigma} f[a] da + n^* \tau^{1-\sigma} \int_0^1 a^{1-\sigma} f[a] da \right]} + \\ & + \frac{\tau p_i^{*-\sigma} \mu I^*}{\left(\frac{\sigma}{\sigma-1}w\right)^{1-\sigma} \left[n \tau^{1-\sigma} \int_0^1 a^{1-\sigma} f[a] da + n^* \int_0^1 a^{1-\sigma} f[a] da \right]}. \end{aligned} \quad (22)$$

Solving the integrals yields:

$$\int_0^1 a^{1-\sigma} f[a] da = \int_0^1 a^{1-\sigma} \beta \frac{a^{\beta-1}}{a_0^\beta} da = \eta, \quad (23)$$

where $\eta \equiv \frac{\beta}{1-\sigma+\beta}$. Notice that in order for the price indexes to be positive it is necessary that $\eta > 0$, that is when $\sigma < 1 + \beta$.

Substituting (23) into (22) and handling we have:

$$\frac{Fw^2\sigma}{a^{1-\sigma}}\eta = \frac{\mu I}{n + n^*\varphi} + \frac{\varphi\mu I^*}{n\varphi + n^*}, \quad (24)$$

where $\tau^{1-\sigma} = \varphi$ and $\eta > 0$. Note that φ measures the "free-ness" of trade. That is, the free-ness of trade rises from $\varphi = 0$, with infinite trade costs, to $\varphi = 1$, with zero trade costs (see Baldwin et al. 2003, p. 20).

Likewise for the South country we have:

$$\frac{Fw^{*2}\sigma}{a^{1-\sigma}}\eta = \frac{\mu I^*}{n^* + n\varphi} + \frac{\varphi\mu I}{n^*\varphi + n}. \quad (25)$$

In the symmetric equilibrium $w = w^*$, so we can choose $w = 1$ and we can write:

$$\frac{\mu I}{n + n^*\varphi} + \frac{\varphi\mu I^*}{n\varphi + n^*} = \frac{\mu I^*}{n^* + n\varphi} + \frac{\varphi\mu I}{n^*\varphi + n}. \quad (26)$$

After some manipulations we obtain:

$$s_n = \frac{1 + \varphi}{1 - \varphi}s_L + \frac{\varphi}{\varphi - 1}, \quad (27)$$

or equivalently:

$$s_n - \frac{1}{2} = \frac{1 + \varphi}{1 - \varphi} \left(s_L - \frac{1}{2} \right), \quad (28)$$

where s_n and s_L denote the shares of firms and of industrial workers in the North, respectively.

Equation (28) is the so called "Home Market Effect" (HME), according to which the location with the larger home market has a more than proportionately larger manufacturing sector. The HME implies that the location with larger demand succeeds in attracting a more than proportionate share of firms in imperfectly competitive industries. This pattern of demand-driven specialization generates the theoretical prediction that large regions should be net exporters of goods produced under increasing

returns and imperfect competition². This result is consistent with the findings of the standard CP model with homogeneous firms.

The above result can be summarized in the following proposition.

PROPOSITION 1: *Heterogeneity has no impact on the HME. The larger market ends up with a share of manufacturing firms more than proportional to its size.*

2.4 Relocation tendencies

According to the standard literature on agglomeration of economic activity, manufacturing workers migrate to the region that provides them with the highest level of indirect utility. As in CP models migration is governed by the *ad hoc* migration equation:

$$\dot{\lambda} = (\omega - \omega^*)\lambda(1 - \lambda), \quad (29)$$

where $\lambda = s_L$ is the share of northern industrial workers and $\omega = w/P^\mu$ and $\omega^* = w^*/P^{*\mu}$ are the indirect utility levels in North and South region, respectively (see Baldwin et al. 2003, pag. 15)³.

A spatial equilibrium is verified when no migration occurs, so that $\dot{\lambda} = 0$. Inspection of the migration equation (29) shows that there are two types of equilibria: interior outcomes, $0 < s_L < 1$, where industrial workers achieve the same level of utility, $\omega - \omega^* = 0$, wherever they reside; core-periphery outcomes, $s_L = 0$ ($s_L = 1$), where $\omega - \omega^* \leq 0$ ($\omega - \omega^* \geq 0$). Nevertheless when regions are intrinsically symmetric, that is for $s_L = \frac{1}{2}$ and $\omega = \omega^*$, $s_L = \frac{1}{2}$ is always an equilibrium.

