Monetary Policy and Stock Prices in a DSGE Framework*

Salvatore Nisticò†

This draft: May 2004

COMMENTS WELCOME.

Abstract

This paper uses a small structural general equilibrium model to highlight the role of stock prices in affecting real activity and to provide a theoretical rationale for looking at equity prices in conducting Monetary Policy. The focus is on the effects that stock market’s booms and busts might have on consumption: a cashless Dynamic Stochastic General Equilibrium (DSGE) model of the business cycle is set up, in which households are finitely-lived, consumer prices are sticky and an explicit role for equity prices is allowed for in the household optimization problem. Short-term dynamics is injected by a productivity shock and a stochastic stationary stock-price misalignment. Monetary Policy is implemented by means of an augmented forward-looking Taylor Rule.

The main results are twofold. First, the Wicksellian natural rate of interest which grants zero-inflation and zero-output gap is a function of all structural shocks. If stock prices exhibit also stochastic misalignments, therefore, a Central Bank pursuing price stability should take them into active account. Second, while the Taylor principle retains its validity, an accommodative stance towards stock prices does not per se yield endogenous instability of the equilibrium, as long as a proportionately stronger commitment is granted towards real output stabilization.

JEL classification: C32, E32, E52
Key words: Monetary Policy Rules, DSGE Models, Sticky Prices, OLG.

*This paper revises a Section of my PhD Thesis, defended at University of Rome Tor Vergata. I wish to thank Giorgio Di Giorgio for his trust and constant backup since my graduation at Luiss University and his precious comments on earlier drafts, Peter Ireland for crucial suggestions in terms of notes and MATLAB programs, Lucio Sarno and Jan Woodley for the hospitality at the Warwick Business School. Fabrizio Coricelli, Francesco Lippi, Fabrizio Mattesini and Gustavo Piga for very helpful comments and discussions. Any remaining errors are, of course, my sole responsibility.

†Luiss-Guido Carli University, Rome. Address for correspondence: Luiss-Guido Carli University, Via Tommasini 1, room 512, 00162 Rome, Italy. E-mail: snistico@luiss.it.
Contents

1 Introduction 2

2 The Theoretical Model: the Demand-Side. 5
   2.1 The population structure .................................. 5
   2.2 Consumers’ behavior ..................................... 6
   2.2.1 Aggregation across cohorts ............................ 8
   2.3 Equilibrium ............................................. 9

3 The Theoretical Model: the Supply-Side. 11
   3.1 The retail sector ......................................... 11
   3.2 The wholesale sector .................................... 12
   3.2.1 Aggregation across firms ............................. 14
   3.3 Flexible prices .......................................... 15

4 The Complete Linear Model 16

5 Price Stability and Equilibrium Determinacy. 19
   5.1 The goal of price stability ................................. 19
   5.2 Monetary policy and equilibrium determinacy ........... 21

6 Concluding Remarks 26
1 Introduction

Over the past two decades, with inflation successfully kept under control after the tumultuous 1970’s, one of the major issues that Central Bankers had to learn to cope with was financial stability. The events of the past fifteen years in the United States gave new scope for a debate in the literature about the desirability that Central Banks be directly concerned about the stock market dynamics.¹

The beginning of the third millennium was characterized by a global economic and financial U-turn with respect to the previous twenty years, with the crash of the stock markets and the first serious recession that the U.S. experienced since 1982, when a process of extraordinary real growth began, driven by the technological revolution in the way of producing, spending and consuming. In the last years of the Long Boom, when most observers were recognizing a speculative bubble in the acceleration of the stock market indexes, the issue of what Monetary Policy was the best response to manage the disrupting effects that might derive from an exogenous misalignment started to gain the interest of the economic literature.

The issue was analyzed in a variety of set-ups, both theoretical and empirical, and the consequent debate is still very controversial, under many respects. The main positions can be roughly represented by two main contributors, which engaged in a very lively debate over the past few years: Cecchetti et Al. [14], [15], [13] though emphasizing the difference between targeting stock-price stability and reacting to stock-price misalignments, strongly recommend that a Central Bank that recognizes a bubble in the dynamics of the stock market react to it; Bernanke and Gertler [3], [4] on the other side, maintain that the only desirable reaction to stock prices is the one implicit in the response to expected inflation and output, and that a flexible inflation targeting is, in itself, enough to pursue also the aim of financial stability.²

¹Mishkin and White [38] highlight the difference between financial instability and stock market crashes, maintaining that the real concern of monetary policy makers should be the former, rather than the latter. The strong point they make is related to firms’ balance sheets conditions and seems weaker when it comes to the possible real effects through households’ wealth. Truth is, anyhow, that the stock market fragility is more often than not a highly sensitive indicator of financial instability, especially in periods of financial sophistication like the ones we live in.

²Other contributions using different approaches (and drawing different conclusions) from each other can be found in Becchetti and Mattesini [2], Carlstrom and Fuerst [12], Chadha, Sarno and Valente [18], Cogley [21], Filardo [23], Gilchrist and Leahy [26], Goodhart [27], Ludvigson and Steindel [34], Miller, Weller and Zhang [35], Mishkin [37], Rigobon and Sack [42] and Schwartz [49].
Bernanke and Gertler [3] also empirically test whether the Federal Reserve and the Bank of Japan have reacted to stock prices within the last two decades, and find opposite results for the two cases. The Federal Reserve is claimed to have totally disregarded stock-price dynamics in the design of its recent policy actions; for the Bank of Japan, instead, the authors provide evidence of a break in the way of dealing with the stock market: the BoJ supposedly became actively “leaning against the bear” in the 1990’s while during the so-called “bubble economy” of the 1980’s it was found to be accommodating the “bull”. This accommodating stance was by many labelled as destabilizing for both the stock market itself and the economy as a whole.

The variety of models used and the differences in the conclusions drawn suggest the need of a simple structural framework that generalize the one usually employed for monetary policy analysis, with which to be compared. The econometric methodology most often used so far, moreover, (namely GMM) seems severely challenged by an endogeneity problem. A structural model, instead, would make possible to shift the focus from a single-equation instrumental-variable procedure to a structural likelihood-based one, which would overcome the identification issue.

In the present paper we present a cashless Dynamics Stochastic General Equilibrium (DSGE) framework for the analysis of Monetary Policy when the real economy is affected by the stock-price dynamics through wealth effects.

We move from the basic Dynamic New-Keynesian model widely used in the literature for monetary policy analysis, in which nominal rigidities assign a central role to monetary policy in driving output and inflation. The only but substantial extension to this baseline model proposed here is an explicit consideration and micro-foundation of stock-price dynamics and its effects on real activity, analyzing what are the implications for price stability and the role of monetary policy.

The mentioned extension is carried out by modifying the baseline model in two ways. The first one is by assuming that the firms producing intermediate goods issue equity shares to the public, which then can choose to allocate their savings by either buying a riskless bond or a portfolio of private stocks. This assumption follows the structure of Lucas’ [33] tree model, and allows for a micro-foundation of stock-price dynamics.

---

3See Rigobon and Sack [42] for a critique in this sense.
4For models of this kind see, among the others, Rotemberg and Woodford [46], [47] and Yun [59]. For a discussion of the role of monetary aggregates for the analysis of monetary models see Woodford [57]. For a thorough analysis using this baseline model and a detailed discussion see Woodford [56], Ch. 4.
The second assumption modifying the baseline model concerns the demand-side of the model, which consists of an indefinite number of overlapping generations of finitely-lived consumers with no relevant bequest motives, along the lines traced by Yaari [58] and Blanchard [7]. The demand-side hence takes the form of a stochastic OLG model, for which a closed-form solution for aggregate consumption within a closed economy is derived by Chadha and Nolan [17] and Piergallini [41]. This assumption allows for an explicit role of financial wealth in driving real activity.

