Managerial Incentives, Derivatives and Stability

By

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Abstract

In this paper we model the derivative strategies optimally undertaken by a manager (or head of a profit center in a hedge fund) when the detailed derivative positions taken are not contractible. We show that the commonly-used incentive features in the compensation structure of top-management in firms or hedge-fund managers may induce them to implement complex derivative strategies that lead to the following pay-off pattern: a high probability (close to one) of a slight increase in performance measures at the cost of allowing for the possibility of disaster states involving large losses, although with a very small probability. These disaster states cause systemic instability (similar to the experience of Long-Term Capital Management in September 1998). We discuss possible audit strategies, governance mechanisms and incentive structures that will ameliorate the probability of systemic instability arising from such incentives in a market with a rich enough menu of derivatives. We characterize the optimal intensity of audit effort with and without the presence of such derivative strategies. The dependence of the optimal audit intensity on the legal liability regime and different rules for apportioning the auditor’s liability is derived. Our results also relate the optimal audit intensity to the cost and efficiency parameters of the audit firm.

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Managerial Incentives, Derivatives and Stability

Since the 1980's, the derivative markets experienced substantial growth both in terms of the volume of transaction and the types of instruments available. (As of year-end 1994, well over 70% of the U.S. Fortune 500 firms reported using derivative instruments\(^1\).) These derivative instruments enable the firm to hedge, speculate, and in other ways modify the distribution of cash flows significantly at relatively low cost \(^2\). On one hand, the availability of these new markets and instruments has made it possible for the firms to hedge specific risks (for example, commodity price, exchange rate and interest rate risks) at low transactions costs. On the other hand, the possibility of modifying the probability distribution of cash flows of the firm in specific custom-made ways at low cost opens up some important incentive problems. For example, the manager of a hedge fund, or the CEO of a firm, compensated by a conventional performance-based compensation contract, can modify the probability distribution of the cash flows of his profit center to “game” very closely his compensation contract. A bank, which is subject to regulatory supervision, can modify its cash flow structure to specifically “game” the rules of the particular regulatory regime in place. Although the presence of these incentive problems is not a new phenomenon, we will argue that the low-cost customization of probability distributions using complex derivative positions has made it possible to implement strategies specifically designed to “game”


These strategies constitute a general version of the risk-shifting strategies implemented by the manager of a limited-liability corporation with risky debt outstanding, when his compensation aligns him with equity holders, and there is incomplete contracting vis-a-vis the riskiness of investment choices. See, e.g., John and John (1993), and John, John, and Senbet (1991), and John (1987).

There are countless strategies which can be implemented to “game” compensation structures and regulatory regimes in place, using derivatives. (See, e.g., Bookstaber (1990).) The detailed positions in the different instruments and the resulting pay-off structures will be discussed later. However, a simple characterization of the generic strategy which can be implemented by a profit center in a manufacturing firm or a financial institution is as follows: hold appropriate derivative positions to synthesize an incremental cash flow distribution such that with 99.99% probability, it adds in a modest way to its earnings from the underlying activity, but with .01% probability, it causes large losses. The relative size of the modest increase and the large losses can be chosen such that the above strategy is a self-financing one. Moreover, since the large-loss state may not “occur” for a number of periods, the profit center can show superior performance for a number of periods prior to the realization of the large-loss outcome.

The use of derivatives to customize cash flow distributions in order to game incentive structures and regulatory regimes raises several auditing and monitoring issues:

(a) For the top management of a manufacturing firm or a financial institution, a crucial challenge may be to set up and implement optimal internal audit systems and procedures to ensure that a particular profit center within the organization does not synthesize and maintain

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The explosive growth of the volume of transactions and menu of derivative products in the last fifteen years and some recent highly publicized derivative losses (e.g., Proctor and Gamble) have fueled debate on the advisability of regulatory monitoring of the use of these instruments. The counterargument has been that firms can monitor their own derivative use without regulatory intervention. However such an argument rests crucially on the effectiveness and viability of internal and external audit systems even in the presence of complex derivative pay-offs masking the nature of the cash flow distribution of the normal business activity. Aspects of the design of such auditing systems is the focus of this paper.

See Alderman and Tabor (1989).

(b) For the regulator or a bank examiner the central question may be the use of optimal monitoring and audit systems such that (i) the bank does not expose the depositors to the large-loss scenario (even with a small probability), (ii) the true-risk of the cum-derivative portfolio of the bank assets can be determined for regulatory purposes, e.g., for charging the appropriate FDIC premiums and undertaking the appropriate level of routine monitoring and corrective actions.

(c) From the point of view of an auditing firm which will be exposed to a large legal liability in the large-loss state when it happens, the impact of derivative usage should be incorporated in the design of the optimal procedures for auditing.

The analysis in this paper will take the perspective of an audit firm designing its optimal audit strategy. The client firm is levered and the cash flow from its operations is random. Its current level of debt and its random cash flows from operations endow the firm with a certain probability of default. This client firm may be simply hedging using derivatives or implementing the following “complex derivative strategy.” The strategy is to hold a self-financing portfolio of derivatives such that the sum of the returns from the

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derivative portfolio and the cash flows from operations reduces the probability of default; however, when default does occur, it occurs with very large losses. The losses in the default state are so large that the legal liability of the audit firm could also be very large. How should the audit firm design its audit strategies? Prior to the audit, the derivative usage of the auditee may be observationally equivalent to that of a firm doing normal hedging using derivatives. What modifications in the audit strategies are optimal for client firms using derivatives when the audit firm does not know whether the client firm is using derivatives for normal hedging or in “complex derivative strategies?”

An argument that has been made in the finance literature is that debt-holders have incentives to monitor and detect the presence of such strategies. Debt holders may not have the incentives if the implemented strategy makes them better off as well in most states of the world. Consider the following example: Suppose there is a firm with debt of promised payment of $10 million. Assume the cash flow from operations is sufficient to meet the promised payment with probability 90%; the firm will be in default with probability 10%. Using derivatives, it is possible for the firm to construct a distribution of incremental cash flows with the following property: the total cash flows from operations and derivative positions will be such that the firm will be solvent with probability 99.99%, and it will incur losses of $10 billion with a probability .01%. Depending on the activities of the firm, the required cash flow distribution may be constructed from a particular complex configuration of commodity, exchange rate, and interest rate derivatives. We will show that such a

5Henceforth in this paper, a firm using derivative strategies similar to the one mentioned above will be said to be using a “complex derivative strategy”. The nature of the strategy is made more concrete in the example formally described in Section 2.2.
incremental cash flow can be constructed in a self-financing fashion by using an appropriate set of derivative positions. Note that under the “complex derivative strategy,” the firm has reduced the probability of bankruptcy from 10% in exchange for 0.1% probability of incurring catastrophic losses. The state of the world in which the catastrophic losses occur may not happen for a number of years. It is also possible that generally accepted audit strategies may not be able to uncover the possibility of such a state.

How should the audit strategy be adapted to deal with a situation like this? If the auditor could precisely monitor all derivative positions taken by the firm throughout the reporting period and assess all the attendant risks, there would be no possibility of an audit failure due to failure to detect the firm’s exposure to the disaster loss state. However, in reality, fashioning the required distribution is accomplished with dynamically changing portfolio positions in the different derivative instruments such that it becomes impossible to audit the firm’s risk exposure without the possibility of error. In this scenario the derivative strategies implemented by the firm may have components of both hedging and speculation. More importantly, from the perspective of the auditor, the dynamically adjusted portfolio of

“Although a spate of spectacular losses stemming from “rogue” derivative positions (e.g., Barings PLC) has led to increased reporting and disclosures standards for derivative policies and positions maintained by firms, it is assumed that the increased regulation does not eliminate the possibility of an audit failure (i.e., the failure to qualify wrong statements) due to the existence of a “rogue” derivative strategy or otherwise. Such standards would include: (1) Statement of Financial Accounting Standards (SFAS) No. 105 “Disclosure of Information About Financial Instruments with Off-Balance-Sheet Risk and Financial Instruments with Concentrations of Credit Risk,” (2) SFAS No. 133, "Accounting for Derivative Instruments and Hedging Activities” which increases and clarifies reporting and disclosure requirements for firms holding derivatives for fiscal years beginning on or after June 15, 1999, and (3) the proposed Statement of Auditing Standards entitled “Auditing Financial Instruments,” which provides a framework for auditors to use in planning and performing auditing procedures for assertions about all financial instruments.
derivative positions adds considerable "noise" to the earnings distribution resulting from the underlying normal activity of the firm\textsuperscript{7}. The high costs of verifying the risk exposure of all the firm’s derivative positions make the underlying problem one of “incomplete contracting.” That is, it is assumed that audit tests are not “foolproof” and may thus fail to detect the auditee’s exposure to the disastrous loss state (or the firm’s insolvency even in the absence of derivatives). The optimal auditing strategies in such an environment is a focal thrust of this paper.