²The intuition behind the HME is the following: since a profit-maximizing firm also minimizes the transport costs it incurs when delivering its output, everything else equal it will locate in the larger market. From this point of view, firms have a high propensity to settle at places where economic activities are already established.

³Substituting the demand functions for M and A into the utility function, we obtain the indirect utility function for North region: $u^N = \mu^\mu(1 - \mu)^{1-\mu} \frac{I}{P^\mu}$. Since $I = wL$, it follows that: $u^N = \mu^\mu(1 - \mu)^{1-\mu} \frac{Lw}{P^\mu}$.

Similarly for South country we have: $u^S = \mu^\mu(1 - \mu)^{1-\mu} \frac{L^*w^*}{P^{*\mu}}$.

The corresponding indirect utility differential is: $\Delta u = \mu^\mu(1 - \mu)^{1-\mu} \left[\frac{Lw}{P^\mu} - \frac{L^*w^*}{P^{*\mu}} \right]$.

The change in indirect utility moves the labor from South to North or from North to South and this change is function of firm's marginal cost, a :

$$\omega[a] - \omega^*[a] = \frac{w}{P^\mu} - \frac{w^*}{P^{*\mu}} = \frac{w^{(1-\sigma)/\mu}}{P^{1-\sigma}} - \frac{w^{*(1-\sigma)/\mu}}{P^{*1-\sigma}}. \quad (30)$$

The above expression can be rewritten as:

$$\omega[a] - \omega^*[a] = N \left(\frac{\sigma}{\sigma - 1} w \right)^{\sigma-1} \left[\frac{w^{(1-\sigma)/\mu}}{\Delta} - \frac{w^{*(1-\sigma)/\mu}}{\Delta^*} \right], \quad (31)$$

where $\Delta = \eta [s_n + (1 - s_n)\tau^{1-\sigma}]$ and $\Delta^* = \eta [\tau^{1-\sigma}s_n + (1 - s_n)]$.

Intuitively, the first workers that will find it profitable to pay the quadratic relocation costs will be those that have the most to gain, namely the most efficient workers (i.e. employees of the more productive manufacturing firms).

We define the threshold level of marginal costs for migration as a_R , where R stands for relocation. We note that the migration of the most efficient workers will affect the equilibrium levels of Δ and Δ^* :

$$\Delta[a_R] = s_n \int_0^1 a^{1-\sigma} f[a] da + (1 - s_n) \left[\int_0^{a_R} a^{1-\sigma} f[a] da + \tau^{1-\sigma} \int_{a_R}^1 a^{1-\sigma} f[a] da \right], \quad (32)$$

$$\Delta^*[a_R] = \tau^{1-\sigma} s_n \int_0^1 a^{1-\sigma} f[a] da + (1 - s_n) \left[\tau^{1-\sigma} \int_0^{a_R} a^{1-\sigma} f[a] da + \int_{a_R}^1 a^{1-\sigma} f[a] da \right]. \quad (33)$$

Equation (32) denotes the North's degree of local competition, where the first term is the price index of the varieties sold in the North; the second term denotes the price index of southern firms' varieties that are produced in the North (southern firms with $a \in [0, a_R]$ have relocated) and the third term indicates the prices of varieties that are made in the South and exported to the northern market.

Solving the integrals we obtain:

$$\Delta[a_R] = \eta \left[s_n + (1 - s_n) a_R^{1-\sigma+\beta} + \tau^{1-\sigma} (1 - s_n) \left(1 - a_R^{1-\sigma+\beta} \right) \right], \quad (34)$$

$$\Delta^*[a_R] = \eta \left[\tau^{1-\sigma} s_n + \tau^{1-\sigma} (1 - s_n) a_R^{1-\sigma+\beta} + (1 - s_n) \left(1 - a_R^{1-\sigma+\beta} \right) \right]. \quad (35)$$