The theoretical framework thus combines the basic features of standard DSGE models with no physical capital, as far as the supply-side is concerned, with an Overlapping-Generations structure of the demand-side. The focus is an explicit consideration of stock-price dynamics in the consumers’ decision making and the real output determination.

The main results are twofold.

We derive what Woodford [56] calls the \textit{Wicksellian natural rate of interest}, which is defined as the real rate of return that ensures price stability and a zero-output gap, and which results in a function of all assumed structural shocks. The model, therefore, shows that if stock prices are also driven by exogenous stochastic misalignments, then a monetary policy maker pursuing price stability through the setting of a short-term interest rate should keep her instrument closest to the \textit{Wicksellian natural rate of interest}, hence taking active account of the stock-price dynamics.

Analysis of equilibrium determinacy, moreover, while confirming the Taylor principle as a necessary condition for a unique stable equilibrium, also shows that an accommodative stance towards stock prices is not \textit{in itself} producing instability, as long as a proportionately stronger reaction is granted towards the output gap.

The remaining of the paper is structured as follows. Sections 2 through 4 present a DSGE model of the business cycle which allows for a specific consideration of any possible wealth effect on consumption induced by the stock-price dynamics. Section 5 defines the goals of economic policy makers, characterizing their behavior, and analyzes the conditions for equilibrium determinacy. The policy maker is assumed to be in charge of monetary policy, implemented through an augmented forward-looking instrument rule which generalizes Taylor’s [55]. Fiscal policy is totally disregarded. Section 6 finally summarizes and concludes.

\footnote{Cardia [11] derives an analogous solution within a small open economy framework.}
2 The Theoretical Model: the Demand-Side.

In this section we follow the methodology described in Piergallini [41] to derive a closed-form solution for aggregate consumption.

Consumers are finitely-lived, demand consumption goods and financial assets, and supply labor to the productive sector. The demand-side of the economy that their behavior determines is modelled according to a discrete-time version of the Blanchard-Yaari framework, featuring overlapping generations with no bequest motive.\(^6\)

2.1 The population structure

Each period a cohort of constant size \(n\) of finitely-lived consumers is born, and faces a constant probability \(\gamma\) of dying before the next period begins. The expected lifetime of each cohort is therefore

\[
\sum_{\tau=t}^{\infty} (1 - \gamma)^{\tau-t} = \gamma^{-1}.
\]

Thus each period \(t\) the population consists of an indefinite number of cohorts, indexed \(s\) \((s = -\infty, \ldots, t-1, t)\), where \(s\) is the period of birth. Since each cohort faces a constant probability \((1 - \gamma)\) of surviving each period and its size at the time it was born was \(n\), the size of a generic cohort \(s\) at time \(t\) is \(n_{s,t} = n(1 - \gamma)^{t-s}\).\(^7\) Accordingly, the aggregate population size at time \(t\) is

\[
\sum_{s=-\infty}^{t} n_{s,t} = \sum_{s=-\infty}^{t} n(1 - \gamma)^{t-s} = \frac{n}{\gamma}.
\]

The choice here is to abstract from population growth and to normalize aggregate population size to 1 \((n_t = 1)\). To that aim, we set the size of newborn cohorts to \(\gamma\) \((n = \gamma)\) and, as a consequence, the size of cohort \(s\) at time \(t\) will be

\[
n_{s,t} = \gamma(1 - \gamma)^{t-s}.
\]

This is therefore the weight that will be assigned to cohort \(s\) in the aggregation of all relevant economic variables as of time \(t\), as described in section 2.2.1.

---

\(^6\)For other stochastic discrete-time versions of the OLG model see Annicchiarico, Marini and Piergallini [1], Cardia [11], Chadha and Nolan [16] and [17], Piergallini [41]. For non-stochastic discrete-time versions see, among the others, Cushing [22], Frenkel and Razin [24], Smets and Wouters [53].

\(^7\)Note that the size of cohort \(t\) at time \(t\) (therefore of newborn cohorts) collapses to: \(n_{t,t} = n(1 - \gamma)^{t-t} = n\).
2.2 Consumers’ behavior

Each household is assumed to have preferences over consumption and leisure, described by a log-utility function.\(^8\)

Consumers born in period \(s\), therefore, seek to maximize their expected lifetime utility, discounted to account for impatience (as reflected by the intertemporal discount factor \(\beta\)) and uncertain lifetime (as reflected by the probability of survival across two subsequent periods, \((1 - \gamma))\). To that aim, they choose a pattern for real consumption \((C_{s,t})\), hours worked \((L_{s,t})\) and saving, allocating the latter in financial-asset holdings: risk-free-bond holdings whose face nominal value is \(B_{s,t}\) and a portfolio of shares issued by the intermediate good-producing firms, \(Z_{s,t}(i)\), whose nominal price at period \(t\) is \(Q^*_t(i)\).

Each period, the sources of funds consist of the nominal labor income \((W_tL_{s,t})\) and the nominal gross financial wealth carried over from period \(t - 1\), \(\omega_{s,t-1}\), defined as:

\[
\omega_{s,t-1} \equiv \frac{1}{1 - \gamma} \left[ B_{s,t-1}R_{t-1} + \int_0^1 \left( Q^*_t(i) + D_t(i) \right) Z_{s,t-1}(i) \, di \right]. \tag{1}
\]

The financial wealth of an individual born at time \(s\) includes therefore the gross payoffs of the two assets available. The riskless bond yields \(R_{t-1}\) per unit, the nominal gross interest rate relative to period \(t - 1\); each share included in the risky portfolio, in turn, pays a nominal dividend yield \(D_t(i)\) and is worth its own current nominal market value \(Q^*_t(i)\). Moreover, financial wealth carried over from the previous period also pays off the gross return on the insurance contract that redistributes among survived consumers (and in proportion to one’s current wealth) the financial wealth of the ones who died. Total personal financial wealth is therefore accrued by a factor of \(\frac{1}{1 - \gamma}\).\(^9\)

The optimization problem faced at time \(t\) by the \(s\)-periods-old representative consumer is therefore:

\[
\max_{\{C_{s,\tau},L_{s,\tau},B_{s,\tau},Z_{s,\tau}(i)\}_{\tau=t,i\in[0,1]}^\infty} \mathbb{E}_t \left\{ \sum_{\tau=t}^\infty \beta^{\tau-t} \left(1 - \gamma \right)^{\tau-t} \left[ \log C_{s,\tau} + \eta \log (1 - L_{s,\tau}) \right] \right\}
\]

such that

\[
P_tC_{s,\tau} + B_{s,\tau} + \int_0^1 Q^*_\tau(i)Z_{s,\tau}(i) \, di \leq W_{\tau}L_{s,\tau} + \omega_{s,\tau-1} \tag{2}
\]

---

\(^8\)The assumption that the utility function be logarithmic, which is certainly a restriction, is chosen in order to retrieve time-invariant structural parameters characterizing the equilibrium conditions. See Smets and Wouters [53] for a non-stochastic framework in which the utility is CRRA.

\(^9\)See Blanchard [7].
and such that

\[ C_{s,\tau}, \quad L_{s,\tau} \geq 0, \quad 0 \leq Z_{s,\tau}(i) \leq 1, \quad B_{s,\tau} \geq B^*, \]

for all \( \tau = t, t+1, t+2, \ldots \infty, \) \( s = -\infty, \ldots, t-1, t, \) and \( i \in [0,1], \) and where \( \beta \in [0,1], \) \( \eta > 0 \) and \( B^* > -\infty \) (i.e. \( B_{s,\tau} \) is lower bounded).