A stylized model of the auditing firm and the auditing process forms the basis for deriving optimal audit strategies. The starting point of the analysis is an assumption that honest errors are made in the audit procedure. With some (possibly small) probability, an auditor fails to detect a material error, irregularity, GAAP departure or a position indicating complex derivative strategy in the client's financial statements. To counteract the possibility of an error, the auditing firm flags a predetermined number of items to audit closely. These items could be those on the financial statements or the details of a derivative position. Since careful auditing of these items is costly, the number of items which are audited closely may be a subset of the items flagged initially, depending on the results of the items carefully audited thus far. The results of the multiple audit procedures are aggregated to a decision by the auditing firm to issue a qualified opinion or an unqualified opinion. Of course, any incongruity between the auditing firm's opinion with the (subsequently revealed) historically true financial state of the firm may lead to the imposition of legal penalties and reputation costs on the auditing firm. Thus, the total expected return to the auditing firm is the audit

\textsuperscript{7}See DeMarzo and Duffie (1991).
fees collected plus the expected value of these legal and reputation costs. In this setting, the auditing firm must decide upon the optimal number of auditing procedures to review the statements (i.e., items to flag) and the optimal aggregation of audit test results of different procedures into a overall "opinion" on the financial statements provided. The efficient aggregation of an "optimal" number of audit procedures to formulate the firm's opinion is central to the analysis. This approach provides direct implications relating audit intensity to audit quality, and to the presence or absence of complex derivative strategies.

A limitation of the analysis is the stylistic representation of the auditing process. Each audit test yields a binary signal that either the statements are correct or incorrect. In reality, one might expect audit tests to provide "multidimensional" audit information and assessments upon which are based estimates of the probability that various account balances and related disclosures are accurate or inaccurate. However, given the binary nature (the financial statements are either correct or incorrect) of the statements in the model, a binary signal is a sufficient statistic of a more detailed characterization. Moreover, similar stylistic representations of the auditing process are commonly used in the theoretical research in auditing for purposes of mathematical tractability.

The model allows the optimal number of audit procedures to be characterized; it is shown to be an increasing function of the liability loss exposure and the risk (or complexity) of the auditing engagement. The optimal number is also shown to be a decreasing function of the costs of an audit procedure. We also provide conditions under which derivative usage by the client firm increases the optimal number of audit procedures used.
The precision of an audit procedure is shown to have an ambiguous effect on the optimal number. At very high levels of precision, the optimal number decreases in precision. However, at lower values, precision leads to an increasing optimal number. The model also provides a characterization of the auditor firm technology and its quality endogenously. One of the direct implications of the model is a positive relationship between optimal number of procedures and audit quality.

The remainder of the paper is organized as follows. The related literature is discussed in section 1. In section 2, the basic model of the client firm and the audit firm is presented. In section 2.2, I formally derive a “complex derivative strategy.” In section 2.3 and 2.4, a stylized model of the auditing firm is presented. In section 2.6, its optimal audit strategy is derived. In section 3, the optimal auditing strategies for auditees using complex derivative strategies are derived and compared with those for firms not using such derivative strategies. These optimal strategies will be derived as a function of the expected liability losses of the auditing firm in the normal cases of insolvency vis-a-vis the liability losses in the disaster scenario. Additional results are developed relating the optimal number of audit procedures to audit quality. Section 4 contains concluding remarks.

1. RELATED LITERATURE

To our knowledge optimal auditing strategies for cash flow distributions modified by complex derivative strategies have not been studied formally. However, for conventional symmetric or two-point distributions the optimal auditing strategies have been explored in environments of moral hazard and adverse selection. There are two strands in the theoretical literature on auditing under moral hazard and incentive problems. In one group papers, the
The auditor's role is to generate information about the unobservable (private) actions of the manager, reducing the moral hazard costs. Antle (1982), Baiman, Evans and Noel (1987), Demski and Swieringa (1974) are selected examples. Another group of papers in this category study the agency problems arising from unobservable "auditor effort," where the compensation contract or liability exposure may improve incentives. Balachandran and Ramakrishnan (1987), Nelson, Ronen and White (1988), and Melumad and Thoman (1990) are important examples.

In the second category of articles, the role of auditing in a signaling game is the focus. For example, Titman and Trueman (1986), and Feltham and Hughes (1987) examine the signaling value of audit quality to a firm selling external claims in a market which is less informed than corporate insiders. Sarath (1988) and Melumad and Thoman (1990) study the "strategic certification role" of auditing in facilitating trade in an appropriate game of incomplete information. The role of the legal system on the auditor's incentives was also examined in Melumad and Thoman (1990). Nelson, Ronen and White (1988) is a detailed analysis of various issues relating the optimal degree of precision adopted by the auditor to the auditor's legal liability for damages.

The increase in auditors' legal exposure (due to factors such as the erosion of the privity doctrine, enactment of the RICO statute, wave of S & L failures) has been reflected in the auditing literature. Legal sanctions have been embedded in models of the auditing process in work by Balachandran and Ramakrishnan (1987), Melumad and Thoman (1990), Nelson, Ronen, White (1988), and Sarath (1988) (among others).
In the eighties and early nineties, there were changes in the legal environment which increased the liability exposure of auditors (see Gormley (1984), Minow (1984), Mednick (1987)). In particular, it has been argued that there was a gradual erosion of the privity doctrine under which the liability of accountants to third parties was restricted to cases in which the injured party could prove fraud. Some decisions at that time held that "foreseeable" third parties could bring tort actions against auditors, which was a serious expansion of the auditor's legal exposure. This limitation of the protection that privity used to give auditors and a shift into the "foreseeable" interpretation would tend to increase the expected penalty for an unjustified unqualified (UQ) opinion in the model, with and without large losses. The effect of these changes in auditor liability and hence on the audit processes will have important policy implications. Similarly, as noted in footnote 6, SFAS 133 has increased accounting reporting and disclosure requirements for firms with derivative positions. The audit firm will be responsible for reviewing the financial presentation of and disclosures for derivative transactions and year-end positions of the auditee. This may change the cost and characteristics of the audit process.

A strand of the recent literature has also examined how the strategies of owners, auditors and investors would be affected by the various legal regimes and the rules for apportioning legal damages. Previous research in the area of auditors’ liability assumes either the strict liability rule (Melumad and Thoman (1990), Nelson, Ronen and White (1988)) or the negligence rule in which a clear definition of due care level is specified
Strict liability dictates holding the auditor liable for the losses he caused, no matter what the degree of his negligence is. This regime also corresponds to joint and several liability rule where the audit firm turns out to be the only defendant to whom the plaintiffs can look for monetary redress. In the due care regime, the auditing firm’s liability is apportioned as a function of the negligence of the audit firm in deviating from the exercise of “due care.” Therefore this regime of liability rules is also characterized as negligence rules or proportional liability rules.

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There has been a significant change in the long standing joint-and-several liability rule recently. The Private Securities Litigation Reform Act of 1995 substantially revised the damage apportionment provisions of the Securities Act of 1933 and the Securities Exchange Act of 1934, instituting a hybrid proportional liability rule\textsuperscript{10}. Auditors had lobbied for a long time to eliminate the JS liability and replace it with some proportional liability rule. The auditing industry claimed that exposure to large potential damages under the JS rule had resulted in excessively large settlements and could reduce the supply of auditing in the long run.