Given these expressions we can write the value of re-location for any atomistic worker, denoted as v , as a function of firms' marginal cost and of the range of firms that have already moved:

$$v[a, a_R] = \omega[a, a_R] - \omega^*[a, a_R] = N \left(\frac{\sigma}{\sigma - 1} w \right)^{\sigma-1} \left[\frac{w^{(1-\sigma)/\mu}}{\Delta[a_R]} - \frac{w^{*(1-\sigma)/\mu}}{\Delta^*[a_R]} \right]. \quad (36)$$

The cost of switching region is χ units of labor per worker, as seen above (13). The stock of southern firms in the North is $\beta n^* a_R^{\beta-1}$, thus the flow of migrating firms is:

$$m = \beta n^* a_R^{\beta-1} \dot{a}_R. \quad (37)$$

At the optimum the marginal cost of switching region must be equal to the marginal benefit of migration, that is $\chi = v[a, a_R]$. It follows that the value to the marginal worker of migrating will be:

$$v[a, a_R] = v[a_R] = \gamma \beta n^* a_R^{\beta-1} \dot{a}_R. \quad (38)$$

Clearly, if $\omega[a] - \omega^*[a] > 0$ there is migration from South to North; after labor migration the share of all firms in the large region is:

$$s = \frac{n}{N} + \frac{n^*}{N} \int_0^{a_R} \beta \frac{a^{\beta-1}}{a_0^\beta} da, \quad (39)$$

where s is the share of northern based firms after migration or the share of all firms in the big region. Solving the integral we have:

$$s = s_n + (1 - s_n) a_R^\beta. \quad (40)$$

Allowing for firm heterogeneity adds a spatial selection component to the HME, namely the large market attracts the most efficient firms.

PROPOSITION 2: *Firms that move to the large market are systematically more efficient than firms that stay behind. Heterogeneity has added a spatial selection dimension to the HME.*

The intuition for this spatial selection is the following. The most efficient firms have a stronger preference for location in the large market than inefficient firms.

Efficient firms have higher sales, exploit better increasing returns and thus enjoy greater savings on trade costs. In addition, efficient firms manage to cope better with the fiercer local competition characterizing the largest market.

2.5 The long-run equilibrium

The long-run equilibrium satisfies the location condition $v[a_R] = 0$; in fact when workers (firm) migration has stopped marginal adjustment costs are zero. It follows that the cut-off level of marginal cost, a_R , that is the key variable to be determined in the long run equilibrium, can be characterized as follows using (36):

$$a_R^{1-\sigma+\beta} = -\frac{1}{2} \frac{2s_n - 1}{1 - s_n}. \quad (41)$$

Now we insert (27) into (41):

$$a_R^{1-\sigma+\beta} = -\frac{1}{2} \frac{2s_L + 2s_L\varphi - 2\varphi - 1 + \varphi}{1 - s_L - s_L\varphi}. \quad (42)$$

We note that a_R is increasing in φ implying that more inefficient workers (firms) find it profitable to relocate as trade gets freer. Full agglomeration occurs when $a_R = 1$, that is when trade is costless ($\varphi = 1$).

We can summarise these results in the following way:

PROPOSITION 3: *Heterogeneity dampens the HME and acts as a dispersion force in the sense that a smaller share of firms will relocate from the smallest region to the largest one for any intermediate level of trade free-ness. Full agglomeration will occur only when trade is costless.*

3 Country-specific productivity

In this section we consider the case in which there are costs of trading ideas inter country and northern firms are more productive than southern ones. In particular, we suppose that marginal costs are on average smaller and less dispersed in the North than in the South. In other words, we assume that the shape parameter β is country

specific, that means that we have β_N for North and β_S for South, where $\beta_N < \beta_S$. We can suppose that the difference between β_N and β_S positively depend on cost of trading ideas.

In this case the cumulative distribution and the density function are $G[a]_N$ and $f[a]_N$ for North and $G[a]_S$ and $f[a]_S$ for South. Therefore solving the integrals we obtain two different values of η , η_N and η_S for North and South respectively:

$$\int_0^1 a^{1-\sigma} f[a]_N da = \int_0^1 a^{1-\sigma} \beta_N \frac{a^{\beta_N-1}}{a_0^{\beta_N}} da = \frac{\beta_N}{1-\sigma+\beta_N} = \eta_N, \quad (43)$$

$$\int_0^1 a^{1-\sigma} f[a]_S da = \int_0^1 a^{1-\sigma} \beta_S \frac{a^{\beta_S-1}}{a_0^{\beta_S}} da = \frac{\beta_S}{1-\sigma+\beta_S} = \eta_S. \quad (44)$$

We note that as in the previous case in order for the price indexes to be positive it is necessary that η_N and $\eta_S > 0$ that is when $\sigma < 1 + \beta_N$ ⁴. In this case condition $\eta_N > \eta_S$ always holds because η is increasing in σ ⁵.