The first-order conditions for an optimum consist of equation (2) holding with equality and the following Euler equations:

\[ \eta C_{s,t} = \frac{W_t}{P_t}(1 - L_{s,t}), \quad (3) \]

\[ \frac{1}{P_tC_{s,t}} = \beta R_t E_t \left\{ \frac{1}{P_{t+1}C_{s,t+1}} \right\}, \quad (4) \]

\[ Q_t^*(i) = E_t \left\{ \delta_{t,t+1}(s) \left[ Q_{t+1}^*(i) + D_{t+1}(i) \right] \right\}, \quad (5) \]

in which we made use of the stochastic discount factor \( \delta_{t,t+1}(s), \) defined as:

\[ \delta_{t,\tau}(s) \equiv \beta^{\tau-t} \frac{P_tC_{s,t}}{P_tC_{s,\tau}}, \quad (6) \]

and such that \( E_t \{ \delta_{t,t+1}(s) \} = R_t^{-1} \) (as implied by equation (4)) and \( \delta_{t,\tau}(s) = \prod_{j=t}^{\tau-1} \delta_{j,j+1}(s). \)

Equation (3) nests the optimal conditions with respect to consumption and leisure, and represents the consumer’s current static supply of labor. Equation (4) tracks the intertemporal path of consumption, given the gross return on risk-free bonds, defining a forward-looking IS-type relation between consumption and the interest rate. Equation (5), finally equates the nominal price of an equity share to its nominal expected payoff one period ahead, discounted by the stochastic factor \( \delta_{t,t+1}(s), \) and defines the stock-price dynamics.

One last requirement for an optimum of the problem above is that the possibility of Ponzi schemes be ruled out, that is the present value of last period’s financial wealth, discounted by the stochastic discount factor and conditional upon survival, shrink to zero as time diverges:

\[ \lim_{T \to \infty} E_t \left\{ \delta_{t,T}(s)(1 - \gamma)^{T-t} \omega_{s,T} \right\} = 0. \quad (7) \]

Using equations (4) and (5), and recalling the definition of financial wealth (1), the equilibrium budget constraint (2) can be given the form of the following stochastic difference equation in the financial wealth \( \omega_{s,t}: \)

\[ P_tC_{s,t} + E_t \left\{ \delta_{t,t+1}(s)(1 - \gamma)\omega_{s,t} \right\} = W_tL_{s,t} + \omega_{s,t-1}. \quad (8) \]
Let’s now define the human wealth for cohort $s$ at time $t$ ($H_{s,t}$) as the expected stream of future labor income, discounted by the stochastic discount factor and conditional upon survival:

$$H_{s,t} \equiv E_t \left\{ \sum_{\tau=t}^{\infty} \delta_{t,\tau}(s)(1-\gamma)^{\tau-t}W_{\tau}L_{s,\tau} \right\}.$$  \hfill (9)

As described in Piergallini [41], solving equation (8) forward, and using the No-Ponzi-Game condition (7) and the definition of stochastic discount factor (6) and of human wealth (9) allows to retrieve the equation that describes nominal consumption as a linear function of total nominal financial and human wealth:

$$P_tC_{s,t} = \varphi(\omega_{s,t-1} + H_{s,t}),$$  \hfill (10)

where $\varphi \equiv [1 - \beta(1 - \gamma)]$ is constant across cohorts and over time.

### 2.2.1 Aggregation across cohorts

The aggregate value of the variables at stake are simply computed as a weighted average of the corresponding generation-specific counterpart, where the weights are given by the cohort size, as anticipated in section 2.1:

$$X_t \equiv \sum_{s=-\infty}^{t} n_{s,t}X_{s,t} = \sum_{s=-\infty}^{t} \gamma(1 - \gamma)^{t-s}X_{s,t},$$  \hfill (11)

for all $X = C, L, B, Z(i)$.

The solution of the consumers’ problem provides four relevant equilibrium conditions for each generic cohort $s$: the static labor supply of equation (3), the stock-price dynamics implied by equation (5), the budget constraint holding with equality (equation (2)) and the relation linking personal consumption to total personal wealth, equation (10).

Note that all the equilibrium conditions above are linear in the cohort-specific variables. As a consequence, applying the aggregator (11) to such conditions yields a set of aggregate relations identical in the functional form to their generation-specific counterparts:

$$\eta C_t = \frac{W_t}{P_t}(1 - L_t),$$  \hfill (12)

$$Q^*_t(i) = E_t \left\{ \delta_{t,t+1} [Q^*_{t+1}(i) + D_{t+1}(i)] \right\},$$  \hfill (13)

$$P_tC_t + B_t + \int_0^1 Q^*_t(i)Z_t(i) \, di = W_tL_t + \omega_{t-1}$$  \hfill (14)

$$P_tC_t = \varphi(\omega_{t-1} + H_t),$$  \hfill (15)
in which \( C_t, L_t, B_t \) and \( Z_t(i) \) fulfill equation (11), and \( \delta_{t,t+1} \) is such that

\[
E_t\left\{ \delta_{t,t+1} \right\} = R_t^{-1}. \tag{16}
\]

Some more caution is needed as to the financial wealth. In this case, in fact, an appropriate aggregation needs to take into consideration that the aggregate value of the gross return on the insurance contract must be 1, since the only effects it has are redistributive ones, which cancel out in aggregate terms. As a consequence aggregate financial wealth is redefined in terms of aggregate variables as:

\[
\omega_{t-1} \equiv B_{t-1}R_{t-1} + \int_0^1 \left( Q_t^*(i) + D_t(i) \right) Z_{t-1}(i) \, di. \tag{17}
\]

Analogously to its generation-specific counterpart, we can write the aggregate budget constraint (14) as a stochastic difference equation in aggregate wealth:

\[
P_tC_t + E_t\left\{ \delta_{t,t+1}\omega_t \right\} = W_tL_t + \omega_{t-1}. \tag{18}
\]

Finally, the above equation (18) and equation (15) make up a system that yields as a solution the equation that determines the dynamic path of aggregate consumption, as shown by Piergallini [41]:

\[
P_tC_t = \frac{\gamma\varphi}{1-\varphi} E_t\left\{ \delta_{t,t+1}\omega_t \right\} + \frac{1}{\beta} E_t\left\{ \delta_{t,t+1}P_{t+1}C_{t+1} \right\}, \tag{19}
\]

where \( \gamma\varphi \frac{1}{1-\varphi} \) is the aggregate propensity to consume out of financial wealth.

### 2.3 Equilibrium

To derive the ultimate relations that describe the demand side of the economy, we need to impose some final market-clearing conditions, descending from the assumptions about the structure of the model.

The complete disregard for fiscal policy implies two additional conditions holding in equilibrium. The first implies that total output equals total aggregate consumption, while the second requires that the net supply of riskless bonds be nil:

\[
Y_t = C_t \tag{20}
\]

\[
B_t \equiv \sum_{s=-\infty}^{t} \gamma(1-\gamma)^{t-s}B_{s,t} = 0. \tag{21}
\]
Moreover, in equilibrium the aggregate stock of outstanding equity for each intermediate good-producing firm must equal the corresponding total amount of shares, which by definition sum up to 1:

\[ Z_t(i) \equiv \sum_{s=-\infty}^{t} \gamma(1-\gamma)^{t-s}Z_{s,t}(i) = 1, \]  

for all \( i \in [0,1] \).