Recently, Datar and Alles (1999) examine the role of audit reputation on the interaction between the auditor and the manager, in the light of recent changes limiting auditors’ legal liabilities. They show that the auditor has an endogenous motivation to build a reputation for being diligent, and that the reputation formation will ensure high audit standards even after the current litigation reform that eases penalty standards.

2. THE MODEL

In this section, the basic features of the model are presented. In the model, a representative firm may use a derivative strategy to modify its cash flow from operations. The portfolio of derivatives that the firm can hold to modify its cash flow distribution will be characterized. A simple model of the audit firm and process is developed which leads to a determination of the optimal audit strategy.

\textsuperscript{10}See King and Schwartz (1997) for more on the 1995 Reform Act.
2.1 The Client Firm

The cash flows from operations for a representative (client) firm is as follows. The firm obtains a cash flow $H$ with a probability $q$ and $L$ with a probability $(1 - q)$, where $H > L > 0$. The firm has outstanding debt of face value $F$, where $H > F > L$. Thus, the firm will default on its debt with a probability $(1 - q)$. Since the firm is a limited liability corporation, if it defaults on its debt, the debtholders are paid $L$ in the default state and $F$ in the solvent state, and equity holders get zero in the default state and $(H - F)$ in the solvent state. (See Figure 1 below.)

<table>
<thead>
<tr>
<th>Firm cash flows</th>
<th>Pay-offs to debt holders</th>
<th>Pay-offs to equity holders</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H$</td>
<td>$F$</td>
<td>$H - F$</td>
</tr>
<tr>
<td>$L$</td>
<td>$L$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

Figure 1. Cash flows and $t = 1$ pay-offs for the representative firm without derivatives

2.2 A Derivative Strategy

The firm can implement derivative strategies to alter the nature of its cash flow distribution from operations. Although there are infinitely many possible strategies, I will characterize in detail a particular self-financing strategy available to the firm.

Let $S = \{1, 2, 3, ..., n, ..., s\}$ be the set of states of the world where the client firm gets a cashflow $L$. Let $K$ be an arbitrarily large positive number. Consider the relevant $s$ Arrow-Debreu securities which are either available in the market or can be synthesized from the
existing securities such as options and other derivatives. See, for example, Ross (1976), John (1981), and Breeden and Litzenberger (1978). The payoff on the \( n \)th Arrow-Debreu security can be characterized as follows: a $1 payment in the \( n \)th state and a zero payoff in all the other states.

Now consider a portfolio of these securities with weight 1 unit on the first \((s - 1)\) securities and - \(K\) on the \(s\)th security. Such a portfolio produces the following vector of returns: the first \((s - 1)\) components are equal to 1 and the \(s\)th component is - \(K\). If \(p_n\) is the market price of the \(n\)th security for \(n = 1, 2, 3, ..., s\), the total cost of constructing the portfolio equals:

\[
C(K) = p_1 + p_2 + p_3 + \ldots + p_{(s-1)} - Kp_s
\]

The above portfolio is self-financing if \(C(K) = 0\), i.e., if:

\[
KP_s = \sum_{n=1}^{(s-1)} p_n
\]

For example, let \(K = 10\) billion, \(s = 10\) million. If

\[
p_s \geq \left( \sum_{n=0}^{10\text{million}} p_n \right)/(10\text{ billion})
\]

there is a self-financing derivative strategy which produces the following pay-off vector,
[1 1 1 1 1.....1, -10 billion]. Since the position is self-financing, the firm can implement \((F - L)\) times the portfolio at zero cost, and the portfolio return from the derivative strategy would be the vector 
\[[(F - L), (F - L),……,(F - L), -((F - L) 10 billion)]\]
Implementing the above derivative strategy, the firm is now solvent in all states except state s. State s is now a disaster state where the firm suffers an incremental loss of $\{(F - L)10 billion\}. If all states \(n, n = 1, 2, 3, \ldots, s\), are of similar probability, then the probability of state s is minuscule, say, roughly equal to \((1 - q)/(10 million) = \varepsilon\) (say).

Now the total cash flow distribution of the firm (including the return from the derivative position) is as follows: \(H\) with a probability \(q\), \(F\) with a probability \((1 - q - \varepsilon)\), and \(\{L - (F - L) 10 billion\}\) with a probability \(\varepsilon\). The firm is now solvent in all states except state s, i.e., it is solvent with probability \((1 - \varepsilon)\) and will default with probability \(\varepsilon\). The firm has reduced its probability of default from \((1 - q)\) to \(\varepsilon\) but the default state entails huge losses of \(\{L - (F - L)10 billion\}\). In the disaster state, the firm will evoke limited liability and walk away from its losses. The audit firm will be sued for damages of \(L_D\) in state s. If the default is normal, i.e., without the derivative strategy on the part of the firm, let the liability be \(L_N\).

(See Figure 2 below for cash flows with derivatives.)
Figure 2. Cash flows and $t = 1$ pay-offs for the representative firm with derivatives

It should also be noted that the expected pay-off to the debtholders have actually increased under the derivative strategy. Even if they were aware of the implementation of the derivative strategy, the firm defaults with a lower probability under that strategy.

2.3 The Audit Process

The audit environment can now be specified as follows: the prior probability that the firm is solvent is given by $\alpha$. When the firm is insolvent and its internal systems do not detect it, the financial statements of the firm may not present the state of insolvency. If the audit firm does not qualify its opinion on the financial statements of the firm, and the insolvency is revealed, the audit firm will be sued for damages and its expected liability is $L_2$. 
The audit environment is characterized by two parameters \( \{\alpha, L_2\} \): the prior probability that the financial statements are correct, and the legal liability exposure of the audit firm if the audit firm does not qualify the financial statements of the firm and the firm is revealed to be insolvent. The audit environment depends on whether or not the client firm has used the derivative strategy (described above in Section 2.1). The audit environment without the use of derivatives will be denoted as \( \{q, L_N\} \) and that with the use of the derivative strategy as \( \{(1 - \varepsilon), L_D\} \).

In this section, a model of the audit firm and the audit process is presented which provides the setting for the derivation of optimal audit strategies\(^{11} \). Before I specify the formal model, an informal overview is given below.

The model captures the essential role of an external auditor. Here, a client's (auditee's) financial reporting system produces a financial statement which describes firm characteristics important to the client and the investors in the capital markets. Because of the complexity or ambiguity of GAAP, and internal recording and processing errors, the output of the financial reporting system might differ from the correct numbers. The external auditor is hired to evaluate the firm's operations and accounting system and render an "opinion" about whether the reported numbers fairly represent the correct value of the financial characteristics. The auditing firm's issues its opinion based on its prior beliefs about the auditee's financial reporting system as well as its own optimally designed audit procedures. The audit procedures can be implemented at varying intensities. The intensity can be

\(^{11}\)The model developed here is applicable to the audit of the client firm in general, whether or not the firm uses derivative strategies. We will characterize the effect of client derivative usage on the optimal auditing strategies.
thought of as the degree of sampling undertaken, the size of the audit team, and or the
different types of information generated. The cost of audit is increasing in the intensity\textsuperscript{12}.

The audit process produces a noisy signal of the status of financial statement (and the
presence of the disaster state for a firm implementing the complex derivative strategy). The
precision of the signal is increasing in the level of intensity of the audit process. Eventually,
the true status of the statements (i.e., whether or not they are correct and whether or not the
disaster state occurs) becomes known. The returns to the auditing firm consists of the
auditing fee (which is not contingent on the audit opinion) minus the expected costs resulting
from the impositions of legal liabilities and sanctions, if any. The expected legal liability as
characterized by the ease of initiating lawsuits, likely size of awards/settlements, set of
permissible defenses and the level of losses for the firm with and without the disaster event
are shown to be important determinants of the intensity of audit procedures. The pay-off
distribution of the implemented derivative strategy and its characteristics will also have
implications for the optimal intensity and the type of information generated during the audit
process. The model yields testable implications which relate the intensity of audit to key
model parameters such as liability loss exposure in the existing legal environment, risk and
complexity of the audit engagement, parameters of the audit technology and costs of audit
procedures, and whether or not the firm uses the derivative strategy.