3.1 The Home Market Effect

In this section we analyze whether the existence of efficiency gaps between the two countries influences the share of world production moving to the larger region. We note that in this case (27) becomes:

$$s_n = -\frac{(\varphi+1) s_L \eta_S + \varphi \eta_S}{s_L [(\varphi+1)(\eta_N - \eta_S)] + \varphi \eta_S - \eta_N}. \quad (45)$$

We note that the HME depends on the share of industrial workers located in the largest region s_L and on η_N and η_S . Since both η_N and η_S , in turn, depend on σ from the above result we notice that the HME crucially depends on σ ⁶. In particular we observe the following:

PROPOSITION 4: *If $s_L = \frac{1}{\varphi+1}$ then the HME is as in the case in which countries have the same productivity distribution functions and is independent of σ . For $\sigma <$*

⁴In fact $\sigma < 1 + \beta_N$ involves $\sigma < 1 + \beta_S$.

⁵Note that $\frac{\delta \eta}{\delta \sigma} = -\beta(1-\sigma+\beta)^{-2}(-\beta) > 0$.

⁶Note that for $\sigma > 1$ then $\frac{\delta \eta_N}{\delta \sigma} = \beta_N^2(1-\sigma+\beta_N)^{-2} > 0 > \frac{\delta \eta_S}{\delta \sigma} = \beta_S^2(1-\sigma+\beta_S)^{-2}$.

$1 + \beta_N$ if $s_L < (>)\frac{1}{\varphi+1}$ the HME is weaker (stronger) than in the case in which countries have the same productivity distribution functions and becomes weaker (less strong) the larger σ .

PROOF: See Appendix.

The intuition behind this result is that for a lower degree of substitution between varieties it is convenient to relocate in the largest market in order to take advantage of increasing returns. By contrast, for a larger degree of substitution between varieties competition among firms is stronger and location of economic activity in the large market becomes less attractive for firms.

3.2 Relocation tendencies

An important implication of the assumption of the country-specific productivity is that we can rewrite the price indexes as:

$$P^{1-\sigma} = \left(\frac{\sigma}{\sigma-1} w \right)^{\sigma-1} (n\eta_N + \varphi n^* \eta_S), \quad (46)$$

$$P^{*1-\sigma} = \left(\frac{\sigma}{\sigma-1} w \right)^{\sigma-1} (\varphi n \eta_N + n^* \eta_S). \quad (47)$$

The price indexes are positive when η_N and $\eta_S > 0$, that is when $\sigma < 1 + \beta_N$.

It follows the next Proposition.

PROPOSITION 5: *When there are costs of trading ideas, the northern firms are more productive than the southern firms (that is when $\beta_N < \beta_S$), there are transport costs and $\sigma < 1 + \beta_N$, the North country is more attractive than the South for all firms and the symmetric equilibrium is not a possible configuration.*

PROOF: See Appendix.

Since symmetry is possible if and only if trade is free the cost of trading ideas is a powerful centripetal force.

3.3 The long-run equilibrium

In the long-run equilibrium the location condition $v[a_R] = 0$ will be satisfied ; in this case condition (36) becomes:

$$v[a, a_R] = N \left(\frac{\sigma}{\sigma - 1} w \right)^{\sigma-1} \left\{ \frac{w^{(1-\sigma)/\mu}}{\eta_N s_n + \eta_S (1 - s_n) \left[a_R^{1-\sigma+\beta_S} + \varphi (1 - a_R^{1-\sigma+\beta_S}) \right]} - \frac{w^{*(1-\sigma)/\mu}}{\varphi \eta_N s_n + \eta_S \left[\varphi (1 - s_n) a_R^{1-\sigma+\beta_S} + (1 - s_n) (1 - a_R^{1-\sigma+\beta_S}) \right]} \right\}. \quad (48)$$

The cut-off level of marginal cost a_R becomes country specific: a_R^S for southern firms and a_R^N for northern ones. In particular, a_R^S can be characterized as follows using (48):

$$a_R^{S(1-\sigma+\beta_S)} = -\frac{1}{2} \left(\frac{\eta_N}{\eta_S} \frac{s_n}{1 - s_n} - 1 \right). \quad (49)$$

An isomorphic formula holds for northern firms.