Finally, let’s define total dividend payments provided by the wholesalers and the general nominal stock-price index as the simple integration through the continuum of firms, under the assumption that their distribution over the interval \([0,1]\) is uniform:

\[ D_t \equiv \int_{0}^{1} D_t(i) \, di \]  

\[ Q^*_t \equiv \int_{0}^{1} Q^*_t(i) \, di. \]  

As a consequence of the above conditions, the aggregate labor supply now reads:

\[ \eta Y_t = \frac{W_t}{P_t} (1 - L_t). \]  

Note that in equilibrium, given conditions (13), (21), (22) and (24), the present discounted nominal value of future financial wealth, \( E_t\{\delta_{t,t+1}\omega_t\} \), is sufficiently represented by the current level of the nominal stock-price index:

\[ E_t\{\delta_{t,t+1}\omega_t\} = E_t\left\{ \delta_{t,t+1} \left[ B_t R_t + \int_{0}^{1} \left( Q^*_{t+1}(i) + D_{t+1}(i) \right) Z_i(i) \, di \right] \right\} = \int_{0}^{1} E_t\{\delta_{t,t+1} \left( Q^*_{t+1}(i) + D_{t+1}(i) \right) \, di \} = \int_{0}^{1} Q^*_t(i) \, di = Q^*_t. \]  

As a consequence, and given also condition (23), the demand-side of the economy is summarized by the following two aggregate Euler equations

\[ P_t Y_t = \frac{\gamma \varphi}{1-\varphi} Q^*_t + \frac{1}{\beta} E_t\{\delta_{t,t+1} P_{t+1} Y_{t+1}\} \]  

\[ Q^*_t = E_t\{\delta_{t,t+1} [Q^*_{t+1} + D_{t+1}]\}, \]  

and by the aggregate resource constraint

\[ Y_t = L_t \frac{W_t}{P_t} + \frac{D_t}{P_t}. \]
Equation (27) defines the dynamic path of aggregate demand, in which an explicit and determinant role is played by the dynamics of stock prices. The latter is defined by equation (28), which is micro-founded on the consumers’ optimal behavior.

Finally, note that the traditional set-up of infinitely-lived consumers is a particular case of the one presented here, and corresponds to a zero-probability of death, $\gamma = 0$. In this case, in fact, equation (27) loses the term depending on stock prices and collapses to the usual forward-looking IS-type schedule relating real output only to the long-run real interest rate:\[^{10}\]

$$\beta P_t Y_t = E_t \{ \delta_{t,t+1} P_{t+1} Y_{t+1} \}.$$  

3 The Theoretical Model: the Supply-Side.

The supply-side of the economy consists of two sectors of infinitely-lived agents: a retail sector operating in perfect competition to produce the final consumption good and a wholesale sector hiring labor from a competitive market to produce a continuum of differentiated perishable intermediate goods.

To allow for price stickiness, we follow broad convention assuming that each period only a sub-set of wholesalers is able to set their price optimally, while the others charge the last-period’s price, according to the mechanism introduced by Calvo [9].

3.1 The retail sector

The representative final good-producing firm chooses the optimal level of input (the intermediate good $Y_t(i)$) to produce the final consumption good $Y_t$. Since it acts in perfect competition, though, it has no decision power over the price at which to sell it. Its problem is then to maximize its profit

$$P_t Y_t - \int_0^1 P_t(i) Y_t(i) \, di$$

subject to the following Constant-Return-to-Scale technology

$$Y_t \leq \left[ \int_0^1 Y_t(i) \frac{i-1}{\epsilon} \, di \right]^{\epsilon-1},$$

\[^{10}\text{In this case, moreover, equation (28) becomes completely redundant and can be disregarded, since any explicit dynamics of the nominal stock-price index does not affect in any way whatsoever not only the level of equilibrium real output but also the conditions for a determinate equilibrium of the macroeconomic system.}\]
where $\epsilon > 1$ is the elasticity of substitution of the intermediate goods with one another, and reflects the degree of competition in the market for the factors of production.

The above problem, within a regime of perfect competition that drives all profits to zero, yields as optimality condition the input demand function by the final good-producing firm (along with the technological constraint holding with equality):

$$Y_t(i) = \left[ \frac{P_t(i)}{P_t} \right]^{-\epsilon} Y_t. \quad (30)$$

The selling price is finally and uniquely determined by the zero-profit condition:

$$P_t = \left[ \int_0^1 P_t(i)^{1-\epsilon} \, di \right]^{\frac{1}{1-\epsilon}}. \quad (31)$$

In this setting, any assumption about the property of this kind of firms is totally equivalent to one another, since there can’t be any dividend on profits, being the latter driven to zero by perfect competition.

### 3.2 The wholesale sector

As they produce a continuum of differentiated perishable goods, which are imperfectly substitutable with one another in the consumption good’s production, the intermediate good-producing firms take advantage of some monopoly power, reflected by $\frac{1}{\epsilon}$. Therefore each firm chooses both the level of input $L_t(i)$ (labor hired from the household) and the selling price $P_t(i)$ to maximize its objective function.

In producing their output, each wholesaler uses a Constant-Return-to-Scale technology

$$Y_t(i) = A_t L_t(i), \quad (32)$$

in which $A_t = \exp\{a_t\}$ reflects a labor-augmenting shock on productivity, distributed as a stationary autoregressive process of order one:

$$a_t = \rho a_{t-1} + u_{a,t}.$$  

In choosing the optimal level of labor to demand, each firm enters a competitive labor market and seeks to minimize total real costs, $TC_t = \frac{W_t}{P_t} L_t(i)$, subject to the technological constraint (32). The optimal real marginal costs, therefore, are constant across firms and
proportional to the real wage, with the factor of proportionality being decreasing in the productivity index $A_t$: 

$$MC_t(i) \equiv \frac{\partial TC_t}{\partial Y_t(i)} = \frac{\partial}{\partial Y_t(i)} \left( \frac{W_t}{P_t A_t} Y_t(i) \right) = \frac{W_t}{P_t A_t} = MC_t.$$  \hspace{1cm} (33) 

As to the selling price, we follow Calvo [9] by assuming that in each period each firm has an exogenous and constant probability $(1 - \theta)$ of getting the chance to set its price optimally; with probability $\theta$, then, each firm will have to charge last period’s price.

If a given firm gets the chance to set its price optimally at time $t$, it does so seeking to maximize the expected stream of future dividends (hence the real value of its outstanding shares, as can be seen iterating forward equation (13)) and taking into consideration that the chosen price will have to be charged up until period $\tau$ with probability $\theta^{\tau-t}$.

The dynamic problem faced by an optimizing firm at time $t$ can therefore be stated as:

$$\max_{P_t(i)} \mathbb{E}_t \left\{ \sum_{\tau=t}^{\infty} \theta^{\tau-t} \delta_{t,\tau} \left[ \frac{P_t(i)}{P_\tau} Y_\tau(i) - \frac{W_\tau}{P_\tau} L_\tau(i) \right] \right\},$$

such that the following constraints are satisfied: the CRS technology (32), the optimality condition from the labor market (33) and the demand for intermediate goods coming from the retail sector (30).