\textsuperscript{12}The precise definition of audit intensity and the costs of the two different stages of
auditing are discussed later in the formal description of the model.
2.4 The Detailed Model of the Auditing Firm

The formal model of the audit firm and the audit procedure will be presented as a four-date, (t = -1, 0, 1 and 2) model. At t = -1, the auditing firm is set up. This stage includes deciding on auditing policies and procedures to be followed, and recruiting auditors. At t = 0, the client firm is identified and the auditing contract between the client firm and the auditing firm is structured.

The client firm's financial statements describe the financial position, operating results and cash flows of the firm. The information is used by market participants to make (probabilistic) predictions about future events, such as the client firm's earnings prospects, cash flows, dividend levels, solvency, etc. The financial statement numbers and disclosures may differ from the correct ones due to unintentional and intentional omissions and errors ("irregularities"). We can assure that the firm’s financial statements and disclosures imply that the firm is solvent. However, such a claim contained in the financial statements is only correct with a probability $\alpha$, $\alpha \in (0,1)$. In other words, the auditing firm starts out with a prior probability $\alpha$, $\alpha \in (0,1)$ that the statements are correct and $(1-\alpha)$ that the statements are incorrect. The final opinion issued by the auditing firm at t = 1, however, is based on the posterior probability assessed by the firm after its elaborate audit procedure carried out between t = 0 and t = 1.

Model parameter $\alpha$ is the auditing firm's prior belief that the financial statements truthfully report the firm's profitability, financial position and cash flows (in conformity with
Factors which cause (or allow) the financial statement numbers to be misstated will cause $\alpha$ to be smaller. These factors include (a) an ambiguous and/or complex GAAP, (b) high degree of operating risk or default risk (c) high degree of complexity of operations of the client firm, (d) low quality of the internal accounting system of the client firm, and (e) whether or not it uses the derivative strategy. In particular $\alpha$ will be affected by the probability of solvency $q$ and the derivative positions held by the firm. From sections 2.1 and 2.2, it is clear that $\alpha = q$ for a firm which does not use complex derivative strategies and $\alpha = (1 - \sigma)$ for a firm which uses the derivative strategy. The differences in $\alpha$ and in the expected legal liabilities $L_D$, $L_N$ are important determinants of the optimal audit strategies for firms with and without complex derivative strategies.

The audit process itself takes place in the time period between $t = 0$ and $t = 1$ and involves several steps. First, a certain number $n$ of possible audit procedures (i.e., various tests of transactions, account balances and financial disclosures) are identified (flagged) for potential audit examination. Identifying or flagging items for potential audit involves a cost $w$ and actual detailed audit examination involves a cost $u$\textsuperscript{14}. Each audit test produces a signal as to the quality or the correctness of the financial statements. The precise nature of the

\textsuperscript{13} It is assumed that conformity to GAAP is essential to truthful reporting. If GAAP permits alternative treatments, any permitted treatment is assumed to be equally correct. If GAAP is ambiguous or silent, it is assumed that correct treatment(s) exist and can be ascertained by and agreed upon by preparers and auditors.

\textsuperscript{14} Real-world audit activities giving rise to cost $w$ would include the cost of performing preliminary audit steps such as undertaking the analytic review, evaluating the internal control system, and preparing an audit plan or budget. Audit cost $u$ is the cost for conducting further tests and investigations of balances and transactions (e.g., gathering receivable and payable confirmations, counting inventory, preparing a “proof of cash”) to gather substantive audit evidence.
signals and how these audit signals are aggregated to formulate an “opinion” on the financial statements are described later. At $t = 1$, the opinion is issued and the agreed-upon audit fee $f$ is paid, independent of the audit opinion issued.

At $t = 2$, nature reveals the historically true financial status of the firm (as of the financial statement date). Depending on the legal structure in place, parties injured by either an inappropriately favorable or unfavorable opinion can seek legal recourse. The legal settlements are assumed to be made at $t = 2$, the final date of the model. These steps are displayed in the time diagram below (Figure 3).

**Figure 3. Sequence of events in auditing environment**

An audit test of a given financial item flagged at $t = 0$ may be conducted between $t = 0$ and $t = 1$. This audit test has the following stylized representation: each audit test generates a noisy signal 's' that can take on two values: SC for "statements are correct" or SW for
"statements are wrong." The precision of the audit test can be represented by a pair of probabilities \( \{r, p\} \) where \( r \in (0.5, 1] \) and \( p \in (0.5, 1) \) represent the conditional probability that the signal \( s \) is correct when the true status of the statements is correct (C) and wrong (W), respectively. More specifically, \( 'r' \) represents the conditional probability that \( s = \text{SC} \) given C, i.e., \( \text{Prob}(s = \text{SC} \mid C) = r \) and \( \text{Prob}(s = \text{SW} \mid C) = (1-r) \). Similarly, \( p \) represents the conditional probability that \( s = \text{SW} \) given W, i.e., \( \text{Prob}(s = \text{SW} \mid W) = p \), and \( \text{Prob}(s = \text{SC} \mid W) = (1-p) \). The precision of all audit tests is assumed to be the same. The tests are assumed to be independent of each other. That is, the distribution of the signals generated by the tests depends only on the true status of the financial statements, and are independent of the outcome of the other tests.

The assumption that \( r \) and \( p \) exceed 0.5 simply denotes that the audit tests are "informative." Clearly, a higher value of \( r \) and \( p \) denote higher "precision." All audit tests are assumed to be of the same quality, i.e., same \( \{r,p\} \).

After suitably aggregating the results of an optimal number of its audit tests (to be derived endogenously), the auditing firm issues either a qualified opinion (Q) or an unqualified (or clean) opinion (UQ). In the context of the model, a qualified opinion is a statement that the financial statements are not correct. The real-world counterparts envisioned for the qualified opinion modelled here are (1) an "except for" qualification (or adverse opinion) due to a material nonconformity to GAAP, and (2) the existence of material uncertainties, including substantial doubts about going concern, which would have formerly

\[15\] This stylized representation of the audit test of the individual auditor is commonly used in theoretical research in auditing. See, for example, Balachandran and Ramakrishnan (1987) and Nelson, Ronen and White (1988).
(pre-SAS nos. 58 and 59) given rise to the issuance of a "subject to" opinion qualification.\textsuperscript{16}

In the context of the model, a qualified opinion (Q) also represents the case in which a threat of a qualification spurs the auditee to make changes which bring the financial statements in conformity with GAAP (such that an unqualified opinion UQ is ultimately rendered.)

At \( t = 2 \), the true status of the financial statements are known (i.e., whether they are correct (C) or wrong (W)). That is, there is a resolution of uncertainty over time which reveals the firm's historically true financial status such that a determination can be made as to whether the financial statements were correct (C) or incorrect (W). Further assume that managerial incentives are such that when managers err, they overstate the true profits (and assets) of the firm.\textsuperscript{17}

If the auditing firm issues an unqualified opinion UQ for wrong statements, it has failed to detect overstated financials (committed a type II error). In this case, the auditors are forced to make payments to injured parties in the amount of legal penalty \( L_2 \). On the other hand, if the auditing firm issues a qualified opinion Q for correct statements (an erroneous assertion that the statements are overstated) it commits a type I error (and incurs penalty \( L_1 \)). Penalties \( L_1 \) and \( L_2 \) are the (conditional expected) monetary values of all penalties auditors incur for

\textsuperscript{16} SAS no. 58 and 59 require an additional explanatory paragraph instead of a "subject to" qualification for reporting the existence of material uncertainties, including substantial doubt about going concern. Consider the inclusion of any such paragraph as a opinion qualification for present purposes.