We note that a_R^S depends on η_N , η_S and σ . In particular, we obtain:

PROPOSITION 6: *When $\beta_N < \beta_S$, the cut-off level of marginal cost a_R^S depends on β_S and on σ . If $\sigma < 1 + \beta_S$ then a_R^S is decreasing in β_S and σ , implying that the lower β_S , the larger number of southern firms that relocate to the northern market; however, the larger σ , the less attractive the North country becomes for southern firms.*

PROOF: See Appendix.

This result depends on the fact that when β_S declines as the productivity of the South region increases; it follows that the cross-country efficiency gaps diminish and a higher number of southern firms can relocate in the North region; workers (consumers) find it convenient to relocate where the price index is smaller. By contrast, when the degree of substitution between varieties is sufficiently high competition among firms is stronger and for less competitive southern firms will be more difficult to penetrate the more productive region. Competition among firms always acts as a dispersion force.

Finally as in the previous analysis full agglomeration will occur only if $a_R = 1$. Substituting into (49) equation (45) and imposing $a_R = 1$ we obtain that in this case full agglomeration is when $\varphi < 0$, but since $0 < \varphi < 1$ it follows that:

PROPOSITION 7: *When northern firms are more productive than southern firms, full agglomeration never takes place. Costs of trading ideas encourage partial agglomeration.*

4 Conclusions

In the past years the "New Economic Geography" literature has relied on the assumption of identical industrial firms. While this was viewed as an assumption of convenience, this paper shows that this standard assumption is not innocuous and that matters in driving location decisions.

In this work we develop a simple two-country NEG model in which we remove the unrealistic assumption of identical firms. In particular, we extend the so called 'core-periphery' (CP) model (Krugman 1991a) with footloose workers to include heterogeneity among firms' marginal production costs and cross-country gaps efficiency levels.

From our analysis it emerges that when different firm-level productivity only depends on the type of varieties produced, heterogeneity acts as a dispersion force in the sense that relocation to the largest region is attractive only for the most productive firms. Different firm-level productivity dampens the "Home Market Effect" introducing a sort "selection effect". This result implies that the standard CP model overestimates agglomeration economies. In the presence of firms heterogeneity, in fact, full agglomeration will occur only when international trade is frictionless.

In the case in which there are costs of trading ideas we can suppose that there exist country-specific productivity distribution functions. Again, we still observe that heterogeneity is a dispersion force and tends to dampen the "Home Market Effect". In fact, also in this case a sort of "selection effect" is at work: the most efficient firms have a stronger preference for location in the larger market than inefficient

firms. In this case, however, the "Home Market Effect" depends on the share of industrial workers located in the largest region; in particular, the effect is stronger than in the case in which regions have not efficiency gaps if the share of industrial workers located in the biggest country is sufficiently large. Moreover, the degree of substitution between varieties matters in driving location decisions when countries differ in their productivity distribution. If the degree of substitution between varieties is sufficiently small the more productive country is more attractive for all firms. By contrast, for a larger degree of substitution among varieties the entry into competitive market is more difficult because competition among firms is stronger. It follows that competition among firms is always a dispersion force. Finally, the cost of trading ideas is a centripetal force and encourage partial agglomeration of economic activity; in fact, in this case symmetric equilibrium is a possible configuration if and only if trade is free, while full agglomeration never takes place.