The first-order condition for the solution of the above problem implies that all firms revising their price at time $t$ will choose a common optimal price level, $P^*_t$, set as a constant mark-up over a weighted average of expected future nominal marginal costs (which are constant across firms):

$$P_t(i) = P_t^* = \mu \mathbb{E}_t \left\{ \sum_{\tau=t}^{\infty} \omega_{t,\tau} P_\tau MC_\tau \right\},$$  \hspace{1cm} (34) 

where

$$\mu \equiv \frac{\epsilon}{\epsilon - 1}$$

is the steady-state gross mark-up reflecting the monopoly power $\frac{1}{\epsilon}$ and the weights $\omega_{t,\tau}$ depend on the relative way in which the firm discounts future cash flows in each period in which the price remains fixed

$$\omega_{t,\tau} \equiv \frac{\theta^{\tau-t} \delta_{t,\tau} Y_\tau P_\tau^{\epsilon-1}}{\mathbb{E}_t \left\{ \sum_{\tau=t}^{\infty} \theta^{\tau-t} \delta_{t,\tau} Y_\tau P_\tau^{\epsilon-1} \right\}}.$$
3.2.1 Aggregation across firms

Recall from section 2.3 that we defined total dividend payments and the nominal stock-price index as the result of an aggregation through the continuum of firms over the interval $[0, 1]$, under the assumption that firms are uniformly distributed over such interval. Here we apply the same aggregator to equation (32) to derive the aggregate production function:

$$A_t L_t = Y_t \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{-\epsilon} di = Y_t \Xi_t,$$

where $L_t$ is defined as the aggregate level of hours worked

$$L_t = \int_0^1 L_t(i) di$$

and

$$\Xi_t = \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{-\epsilon} di$$

is an index capturing price dispersion over the continuum of intermediate goods-producing firms.

Finally, moving from the definition of the general price level (31) and considering that all firms revising their price at $t$ (a fraction $(1 - \theta)$ of all firms) choose the same price $P_t^*$ and that all firms keeping the price constant (a fraction $\theta$) charge last period’s general price level, the aggregate price level as of time $t$ can be written as:

$$P_t = \left[ \theta P_{t-1}^{1-\epsilon} + (1 - \theta) P_t^* \right]^{\frac{1}{1-\epsilon}},$$

with $P_t^*$ fulfilling equation (34).

Hence, the supply-side of the economy under sticky prices can be summarized by three equations.

The first is the aggregate production function

$$A_t L_t = Y_t \Xi_t.$$  \hspace{1cm} (37)

The second descends from the equilibrium in the labor market, combining the aggregate labor supply (25) with the condition for an optimal choice of labor on the part of the firms, (33)

$$1 - L_t = \eta \frac{Y_t}{A_t MC_t};$$

\hspace{1cm} 11See equations (23) and (24).
substituting in the above equation the aggregate production function to eliminate aggregate hours worked yields the relation between equilibrium real marginal costs and total output under sticky prices:

\[ MC_t = \eta \frac{Y_t}{A_t - Y_t \Xi_t}. \] (38)

The last equation describing the supply-side of the economy is the equation determining the aggregate price level:

\[ P_{t}^{1-\epsilon} = \theta P_{t-1}^{1-\epsilon} + (1 - \theta) \left[ \mu E_t \left\{ \sum_{\tau=t}^{\infty} \omega_{t,\tau} P_{\tau} MC_{\tau} \right\} \right]^{1-\epsilon}. \] (39)

### 3.3 Flexible prices

As the goal of economic policy should be to offset the distortions diverting the system from its efficient equilibrium, the benchmark case in this class of frameworks is usually the one in which the distortion affecting prices (i.e. nominal rigidities) is cancelled out. In this section, therefore, we outline the equilibrium of the system under flexible prices, that is when the probability of having to keep the price fixed shrinks to zero: \( \theta = 0 \). In this case each variable converges to its natural level, denoted by a superscript \( n \).

When all firms can undertake optimal price setting with probability 1, each of them sets its own period-\( t \) price as a constant mark-up over current nominal marginal costs, depending on the monopoly power characterizing the market:

\[ P_t(i) = \mu P_t MC_t. \] (40)

Since equilibrium real marginal costs are constant across firms, then each firm’s relative price will be constant as well. This implies that all firms will choose the same price level \( (P_t(i) = P_t \text{ for all } i) \), and that real marginal costs under flexible prices will just be the reciprocal of the steady-state gross mark-up \( \mu \):

\[ MC_t^n = \frac{1}{\mu} = \frac{\epsilon - 1}{\epsilon}. \] (41)

Moreover, since all firms choose the same price level, price dispersion across firms is nil and therefore the gross index \( \Xi_t \) equals 1.

\(^{12}\)As the differentiation among the intermediate goods decreases, so does the monopoly power; the parameter \( \epsilon \) diverges and the gross mark-up rate \( \mu \equiv \frac{\epsilon}{\epsilon - 1} \) converges to 1: \( \lim_{\epsilon \to \infty} \frac{\epsilon}{\epsilon - 1} = 1 \). This is the case of perfect competition, in which prices only cover marginal costs.
Substituting the above results into equation (38), finally, yields the natural (or potential) level of output, that would prevail in the economic system were prices completely flexible:

\[ Y^n_t = \frac{A_t}{1 + \mu\eta}. \]  

(42)

Potential output then is affected positively by the level of technology and negatively by the monopoly power \(1/\epsilon\).

4 The Complete Linear Model

Combining the demand-side represented by equations (27) through (29) and the supply-side summarized by equations (37) through (39), we obtain the complete model of the private sector operating in the economic system described so far.

Using the aggregate resource constraint, the aggregate production function and the optimal condition for labor demand to substitute for the nominal dividends in the stock-price dynamics equation, reduces the system of equilibrium conditions for the case with nominal rigidities to the following four equations:

\[ P_t Y_t = \frac{\gamma^\varphi}{1 - \varphi} Q_t^* + \frac{1}{\beta} E_t \left\{ \delta_{t+1} P_{t+1} Y_{t+1} \right\} \]  

(43)

\[ Q_t^* = E_t \left\{ \delta_{t+1} \left[ Q_{t+1}^* + P_{t+1} Y_{t+1} \left( 1 - \Xi_{t+1} MC_{t+1} \right) \right] \right\} \]  

(44)

\[ MC_t = \eta Y_t A_t - Y_t \Xi_t \]  

(45)

\[ P_t^{1-\epsilon} = \theta P_{t-1}^{1-\epsilon} + (1 - \theta) \left[ \mu E_t \left\{ \sum_{\tau=t}^{\infty} \varpi_{t,\tau} P_{\tau} MC_{\tau} \right\} \right]^{1-\epsilon}. \]  

(46)

In the particular case of full price flexibility (\(\theta = 0\)), instead, the system is summarized by the following three:

\[ MC^n_t = \frac{1}{\mu} = \frac{\epsilon - 1}{\epsilon} \]  

(47)

\[ Y^n_t = \frac{A_t}{1 + \mu\eta} \]  

(48)

\[ Q^n_t = E_t \left\{ \delta_{t+1} \left[ Q^n_{t+1} + \frac{1}{\epsilon} P^n_{t+1} Y^n_{t+1} \right] \right\}, \]  

(49)

where all variables are at their natural level.

In the long-run, the system converges to a non-stochastic zero-inflation steady state, which is common to both specifications about consumer-price setting. The above equations (43) through (49) define the following steady-state relations, in which the absence of
time subscripts denotes the steady-state level of the several variables:\[13\]

\[
\beta R = 1 + \varphi \frac{\gamma}{1 - \gamma} RQ \equiv 1 + \psi
\]

\[
1 - \frac{PY}{eQR} = \frac{1}{R} = \frac{\beta}{1 + \psi} \equiv \tilde{\beta}
\]

\[
MC = MC^n = \frac{1}{\mu}
\]

\[
\Xi = 1
\]

\[
Y = Y^n = \frac{A}{1 + \mu \eta}.
\]

Around the steady-state outlined by the above relations we compute a log-linear approximation of the equilibrium conditions, in which lower-case variables denote log-deviations from the corresponding steady-state level, \(x_t \equiv \log(X_t) - \log(X)\), \(q_t \equiv q^*_t - p_t\) defines the log-deviations of the real stock-price index and \(\pi_t \equiv p_t - p_{t-1}\) defines the period-\(t\) net inflation rate:

\[
y_t = \frac{1}{1 + \psi} E_t y_{t+1} + \frac{\psi}{1 + \psi} q_t - \frac{1}{1 + \psi} (r_t - E_t \pi_{t+1})
\]

\[
q_t = \tilde{\beta} E_t q_{t+1} + (1 - \tilde{\beta}) E_t y_{t+1} - \frac{1 - \beta}{\mu - 1} E_t mc_{t+1} - (r_t - E_t \pi_{t+1})
\]

\[
mc_t = \lambda (y_t - a_t)
\]

\[
\pi_t = \tilde{\beta} E_t \pi_{t+1} + \frac{(1 - \theta)(1 - \theta \tilde{\beta})}{\theta} mc_t,
\]

where \(\lambda \equiv \frac{1 + \mu \eta}{\mu \eta}\) is the elasticity of real marginal costs to real activity, which is a decreasing function of the monopoly power \(\frac{1}{\epsilon}\) in the wholesale sector.