\textsuperscript{17} Nelson, Ronen, White (1988) also make this assumption, in the absence of which legal penalties for type I and II errors are poorly specified.
making type I and II errors (respectively), including in- and out-of-court settlements, reputation costs and lost profits from auditor switching. These costs are incurred

\[\text{Pay-off to auditing firm, excluding } u\]

<table>
<thead>
<tr>
<th>Audit opinion</th>
<th>(t=1)</th>
<th>(t=2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q</td>
<td>(f - wn)</td>
<td>(-L_1)</td>
</tr>
<tr>
<td>C</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\alpha)</td>
<td>UQ</td>
<td>(f - wn)</td>
</tr>
<tr>
<td>(1-(\alpha))</td>
<td>Q</td>
<td>(f - wn)</td>
</tr>
<tr>
<td>W</td>
<td></td>
<td></td>
</tr>
<tr>
<td>UQ</td>
<td>f - wn</td>
<td>(-L_2)</td>
</tr>
</tbody>
</table>

\[\text{Figure 4.}\]
Pay-offs to audit firm at \(t = 1\) and \(t = 2\), given that \(n\) items are flagged (excluding \(u\))

\[\text{18} \text{ Chow and Rice (1982) find an increase in the frequency of auditor-switching by firms receiving a qualified opinion. The loss of client-specific quasi-rents (see DeAngelo (1981)) or reputation effects of such switches may be important.}\]
by the firm at t = 2 as an offset against the fees f received at t = 1. Figure 4 summarizes the auditing firm pay-offs (at t = 1 and t = 2):

The audit strategy consists of two elements: (1) n, the number of items to flag for potential careful audit tests, and (2) the optimal number of audit tests to conduct on the flagged items. Consider the problem of determining n for the optimal audit strategy. The solution involves two stages and has to be solved backwards, as in dynamic programming. For a given number n of items flagged at t = 0 (which sets the maximum number of audit tests which can be conducted), a certain number of audit tests, less than or equal to n, will be conducted between t = 0 and t = 1. The optimal number of tests to be conducted will be determined according to the optimal stopping rule, which will be described later. Given \( \alpha \) (the auditing firm's prior probability that the firm's financial statements are correct) and n (the number of items flagged at t = 0), the audit follows the Bayes procedure. One by one, the audit tests of the flagged items are conducted. Each additional evaluation alters the firm's posterior about the correctness of the auditee's financial statements according to Bayes rule. Given the posterior, the costs of an audit test u and legal costs \( L_1 \) and \( L_2 \), an optimal determination is made whether to stop the audit process or conduct another test. The process continues until it is optimally stopped or all items flagged have been tested. In either case, the firm's audit opinion issued at t = 1 will be based on the posterior incorporating the evaluation of the last audit test conducted.

The stage 1 problem of determining the optimal number of items to flag at t = 0 simply involves optimizing the profit function \( \pi(n) \), which incorporates the optimal stopping rules used in the stage 2 problem. The underlying intuition can be seen by examining the marginal
costs and benefits of an incremental flagged item. The incremental flagged item has a cost \( w \); in addition, if the flagged item is actually examined in an audit (given the endogenous stopping rule in stage 2), an additional audit cost \( u \) must also be paid. The optimal number of flagged items is attained when the last item flagged causes an increase in expected auditing costs which equals the marginal reduction in the expected legal costs resulting from a reduction in the probability of making (Type I and Type II) errors.

2.5 Pay-Offs to the Auditing Firm

For the general case in which an audit test can result in either Type I or Type II errors, the stopping rule is extremely complicated (e.g., see Berger (1980), Cressie and Morgan (1986)). For simplicity, assume that audit tests only commit Type II errors; i.e., if the financial statements are correct, the audit test will reveal it with certainty. This corresponds to restricting \( r = 1 \) for all audit tests. Although this assumption is made primarily for reasons of tractability, there is some empirical and authoritative support for its validity.\(^\text{19}\) (Melumad and Thoman (1990) make a similar assumption.) Parameter 'p' continues to be unrestricted, i.e., \( 0.5 < p < 1 \).

\(^{19}\)There is some empirical evidence in the spirit of the assumption that type II errors (in this framework, a failure to detect overstated financials) are more important than type I errors. Studies of litigation against auditors indicate a much higher frequency of actions in which it is alleged that auditors failed to detect or disclose overstated values rather than understated values in the financial reports. In a study of 129 cases filed against auditors from 1960-76, St. Pierre and Anderson (1984) find "...none of the suits concerned errors in undervaluing assets, recognizing inadequate amounts of revenue, or recognizing excessive expenses" (p. 242). Conversations with Z. Palmrose regarding her (1987 and 1988) studies covering 472 actions against auditors (from 1960-85) confirmed that the sample cases rarely involved allegations that auditors failed to detect or disclose understated values. The definition of audit risk promulgated by the AICPA also minimizes the risk of understated results: "This definition of audit risk does not include the risk that the auditor might erroneously conclude that the financial statements are materially misstated. In such a situation, he would ordinarily reconsider or extend his auditing procedures...These steps would ordinarily lead the auditor to the correct conclusion." [AICPA, AU §312.02]
With this additional assumption, for any given optimal \( n \) determined at \( t = -1 \), the optimal audit procedure is as follows. The first time an audit test obtains a negative test result (signal \( s = \text{SW} \)), the firm should issue a qualified opinion (Q) without further auditing. Otherwise, each successive audit test signal should be solicited until all \( n \) tests have been evaluated. If all \( n \) audit tests yield signal \( s = \text{SC} \), attesting to the correctness of the financial statements, then the firm should issue an unqualified opinion (UQ). In summary, for any given \( n \), the stopping rule is (1) to stop the first time a bad signal (\( s = \text{SW} \)) is received and issue a qualified opinion (Q) or (2) conduct all \( n \) tests if no bad signals are received (and issue an unqualified opinion (UQ)). The reader can verify that (1) and (2) exhaust all possibilities.

The optimality of this stopping rule (as specified in (1) and (2)) used at \( t = 0 \), for any \( n \) optimally chosen at \( t = 0 \), can be established as follows: Given the audit technology with \( r = 1 \), a single bad audit signal is a sufficient condition for the financial statements to be in error. Therefore, to conduct additional tests after the first bad audit signal is received only adds to audit costs \( u \) without any counterbalancing benefit (such as an expected loss reduction). Equivalently stated, continuation would simply result in added audit costs \( u \) without changing the firm's posterior about the auditee. Therefore, the optimality of (1) is self-evident.

The second part of the stopping rule ((2) above) requires that all \( n \) of the items flagged yield a positive audit signal before the firm issues an unqualified opinion. A question naturally arises as to whether a firm could optimally base an unqualified opinion on fewer than \( n \) positive audit signals. For example, would it ever be optimal for a firm which had flagged \( n \) items to stop its auditing procedures after having received, say, \((n - 1)\) consecutively positive audit signals from the first \((n - 1)\) audit tests used on a given audit?
Such a stopping rule would be optimal only if the incremental cost of doing the nth audit test exceeds the marginal reduction in expected legal costs, say, by some amount $\lambda > 0$.

However, since such assessments can be made at $t = 0$, the firm would have optimally flagged only $(n - 1)$ items, at an expected cost savings of $(w + \lambda)$ over the n-item case (i.e., the case of flagging n items and conducting n audit tests). Thus, if n items are optimally flagged at $t = 0$, the optimal stopping rule to be used at $t = 1$ for issuing an unqualified opinion is the rule specified in (2) above.

Given the simplifying assumption $r = 1$ and the optimal stopping rule and number of items n discussed above, there are three possible auditing outcomes: (1) Given correct financial statements, an unqualified opinion (UQ) is given, (2) Given incorrect financial statements, all n flagged items are audited without obtaining a single bad signal, and an unqualified opinion (UQ) is issued, (3) Given an incorrect financial statement, one of the audit tests reveals a negative signal such that a qualified opinion (Q) is issued. The pay-offs to the auditing firm for each of these outcomes and the respective probabilities are as follows:

The audit fees $f$ and the costs of flagging n items equal to $wn$ are paid independent of the outcome of the audit process. Given correct financial statements (which have a prior probability of $\alpha$), an unqualified opinion (UQ) will be given with certainty. This entails testing all n flagged items such that the pay-off to the audit firm is $(f - wn - u_n)$.