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Appendix

PROOF OF PROPOSITION 4: We denote s_n in equation (27) as s_n^A and s_n in equation (45) as s_n^B . We have:

$$\frac{s_n^A}{s_n^B} = \frac{(1 - \varphi) \eta_S + (\eta_N - \eta_S) [1 - s_L (\varphi + 1)]}{(1 - \varphi) \eta_S} = 1 + \frac{1 - s_L (\varphi + 1)}{(1 - \varphi) \eta_S} (\eta_N - \eta_S). \quad (50)$$

It follows that $s_n^B = s_n^A$ if and only if $1 - s_L (\varphi + 1) = 0$, that is when $s_L = \frac{1}{\varphi + 1}$. In this case the HME is the same as in the case in which countries differ in their efficiency levels as in the case in which countries have the same productivity distribution functions. When $s_L \neq \frac{1}{\varphi + 1}$ then $s_n^B \neq s_n^A$. In particular, if $\sigma < 1 + \beta_N$ and $\eta_N, \eta_S > 0$ then $s_n^B < (>) s_n^A$ if $s_L < (>) \frac{1}{\varphi + 1}$, that means that the HME is weaker (stronger) in the case in which regions differ in their efficiency levels than in the case in which countries have the same productivity distribution functions.

Moreover when $s_L \neq \frac{1}{\varphi+1}$ and σ increases then $(\eta_N - \eta_S)$ decreases and $\frac{s_n^A}{s_n^B}$ increases. Hence if $s_n^B < (>)s_n^A$ the difference between s_n^B and s_n^A increases (decreases).

PROOF OF PROPOSITION 5: The results immediately follow from equations (46)-(47):

$$P^{1-\sigma} = \left(\frac{\sigma}{\sigma-1} w \right)^{\sigma-1} (n\eta_N + \varphi n^* \eta_S), \quad (51)$$

$$P^{*1-\sigma} = \left(\frac{\sigma}{\sigma-1} w \right)^{\sigma-1} (\varphi n \eta_N + n^* \eta_S). \quad (52)$$

where $\sigma < 1 + \beta_N$ and η_N and $\eta_S > 0$.

If $n = n^*$ then:

$$\frac{\delta P^{1-\sigma}}{\delta \varphi} > 0 = \left(\frac{\sigma}{\sigma-1} w \right)^{\sigma-1} \eta_S < \frac{\delta P^{*1-\sigma}}{\delta \varphi} > 0 = \left(\frac{\sigma}{\sigma-1} w \right)^{\sigma-1} \eta_N. \quad (53)$$

It follows that if $\tau > 1$ then $P < P^*$.

If $n > n^*$ and $\tau > 1$ we have that:

$$P^{1-\sigma} = \left(\frac{\sigma}{\sigma-1} w \right)^{\sigma-1} (n\eta_N + \varphi n^* \eta_S) > P^{*1-\sigma} = \left(\frac{\sigma}{\sigma-1} w \right)^{\sigma-1} (\varphi n \eta_N + n^* \eta_S). \quad (54)$$

It follows that $P < P^*$; in this case South is condemned to remain a periphery.

If $n < n^*$ and $\tau > 1$ we have:

$$\begin{aligned} P^{1-\sigma} &= \left(\frac{\sigma}{\sigma-1} w \right)^{\sigma-1} (n\eta_N + \varphi n^* \eta_S) > P^{*1-\sigma} = \\ &= \left(\frac{\sigma}{\sigma-1} w \right)^{\sigma-1} (\varphi n \eta_N + n^* \eta_S) \Leftrightarrow n\eta_N > n^* \eta_S. \end{aligned} \quad (55)$$

It follows that $P < P^* \Leftrightarrow n\eta_N > n^* \eta_S$; in this case North can become a core.

Finally independently of n and n^* if $\tau = 1$ each configuration (partial agglomeration or symmetric equilibrium) is possible because $P = P^*$.

PROOF OF PROPOSITION 6:

If $\sigma < 1 + \beta_S$ then η_N and $\eta_S > 0$.

If β_S increases then $\frac{\eta_N}{\eta_S}$ will increase because η_S decreases. But if $\frac{\eta_N}{\eta_S}$ increases then both $a_R^{S(1-\sigma+\beta_S)}$ and a_R^S will diminish:

$$\frac{\delta a_R^{S(1-\sigma+\beta_S)}}{\delta \frac{\eta_N}{\eta_S}} = -\frac{1}{2} \frac{s_n}{1-s_n}. \quad (56)$$

It follows that a_R^S is decreasing in β_S .

If σ increases then $\frac{\eta_N}{\eta_S}$ will rise because both η_N and η_S increase, but η_N increases more than η_S . Hence $\frac{\eta_N}{\eta_S}$ increases and $a_R^{S(1-\sigma+\beta_S)}$ and a_R^S decreases. It follows that a_R^S is decreasing in σ .