From equation (48) emerges that the log-linear approximation of potential output equals the productivity shock, and therefore evolves according to a stationary stochastic autoregressive process of order one:

\[
y^n_t = a_t = \rho_a y^n_{t-1} + u_{a,t}.
\]

Given the equilibrium log-linear expression for real marginal costs (57) and the relation just obtained, therefore, log-linear real marginal costs can be expressed as a linear function of the output gap, that is the deviation of actual output from its natural level: \((y_t - y^n_t)\). Accordingly, the complete linear model can be summarized by the following system of

\[13\]Note that in the long-run, the stochastic discount factor converges to \(R^{-1}\), as implied by equation (16).
stochastic difference equations in three endogenous forward-looking and two exogenous pre-
determined variables:

\begin{align}
y_t &= \frac{1}{1 + \psi} E_t y_{t+1} + \frac{\psi}{1 + \psi} q_t - \frac{1}{1 + \psi} (r_t - E_t \pi_{t+1}) \\
\pi_t &= \tilde{\beta} E_t \pi_{t+1} + \kappa (y_t - y^n_t) \\
qu_t &= \tilde{\beta} E_t q_{t+1} - (1 - \tilde{\beta}) \left( \frac{\lambda}{\mu - 1} - 1 \right) E_t \left\{ y_{t+1} - y^n_{t+1} \right\} - (r_t - E_t \pi_{t+1}) + (1 - \tilde{\beta}) \rho_a a_t + \nu_t \\
y^n_t &= \rho_a y^n_{t-1} + u_{a,t} \\
\nu_t &= \rho_\nu \nu_{t-1} + u_{\nu,t},
\end{align}

where we set \( \kappa = \lambda (1 - \theta) (1 - \tilde{\beta} \theta) \) and \( u_{i,t} \sim N(0, \sigma^2_i) \), for all \( i = a, \nu \).

Equation (60) defines a forward-looking IS-type relation that relates real output to its own expected future values, a short-term real interest rate and assigns an explicit and relevant role to real stock prices in driving its dynamics, through a wealth effect. The widely used case of infinitely-lived consumers requires \( \gamma = \psi = 0 \), and reduces equation (60) to the usual forward-looking IS schedule.

Equation (61) defines a New-Keynesian Phillips Curve, relating the dynamics of inflation to the pressures coming from actual output exceeding its potential level. The coefficient \( \kappa \) on the output gap depends negatively on the degree of nominal rigidities \( \theta \) and positively on the elasticity of real marginal costs to real activity \( \lambda \). Note that also this relation is affected by the presence of wealth effects, displaying a lower weight on future inflation \( \tilde{\beta} \equiv \frac{\beta}{1 + \psi} \) and consequently a higher weight on the output gap compared with standard case, to which anyhow we converge as \( \gamma \) goes to zero.

Equation (62) describes the dynamics of real stock prices. Albeit coming from a General Equilibrium model, it is equivalent to Campbell and Shiller’s [10] approximate present value model, in which a simple arbitrage equation links stock prices to expected variations in future dividends and discount factors. Here the relationship is explicitly stated in terms of underlying real variables like the output gap and the real interest rates. The last term of equation (62) was appended to capture the possible deviations of current real stock prices from their fundamental-based valuation, to account for waves of optimism or short-termism, and evolves according to equation (64). This is the only term not explicitly micro-founded in the set up, and is intended to represent the observed deviations from market efficiency,
allowing for the expectations of part of the investors to be influenced by external, non-fundamental factors like fads or “irrational exuberance”.\textsuperscript{14}

5 Price Stability and Equilibrium Determinacy.

In this section we address the issue of the requirements for price stability, deriving what Woodford \cite{56} calls the \textit{Wicksellian natural rate of interest}; we then characterize monetary policy and discuss the implied properties of a determinate equilibrium.

5.1 The goal of price stability

In order for the system to endogenously yield price stability, the Phillips Curve \eqref{61} requires that actual output be at all times equal to its natural level ($y_t = y^n_t$, for all $t$), that is the system mimic the case of fully flexible prices. Substituting this condition, and the implied zero inflation rate, into equation \eqref{60} allows us to show that the \textit{Wicksellian natural rate of interest}, i.e. the real rate of return prevailing in the case of flexible prices, is a function of the shock on productivity and the \textit{natural} level of real stock prices:\textsuperscript{15}

\begin{equation}
\begin{aligned}
r^n_t &= \psi q^n_t - (1 + \psi - \rho_a) a_t. \\
\end{aligned}
\end{equation}

To derive this latter variable in terms of the underlying structural shocks we write equation \eqref{62} under the hypothesis of price stability, making use of equation \eqref{65}. After some algebra we get:\textsuperscript{16}

\begin{equation}
\begin{aligned}
q^n_t &= \tilde{\beta} E_t q^n_{t+1} + \frac{1 + \psi - \tilde{\beta} \rho_a}{1 + \psi} a_t + \frac{1}{1 + \psi} \nu_t, \\
\end{aligned}
\end{equation}

\textsuperscript{14}Shiller \cite{50, 51, 52} introduces and comment on fads and irrational exuberance, Summers \cite{54} provides a detailed and formal discussion, to which our modelling choice is consistent.

To see this, define actual stock prices, as in Summers \cite{54}, as $q_t = q^f_t + \nu^*_t$, where $q^f_t$ is the fundamental, fully rational real price of stocks, following equation \eqref{56} and $\nu^*_t$ is an exogenous stochastic misalignment capturing the possibility of irrational exuberance, or a fad, driving expectations about the future level of stock prices. Substituting the above definition into equation \eqref{56} yields

\begin{equation}

q_t = \tilde{\beta} E_t q^n_{t+1} + (1 - \tilde{\beta}) E_t y_{t+1} - \frac{1 - \tilde{\beta}}{\mu - 1} E_t mc_{t+1} - (r_t - E_t \pi_{t+1}) + (1 - \tilde{\beta} \rho_c) \nu^*_t,
\end{equation}

which equals equation \eqref{62}, given the equation for real marginal costs and defining $\nu_t \equiv (1 - \tilde{\beta} \rho_c) \nu^*_t$.

\textsuperscript{15}We define the level of the stock-price index prevailing under flexible prices as \textit{natural} or \textit{potential}, in analogy with the corresponding definitions for output and the interest rate.

\textsuperscript{16}This equation is equivalent of course to the log-linear approximation of equation \eqref{49}, augmented to allow for the possibility of a fad (which is independent from any assumption about how consumer-prices are set) and where equation \eqref{65} approximates the stochastic discount factor: $\log(\delta^n_{t+1}) - \log(1/R) \equiv -r^n_t$. 