Given incorrect financial statements (which has a prior probability of $(1-\alpha)$), two outcomes are possible. With probability $(1- p)^n$, n items will be audited without obtaining a single bad signal (i.e., $s = SW$). An unqualified opinion (UQ) will be issued, leading to a pay-off
( f - wn - un - L.) for the auditing firm. With probability \(1 - (1 - p)^n\) one of the audit tests yields a negative signal (i.e., \(s = SW\)) such that a qualified opinion (Q) is issued. In more detail, for some \(i, i \leq n\), \(i\) audit tests are performed where the first \((i - 1)\) audit tests reveal a positive signal and the \(i^{th}\) test reveals a negative signal with a probability \(p(1-p)^{i-1}\). Since this can happen for any \(i\), \((i = 1, 2, ..., n)\), the aggregate probability of this event is

\[
p \sum_{i=1}^{n} (1 - p)^{i-1}
\]

Since \(i\) items are audited in the case where the \(i^{th}\) test reveals a bad signal, the expected number of items audited to qualify incorrect statements is

\[
p \sum_{i=1}^{n} i (1 - p)^{i-1}
\]

Let \(\bar{\gamma}\) be defined such that

\[
p \sum_{i=1}^{n} i (1 - p)^{i-1} = \bar{\gamma} (1 - (1 - p)^n)
\]

or

\[
\bar{\gamma} = \frac{p \sum_{i=1}^{n} i (1 - p)^{i-1}}{p \sum_{i=1}^{n} (1 - p)^{i-1}} - \frac{p \sum_{i=1}^{n} i (1 - p)^{i-1}}{1 - (1 - p)^n}
\]
Thus, \( \bar{N} \) is the expected number of audit tests conducted to qualify incorrect statements divided by the probability of doing so.

Of the total \( n \) items flagged for possible examination, all will be involved in the review of correct statements (since \( n \) positive signals are required for an unqualified opinion). However, the number of items tested (\( n \) available) involved in the review of an incorrect statement depends on the ordering of the first negative signal realization (if any), and therefore is a random variable. The optimal number \( n^* \) of items flagged for potential testing is characterized in the next section.

**2.6 Optimal Audit Strategy**

In this section, we will derive the optimal audit strategy to be followed by the audit firm \{p, w, u\}. For an auditing firm parametrized by the efficiency of its audit test \( p \), and the cost of flagging an item \( w \), and the cost of a detailed audit test \( u \), how many items \( n \) should it optimally flag at \( t = 0 \)? Once \( n \) is optimally determined, the number of items actually subjected to an audit examination depends on the ordering of the first negative signal realization (if any), and is therefore a random variable. The optimal audit strategy \( n \) pursued by an audit firm \{p, w, u\} will also depend on the characteristics of the audit environment \{\( \alpha \), \( L_2 \}\}. Recall that \( \alpha \) is the probability that the statements are correct and \( L_2 \) is the expected legal liability for failing to qualify a wrong financial statement or failing to make a “going concern” qualification for a firm which will turn out to be insolvent. For a client firm which is not using complex derivative strategies, the audit environment is \{\( q \), \( L_N \}\}. For a firm which uses complex derivative strategies, the audit environment is \{(1 - \( \varepsilon \)), \( L_D \)\} as described
in Section 2.2. The optimal audit strategies used by an auditing firm \{p, w, u\} in audit environments \{q, L_N\} and \{(1 - \varepsilon), L_D\} will be derived below. The comparison of the resulting optimal strategies will provide insights on how the optimal audit strategy is affected by the client firm using the derivative strategy. The derivation below will be done for the generic audit environment \{\alpha, L_2\}.

Let \(n^*\) be the required optimal auditing strategy. Then \(n^*\) should maximize the expected net profit \(\pi(n)\), which can be written using the pay-offs characterized in Section 2.5.

\[
\pi(n) = f - wn - \alpha un - (1-\alpha)u \{ \hat{n} [1-(1-p)^n] + n(1-p)^n \} - (1-\alpha)(1-p)^n L_2
\]  

(5)

where \(w\), \(u\) and \(\hat{n}\) are as defined above. It is convenient to define \(\hat{n}\) as the expected number of items audited in the evaluation process of a client firm with incorrect financial statements, as follows:

\[
\hat{n} = \hat{n} [1 - (1 - p)^n] + n(1 - p)^n
\]  

(6)

where the first term in (6) is the expected number of items audited in the evaluation of incorrect financials in which a qualified opinion is given, while \(n(1-p)^n\) is the expected number of items audited in the evaluation of incorrect financials in which an unqualified opinion is given. The conditional costs of audit tests in auditing incorrect financial statements is thus \(\hat{n}u\). The unconditional expected detailed audit test costs is \(\alpha wn + (1 - \alpha)u\hat{n}\).

Rearrange (1) to obtain

\[
\pi (n) = f - [wn + \alpha un + (1-\alpha)u\hat{n}] - (1-\alpha)(1-p)^n L_2
\]  

(7)
This expresses the profit to the auditing firm as the fees minus the costs of flagging items and expected costs of detailed audit tests and the expected legal costs.

To initialize the model, two assumptions are made which are plausible in any scenario in which auditing is viable. The first assumption is that a qualified opinion can only be issued based on some negative evidence. In other words, a firm which conducts zero audit tests has no alternative but issue an unqualified opinion (UQ). The second assumption is that it is optimal to flag and audit at least one item carefully, i.e., \( \pi(1) - \pi(0) > 0 \) so that

\[
(1-\alpha)pL_2 > w + u
\]

The above condition (8) says that for the given structure of costs \((w, u)\), the precision \(p\) of the audits is high enough, the prior probability of incorrect financial statements is high enough or the litigation costs \(L_2\) of an erroneous opinion are serious enough to warrant the careful audit of at least one item. Clearly, these are very plausible assumptions.

The optimal audit strategy \(n^*\) is characterized below.

**Proposition 1:**

*The optimal auditing strategy \(n^*\) in the audit environment \({\alpha, L_2}\) for the audit firm \({p, w, u}\) is given by:

\[
\eta^* = \frac{\ln \left[ \frac{\alpha w + w}{(1 - \alpha)(pL_2 - u)} \right]}{\ln(1 - p)}
\]

**Proof:** See Appendix A.
3. DERIVATIVES AND AUDIT STRATEGY

The optimal audit strategy $n^*$ is a function of audit firm parameters $p$, $w$ and $u$ as well as those of the audit environment $\{\alpha, L_2\}$. Since use of complex derivative strategies affects $\alpha$ and $L_2$, an examination of the effect of $\alpha$ and $L_2$ on the optimal audit strategy $n^*$ is of particular interest.

3.1 Effect of Legal Liability

Several testable implications can be derived from the optimal audit strategy in (9). These implications will be presented in the form of propositions.

Consider first the relationship between the legal environment and optimal audit strategy.

**Proposition 2:**

$n^*$ is increasing in the liability loss exposure $L_2$.

**Proof:** Using (9),

$$\frac{\partial n^*}{\partial L_2} = \frac{-p\ln(1-p)}{pL_2 - u}$$

which is positive, since $\ln(1-p) < 0$ and inequality (8) implies $pL_2 - u > 0$.

If the accountants' liability grows, Proposition 2 implies that one should observe a more elaborate auditing strategy $n^*$. In the eighties and the early nineties, fears were raised within the accounting and legal communities that there have been changes in the legal environment which increase the liability exposure of auditors (see Gormley (1984), Minow (1984), Mednick (1987)). In particular, it has been argued that there has been a gradual erosion of the privity doctrine under which the liability of accountants to third parties was restricted to cases
in which the injured party could prove fraud. Some recent decisions holding that "foreseeable" third parties could bring tort actions against auditors is a serious expansion of the auditor's legal exposure. This limitation of the protection that privity had given to auditors and a shift into the "foreseeable" interpretation directly increases $L_2$, the expected penalty for an unjustified unqualified (UQ) opinion in the model.

Other legal and business trends have also led to a steady rise in the number of lawsuits against auditor, in which very large damage awards are being (or have been) sought. These include:

1. complex derivative strategies which result in large losses, i.e., $L_2 = L_D$.

2. aggressive use of the R.I.C.O. (anti-racketeering) statute to prosecute accountants and other financial executives (see Crovitz (1990) and Minow (1984));

3. increased auditor responsibility for detecting and reporting management fraud, errors and illegal acts under new auditing standards (see Carmichael (1988), Roussey, et. al (1988));

4. increases in the number of and awards in governmental and private lawsuits against auditors for the recovery of savings and loan industry losses (see Berton (1990) and Cowan (1990)). The fallout from the B.C.C.I. scandal portends a new round of lawsuits and standard-tightening for auditors (see Cowan (1991)).

In general, any material change in the legal environment which increases $L_2$ should lead to an increase in the intensity of audits (e.g., the list of flagged items). Such changes which increase $L_2$ might be empirically captured by (1) the frequency of tort actions.

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against auditors by investors (including class action litigation by shareholders and
bondholders), (2) the frequency and size of awards against auditors, (3) the size of out-of-
court settlements by auditors. The model predicts an increasing relationship between these
proxies for \( L_2 \) and a more elaborate audit strategy.

Similarly, the availability of professional insurance coverage for accountants will have
the effect of reducing \( L_2 \), although the premium paid by the auditing firm reduces the
unconditional payment \( f \) received. In other words, if the coverage available reduces the
expected liability losses to \( (L_2 - C) = L \) and the insurance premium is \( \phi \) (paid from \( f \)), the
optimal size \( n^* \) of the auditing firm can be shown to be

\[
n^* = \frac{\ln \left[ \frac{\alpha \mu + \nu}{(1 - \alpha)(\mu L - \omega)} \right]}{\ln(1 - \omega)}
\]

Since \( L < L_2 \), \( n^* < n^* \). Thus, for a given legal environment, increased availability of
malpractice insurance for auditors will reduce the auditing intensity. This is also an
empirically testable implication of Proposition 2. Similarly, the lowering of \( L_2 \) by the
recent Private Securities Reform Act of 1995 by reversing the damage apportionment rules
in favor of the auditing firms (see Section 1), may lead to a reduction in \( n^* \).

### 3.2 Effect of the Reliability of Financial Statements

As argued above, optimal audit strategy \( n^* \) is increasing in \( L_2 \), the expected liability loss
conditional on the issuance of a clean opinion for wrong statements. As shown in (7),
\( (1 - \alpha)(1 - p) L_2 \) is the unconditional expected legal liability of the auditing firm before the audit procedure commences. This expected loss increases in \((1-\alpha)\), the firm's prior belief that the financial statements are wrong. It is therefore intuitive that optimal audit strategy varies in \(\alpha\), which can be alternatively be thought of as the reliability of the financials or "inherent risk" in the audit.

**Proposition 3:**

*Factors which decrease \(\alpha\), the reliability of financial statements, will increase the optimal audit strategy \(n^*\).*

**Proof:** Using (9),

\[
\frac{\partial n^*}{\partial \alpha} = \frac{(u + w)}{\ln(1-p)(1-\alpha)(w + \alpha u)} < 0
\]

All factors which will reduce \(\alpha\) will lead to a more elaborate optimal audit strategy. Ceteris paribus, the following factors tend to reduce \(\alpha\) and lead to increases in \(n^*\): (1) decrease in the quality of the internal control systems, (2) increase in the complexity of the auditee's operations, (3) increase in the business, technological and audit risks of the auditee's operations, (4) increase in the ambiguity or complexity of GAAP.

The quality and the efficiency of the internal control system of the auditee will increase the reliability of the financial statements generated and therefore a higher \(\alpha\) is produced. Empirical proxies of the quality of the internal control system may be the size of the personnel involved, salaries paid to internal auditors per year, the auditee's expenditure on the internal control system, etc.
As the operations of the auditee become more complex, the number of decision centers which need to be monitored increase. For a given level of monitoring and internal control, the financial statements produced is less reliable, i.e., a lower $\alpha$ is obtained. Simunic (1980) has suggested decentralization and diversification measures as proxies for the complexity of the auditee's operations. Proposition 3 predicts a positive relationship between $n^*$ and such proxies.

A third set of factors which affect the reliability of the financial statements are those which affect the underlying exogenous uncertainty in the auditee's operations. Increasing operating risk of the auditee, captured by proxies such as the earnings variability or beta of the assets, should lead to lower $\alpha$'s and higher $n^*$. Similarly, Proposition 3 also implies that higher technological risk (proxied by measures such as research and development expenditure, number of new products introduced, number of patents held by the firm) increases $n^*$. Auditors have also long recognized that certain accounting populations involve a high degree of "relative audit risk" (see Simunic (1980) and Alderman and Tabor (1989)). Receivables and inventories are examples of risky balance sheet components which require complex valuation techniques, often involving the forecasting of future events.

In sum, factors which increase $L_2$ and $(1-\alpha)$ cause an increase in the unconditional expected liability exposure for the auditing firm and an increase in $n^*$. Recent business and legal trends have rendered auditors more legally vulnerable for the audit of complex business operations and deals. Financial statements become more difficult to audit with businesses' introduction of new technologies, contracts and financial instruments in a global
market. The adoption of complex or ambiguous accounting standards may also lower financial statement reliability. Propositions 2 and 3 imply an increase in the intensity of the optimal audit strategy as a response to increased legal vulnerability and audit complexity.

3.3 Auditing of Derivative Positions

Using the results of section 3.1 and 3.2, we can characterize the difference in audit strategies when the client firm does and does not use complex derivative strategies to alter its cash flow distribution from operations. If the client firm switches from not using derivative strategies to that of using derivative strategies, two changes occur. The expected legal liability of the auditing firm $L_2$ increases from $L_N$ to $L_D$ (where it is assumed that the liability $L_D$ in the disaster scenario is much larger than the liability from normal insolvency $L_N$) where the audit firm failed to make a “going concern” qualification. From Proposition 2, it is clear that the increase in $L_2$ from derivative usage will lead to an increase in $n^*$. 

On the other hand, derivative usage can be implemented to reduce insolvency states such that the probability of insolvency is reduced to $\varepsilon$ ($\varepsilon$ is close to zero). $\alpha = (1 - \varepsilon)$ with derivative usage is larger than $\alpha = q$ the probability of solvency without derivative usage. From Proposition 3, the increase in $\alpha$ from derivative usage (i.e., from $q$ to $(1 - \varepsilon)$) would lead to a decrease in $n^*$. In other words, increase in $L_2$ increases $n^*$; but increase in $\alpha$ decreases $n^*$. Since the two effects on the audit environment from derivative usage has opposing effects on $n^*$, it would be useful to characterize the conditions under which the optimal audit strategy should have a higher intensity when the client firm is using derivative strategies.

Proposition 4:
The optimal audit strategy \( n^* \) should be higher for a client firm using derivative strategies, if

\[
\{(1 - \epsilon)w + \omega\}(1 - q)(pL_N - \omega) > \epsilon(q\omega + \omega)(pL_D - \omega)
\]  

(10)

**Proof:** Derivative usage changes the audit environment from \( \{q, L_N\} \) to \( \{(1 - \epsilon), L_D\} \).

Substituting in (9), inequality (10) implies that \( n^*_D \) is higher than \( n^*_N \).

### 3.4 Effect of Audit Firm Parameters:

I will now analyze how the optimal audit strategy \( n^* \) varies as a function of audit firm characteristics, \( p, w \) and \( u \).

**Proposition 5:**

*The optimal audit strategy \( n^*_D \) is decreasing in \( w \), the cost of flagging and \( u \), the audit test costs.*

**Proof:** Using (9),

\[
\frac{\partial n^*}{\partial w} = \frac{1}{(w + \omega)\ln(1 - p)} < 0
\]

Similarly,

\[
\frac{\partial n^*}{\partial \omega} = \frac{\epsilon pL_2 + w}{(pL_2 - \omega)(w + \omega)\ln(1 - p)}
\]

which is negative, since inequality (8) implies \( pL_2 - \omega > 0 \).
The implications of the above proposition are testable. Factors in the economy which tend to decrease \( w \) and \( u \) will lead to more elaborate audit strategies. Technological developments which reduce the audit test costs, e.g., computerized data handling, cheaper and quicker access to geographically dispersed clients, will increase \( n^* \).