19
which can be solved forward, yielding

\[ q_t^n = E_t \left\{ \sum_{\tau=t}^{\infty} \left( \frac{\bar{\beta}}{1 + \psi} \right)^{\tau-t} \left[ 1 + \psi - \bar{\beta} \rho_a \frac{1}{1 + \psi} a_{\tau} + \frac{1}{1 + \psi} \nu_{\tau} \right] \right\}. \]

Given the assumed stochastic dynamics of the structural shocks \( a_t \) and \( \nu_t \), the above equation further simplifies to\(^\text{17}\)

\[ q_t^n = a_t + \frac{1}{1 + \psi - \bar{\beta} \rho_a} \nu_t. \quad (67) \]

The potential level of the real stock-price index, therefore, responds positively to both fundamental shocks on productivity and stochastic misalignments. Note that under the hypothesis of market efficiency \((\sigma_{\nu}^2 = 0)\), the natural level of stock prices corresponds to the natural level of output, ultimately driven solely by shifts in productivity.\(^\text{18}\)

We are now able to write the final reduced form for the Wicksellian natural rate of interest. Plugging equation (67) into (65), in fact, yields (again after some basic algebra):

\[ r_t^n = \frac{\psi}{1 + \psi - \bar{\beta} \rho_a} \nu_t - (1 - \rho_a) a_t. \quad (68) \]

The natural rate of interest, therefore, responds to both structural shocks to the economy, but in opposite ways: negatively to the one coming from a shift in technology that makes labor more productive, and positively to the one coming from some wave of exuberance that pushes up the level of stock prices and hence consumers’ financial wealth.\(^\text{19}\)

Since the natural rate of interest is, by definition and by construction, the real rate of interest that features a system where prices are fully flexible, a monetary policy maker

\(^\text{17}\)We assumed that both shocks be stationary; however, note that also in the case one or both of the structural shocks were random walks, still equation (67) would hold, given \( \psi > 0 \) and \( 0 < \bar{\beta} < 1 \).

For an alternative, maybe more insightful, derivation equation (67), note that equation (66) can also be written as

\[ q_t^n - a_t = \frac{\bar{\beta}}{1 + \psi} E_t \{ q_{t+1}^n - a_{t+1} \} + \frac{1}{1 + \psi} \nu_t \]

from which it is straightforward to see that if the market is not affected by any irrational exuberance the natural level of stock prices is defined only by the level of productivity.

\(^\text{18}\)Note also that in the case we assumed a unit root in the process driving the dynamics of productivity \((\rho_a = 1)\), which is often the case in the literature, then equation (67) would micro-found the random-walk hypothesis about the fundamental price of stocks under market efficiency and consumer-price flexibility, as commonly assumed in financial literature (see Summers [54]).

\(^\text{19}\)Note that equation (68) can be written as

\[ r_t^n = \frac{\psi}{1 + \psi - \bar{\beta} \rho_a} \nu_t + E_t \{ \Delta a_{t+1} \}, \]

where \( E_t \{ \Delta a_{t+1} \} = -(1 - \rho_a) a_t \), and which yields the result commented on in Galí [25] that the optimal policy for price stability “seeks to insulate the price levels from the effects of changes in productivity”.

---

\( q_t^n \) is the natural level of stock prices, \( a_t \) is the natural level of output, \( \nu_t \) is the natural level of consumer prices, \( \rho_a \) is the elasticity of substitution between goods, \( \psi \) is a parameter reflecting the degree of risk aversion, and \( \bar{\beta} \) is a parameter reflecting the degree of risk aversion.

\( E_t \{ \cdot \} \) denotes the expected value at time \( t \) of the expression inside the braces.

\( \Delta a_{t+1} \) is the change in the natural level of output from time \( t \) to \( t+1 \).
pursuing price stability by means of an instrument rule should seek to make the dynamic path of nominal interest rates as close as possible to the implied Wicksellian natural rate.

A Central Banker whose primary objective be price stability, then, should actively consider stock-price dynamics in the design of monetary policy actions, leaning against stochastic misalignments while accommodating productivity gains.

Note that in the standard case of infinitely-lived consumers and no variable physical capital, in which stock prices have no real effects and to which our model converges as $\psi$ goes to zero, equations (65) and (68) both collapse to the expression

$$r^n_t = -(1 - \rho_a)a_t. \tag{69}$$

This means that when stock prices do not affect real activity, a Central Banker pursuing price stability should not care about their dynamics. This is a not surprising and totally expected result.

A more interesting implication comes from the fact that also in the case stock prices did affect real activity ($\psi > 0$) but were only driven by fundamentals ($\sigma^2_v = 0$), then equation (68) would be the same as in the standard no-wealth-effects case. The implication is that when stocks are fully rationally priced, therefore, a monetary policy concerned only about real shocks like the one on productivity is sufficient to achieve output and price stability. This conclusion resembles the one maintained by Bernanke and Gertler [3], according to which a flexible inflation targeting disregarding stock-price dynamics is sufficient to achieve both real and financial stability.

However, when the stock market is affected by fads or irrational bubbles à la Shiller-Summers, then an active role for monetary policy emerges and is, in fact, required in order to hit the target of a zero inflation and output gap. It is important to emphasize that this implication does not descend from any change in the Central Bankers’ targets: throughout the analysis, in fact, their only target is just a zero inflation rate, with no explicit interest in stock-price stability per se.

### 5.2 Monetary policy and equilibrium determinacy

To close the theoretical model we still need to make assumptions as to what drives the Central Banker’s interventions in the system. We assume that the main goal of monetary policy makers be to offset market distortions, seeking to drive the system towards its
natural equilibrium. As a consequence, we assume that the Central Bank controls a short-
term nominal interest rate, according to the following forward-looking reaction function,
generalizing Taylor’s [55]:

$$r_t = \rho r_{t-1} + \phi_\pi E_t \pi_{t+1} + \frac{\phi_y}{4} (y_t - y^n_t) + \frac{\phi_q}{4} (q_t - q^n_t) + u_{r,t}$$  \hspace{1cm} (70)$$

Hence the Central Bank allows a certain degree of inertia in the dynamic path of interest
rates and reacts to expected deviations of the inflation rate from the zero-target as well as
to deviations of both output and the stock-price index from their corresponding potential
levels.\footnote{Following Woodford [56], Ch. 4, and assuming the model as quarterly, we write the coefficients on the
output and stock-price gaps as $\phi_i/4$, for $i = y, q$, so that the coefficients $\phi_i$ are the ones corresponding to a
policy rule written in terms of annualized interest and inflation rates.} The stochastic term $u_{r,t}$ is meant to capture deviations of actual interest rates’
dynamics from the one implied by a mechanical reaction function, either voluntary or invol-
tuntary on the part of the policy makers, and evolves according to a gaussian white-noise.

A useful fashion in which to write the complete linear model to capture what drives the
dynamics relevant for monetary policy analysis is in deviations of each variable from its own
natural level. Denoting in what follows the output gap as $x_t \equiv y_t - y^n_t$ and the stock-price
gap as $s_t \equiv q_t - q^n_t$, and considering that the natural level of the inflation rate is zero, the
system may be written as:

$$x_t = \frac{1}{1 + \psi} E_t x_{t+1} + \frac{\psi}{1 + \psi} s_t - \frac{1}{1 + \psi} (r_t - r^n_t - E_t \pi_{t+1})$$  \hspace{1cm} (71)$$

$$s_t = \tilde{\beta} E_t s_{t+1} - (1 - \tilde{\beta}) \left( \frac{\lambda}{\mu - 1} - 1 \right) E_t x_{t+1} - (r_t - r^n_t - E_t \pi_{t+1})$$  \hspace{1cm} (72)$$

$$\pi_t = \tilde{\beta} E_t \pi_{t+1} + \kappa x_t$$  \hspace{1cm} (73)$$

$$r_t = \rho r_{t-1} + \phi_\pi E_t \pi_{t+1} + \frac{\phi_y}{4} x_t + \frac{\phi_q}{4} s_t + u_{r,t},$$  \hspace{1cm} (74)$$
in which any short-term dynamics is injected by $r^n_t$, fulfilling equation (68).