In the next section, the relationship of optimal audit strategy \( n^* \) to audit test precision \( p \) (the probability an audit test signal is negative given the financial statements are wrong) is examined.

The impact of an increase in the precision of the audit test on the optimal audit strategy \( n^* \) is not a monotonic relationship. An increase in \( p \) has two effects on \( n^* \). On the one hand, for any given \( n \), an increase in \( p \) lowers expected audit test costs \( \hat{\mu} \), since for any \( n \), \( \hat{\epsilon} \) is decreasing in \( p \) (see equation (6)). This in itself tends to raise \( n^* \). On the other hand, with the increased accuracy of each audit test, the marginal contribution of the last audit test actually done is reduced, thereby lowering \( n^* \).

**Proposition 6:**

*With increasing accuracy of audit tests, the optimal audit strategy \( n^* \) will be increasing for low values of accuracy but decreasing for high values of accuracy. In fact, \( n^* \) will rise (fall) in \( p \) as \( 1 - p - n^*(p - u/L) > (<) 0 \)*

**Proof:** See Appendix A.

An increase in \( p \) can empirically be proxied by increases in the training or experience of auditors. There may be technological advances which increase audit precision by improving the auditor's ability to access and evaluate the relevant data. Proposition 6
shows that an increase in \( p \) does not necessarily imply smaller \( n^* \). The ambiguous relationship of \( n^* \) to \( p \), (where it is increasing in the low \( p \) region and decreasing in the high \( p \) region) makes it necessary to parametrize the model in some detail before a direct test of the relationship can be implemented.

4. CONCLUSION

Using the rich menu of derivative instruments, firms can hedge, speculate, and in other ways modify the distribution of cash flows from operations significantly at relatively low cost. However, the possibility of modifying the probability distribution of cash flows of the firm using complex derivative strategies in specific custom-made ways opens up important monitoring problems. In this paper, we examine ways in which the optimal strategies of an auditing firm would be affected by the possibility that the audited firm may be using these complex derivative strategies.

Using a model of the audit firm and process, we characterize the optimal audit strategy in the presence and absence of derivative usage by the auditee. The optimal audit intensity is related to the legal liability regime and the specific rules for allocating the auditor’s liability when the firm is insolvent and the auditor failed to qualify the audit opinion. Our other results relate the optimal audit intensity to the cost and efficiency parameters of the audit firm.
APPENDIX A: Proofs of Propositions

Proof of Proposition 1

Define

\[ \Delta(n) = \pi(n+1) - \pi(n) \]

From equation (5) in the text,

\[ \Delta(n) = \pi(n+1) - \pi(n) = (1-\alpha)p(1-p)^nL_2 - \{w + \alpha u + (1-\alpha)(1-p)^n u\} \]

(A1)

The first term is the marginal saving in expected litigation costs of a missed qualification by having flagged an additional item for audit. The second term is the incremental cost of flagging an additional item and the expected cost of doing a detailed audit test on that item. It can be seen that (a) \( \Delta(n) \) is decreasing in \( n \) (and equivalently, \( \pi(n) \) is concave in \( n \)). To see this, regroup terms in (A1) to obtain,

\[ \Delta(n) = (1-\alpha)(pL_2 - u)(1-p)^n - (w + \alpha u) \]

(A2)

Increasing \( n \) reduces \( (1-p)^n \), thereby reducing \( \Delta(n) \).

It should also be noted that

(b) \( \Delta(n) < 0 \) for a large enough \( n \), and

(c) \( \Delta(0) = \pi(1) - \pi(0) > 0 \) from equation (8) in the text.

Given properties (a), (b) and (c) for \( \Delta(n) \), there is a value of \( n \), call it \( n^* \), such that \( \Delta(n^*) > 0 \) and \( \Delta(n+1) \leq 0 \). With \( n^* \) flagged items, the firm profits can be increased by flagging an additional item, whereas with
(n + 1) flagged items, the effect of adding a flagged item is to reduce firm profits. Therefore, the optimal audit intensity (i.e., the optimal number of items to flag) is n°, where

\[ n° = n + 1 \]

(A3)

Thus far π(n) and Δ(n) have been carefully defined as functions with the domain of nonnegative integers, and n° as the optimal number of flagged items is also an integer. However, for purposes of comparative statics on the optimal audit intensity, it will be convenient to define a real-valued counterpart to Δ(n) and n°. Formally, Δ : Z → R. The mapping defined in (A1) was defined on the domain Z (the set of all nonnegative integers) into the range, the set of real numbers. Now define: Δ : R → R, a new function (which is a mathematical extension of Δ) where the domain is now the set of all nonnegative real numbers. Since (1-p) in equation (A2) is well-defined for all positive real numbers n, the definition of Δ(n) = Δ(n) as already specified in (A2) is well specified for all \( n \in R \). The only change is that Δ(n) is defined for all nonnegative numbers, whereas Δ(n) was originally defined only for nonnegative integers.

Now denote by n* a real number which satisfies the difference equation \( Δ(n) = 0 \). From the definitions of Δ(n), Δ(n), \( n \) and n°, it follows that

\[ n < n° \leq n + 1 \]

(A4)

That is, n° (the optimal audit intensity, defined in (A3)) is the smallest integer greater than or equal to n*.

The discussion in the text deals with n* rather than n°. It is technically convenient to employ the real-valued number n* satisfying the difference \( Δ(n°) = 0 \) rather than the integer valued n°. However, all results that are shown for n* will also hold for a corresponding n°. In the text, n* is referred to as the optimal audit intensity. Using (A1), write the difference equation \( Δ(n°) = 0 \) as follows:
(1-\(\alpha\))p(1-p)^*L_2 = w + \alpha u + (1-\(\alpha\))u(1-p)^*

(A5)

\(n^*\), the optimal audit intensity, is such that the reduction in expected costs of litigation (by a marginal decrease in the probability of type II error (on the LHS of (A5)) equals the expected additional cost \(w\) of flagging an item plus the expected additional cost of performing a detailed audit test, \(\alpha u + (1-\(\alpha\))u(1-p)^*\). Since the fee \(f\) is paid independently of the audit opinion, it does not enter into the above argument characterizing \(n^*\).

Simplifying and rearranging terms yields:

\[(1-\(\alpha\))(1-p)^*(pL_2 - u) = \alpha u + w\]  

(A6)

\[(1 - p)^* = \frac{\alpha u + w}{(1 - \alpha)(pL_2 - u)}\]  

(A7)

\[n^* = \frac{\ln\left[\frac{\alpha u + w}{(1 - \alpha)(pL_2 - u)}\right]}{\ln(1 - p)}\]  

(A8)

Proposition 1 has been proved by construction.

**Proof of Proposition 6:**

Using equations (8) and (9) in the text, it can be shown that

\[\text{sign} \frac{\partial n^*}{\partial p} = -\text{sign}\left[\frac{n^*}{(1 - p)} - \frac{L_2}{(pL_2 - u)}\right]\]  

(A9)
Given (8), \( p > \frac{u}{L_2} \). For high values of \( p \), as \( p \to 1 \), \( (1 - p) \to 0 \) and the sign of \( \frac{\partial n^*}{\partial p} \) is negative because \( p - \frac{u}{L_2} > 0 \) from (8).

Now consider low values of \( p \). Given (8), define

\[
\bar{p} = \inf \{ p : (1 - \alpha)p L_2 > w + u \}.
\]

It is easy to verify that

\[
\left[ \frac{\alpha u + w}{(1 - \alpha)(\bar{p} L_2 - u)} \right] = 1 \text{ such that } n^*(\bar{p}) = 0. \text{ Evaluate sign of } \frac{\partial n^*}{\partial \bar{p}}
\]
\[ = \text{sign}[(1 - \bar{p})] > 0. \]

The continuity of \( \frac{\partial n^*}{\partial p} \) obtains \( \frac{\partial n^*}{\partial p} > 0 \) for a range of values of \( p \) around \( \bar{p} \).

Q.E.D
References