Within such a framework, an equilibrium consists of a dynamic path for the endogenous
variables $\{x_t, s_t, \pi_t, r_t\}$, given the exogenous stochastic processes $\{r^n_t, u_{r,t}\}$. Such equilibrium
is said to be determinate if it is stable and unique.

Considering system (71)–(74) in matrix form, as

$$E_t z_{t+1} = A z_t + B \varepsilon_t,$$  \hspace{1cm} (75)$$
where we set $z_t \equiv [x_t \quad s_t \quad \pi_t \quad r_{t-1}]'$ and $\varepsilon_t \equiv [r^n_t \quad u_{r,t}]'$, the conditions for equilibrium
determinacy hinge entirely on the magnitude of the four eigenvalues of matrix $A$.\footnote{Following Woodford [56], Ch. 4, and assuming the model as quarterly, we write the coefficients on the
output and stock-price gaps as $\phi_i/4$, for $i = y, q$, so that the coefficients $\phi_i$ are the ones corresponding to a
policy rule written in terms of annualized interest and inflation rates.}
Blanchard and Kahn [8] prove that a system of stochastic difference equations like (75) is characterized by a unique stable solution (i.e. the equilibrium is determinate) as long as the number of stable eigenvalues of matrix $A$ match exactly the number of endogenous variables that are predetermined. Otherwise the system exhibits no stable solution (if the number of stable eigenvalues is lower than required) or else a multiplicity of stable solutions and hence an indeterminate equilibrium (if the number of stable eigenvalues exceeds the number of predetermined endogenous variables).

In the present framework, since we assumed that the Central Bank pursues a certain degree of interest rates’ smoothing, the only endogenous predetermined variable is the lagged interest rate $r_{t-1}$. Consequently, the condition for a determinate equilibrium in our case is that matrix $A$ have just one eigenvalue within the unit circle. No stable eigenvalues would in this case produce no stable solutions and more than one would imply an indeterminate equilibrium.

The reason why an indeterminate equilibrium is undesired in economic models like the one in hand is that in such cases innovations in the sunspot variable(s) are sufficient to produce potentially large and persistent endogenous fluctuations in the system. A simple revision in expectations in these cases becomes self-fulfilling and induces a source of endogenous macroeconomic instability.

In rational-expectations models like the one analyzed here, the issue is typically addressed in order to see whether different policy regimes can affect in some way the achievement of a determinate equilibrium. Within this conceptual framework lies the so-called Taylor principle, according to which, in order for the system to show equilibrium determinacy, interest rates should respond more than proportionately to a sustained increase in the inflation rate.

Failing of such condition has been recently advocated as a possible explanation for the high macroeconomic instability that featured the U.S. economy during the 1960’s and 1970’s, before Volcker was appointed as Chair of the Federal Reserve. Estimates of monetary policy rules for that period, in fact, provided by Clarida, Gali and Gertler [20], show that the pre-Volcker period was characterized by an apparent accommodative stance of the Fed with respect to inflation, in contrast with the Taylor principle, as opposed to the Volcker-Greenspan era, which in contrast featured high stability of both prices and output.

Woodford [56] summarizes the results in the literature about determinacy in monetary
models where stock prices play no role. He especially shows the effects on equilibrium deter-
minacy of allowing for a certain degree of inertia in the path of interest rates, which implies
a relaxation of the Taylor principle, the more so the stronger the inertia. Nonetheless, it is
shown that the Taylor principle in this case still applies if the coefficients are transformed
to control for the inertia: considering $\Phi_i \equiv \frac{\phi_i}{1-\rho}$, for all $i = \pi, x$, in fact, implies identical
conditions for determinacy as in the case of no interest rates’ smoothing.

A second result, already showed and commented on by Clarida, Gali and Gertler [20],
is that within a monetary policy regime that entails a forward-looking rule, the parametric
space for $\phi_y$ that ensures equilibrium determinacy is not only lower- but also upper-bounded.

Within our framework, which implies a 4-by-4 coefficient matrix $A$ in system (75),
retrieving analytical conditions for determinacy is a non-trivial exercise and looses much of
the usual appeal in terms of the power to draw clear conclusions. We found here much more
powerful to plot the regions of determinacy by numerically simulating the model within a
wide parameter sub-space for the policy rule’s coefficients.

Consequently, all the figures below show the regions (depicted in white) for which the
conditions on the eigenvalues of matrix $A$ hold. The shaded areas indicate the regions in
which more than one eigenvalues are within the unit circle, and therefore the equilibrium
is indeterminate; in any case, within the parameter sub-space considered, albeit very wide,
no regions of unstable solutions were detected. The chosen calibration of the structural
parameters, under which the exercise is conducted, were taken from widespread convention.
The steady-state gross quarterly interest rate $R$ was calibrated at 1.015, implying a long-
run nominal annualized interest rate of 6%; the intertemporal discount factor $\beta$ was set at
0.99; $\epsilon$, the elasticity of substitution among intermediate goods was set at 6, implying a
steady-state net mark-up rate of 20%, and $\eta$ was chosen to be 1; finally, the probability for
firms of having to keep their price fixed for the current quarter was set at 0.75, implying
that prices are revised on average once a year.\textsuperscript{21}

The analysis of determinacy confirms both results mentioned above and provides a
further interesting insight about the links between monetary policy and stock prices.

Fig. 1 shows the regions of determinacy in the case of no interest rates’ smoothing, for
different values of $\phi_y$ and $\phi_q$. The conclusion drawn is that in this framework the Taylor
principle retains its relevance, confirming that $\phi_\pi = 1$ remains a critical value to ensure

\textsuperscript{21}To check for robustness, the determinacy analysis illustrated in what follows was conducted also for
different sets of calibrated structural parameters, yielding no significant differences in the results.
equilibrium determinacy.

In Fig. 2 we checked what are the effects of allowing for different degrees of inertia in the path of interest rates, simulating the model under different stances towards the output and stock-price gaps. The top panels of the figure show that both under an active and inactive stance towards output and stock prices, the Taylor principle is weakened, the more so the stronger the inertia (the higher $\rho$). If we consider the transformed coefficients $\Phi_i \equiv \frac{\phi_i}{1-\rho}$, though, which discount the effect of the inertia, the latter becomes ineffective and the Taylor principle is again a necessary condition for determinacy.

To check for the existence of other bounds to the region of determinacy, Fig. 3 and Fig. 4 widen the parameter sub-space in which the analysis is carried out, and confirm that our specification of the reaction function, being forward-looking, implies also an upper-bound for equilibrium determinacy, which is anyhow well above empirically plausible values for $\phi_\pi$. Another insight of Fig. 3 is that an accommodative stance towards either real activity or stock prices (i.e. negative values of either $\phi_y$ or $\phi_q$, when the other one is zero) produces an indeterminate equilibrium and therefore endogenous macroeconomic instability.

Although the determinacy analyses of this kind usually consider only the positive semi-space for the coefficients of the policy rule, the above result about a negative $\phi_y$ is common to all basic models used elsewhere for monetary policy analysis. The interpretation is straightforward: reacting to a positive output gap by lowering the interest rate further reinforces the inflationary pressures that it already implies, amplifying the consequent macroeconomic instability.

A similar argument is often used about the stance towards stock prices. Exploring the links between asset prices and monetary policy for the American and Japanese recent past, Bernanke and Gertler [3] find that the Bank of Japan, during the “bubble economy” of the 1980’s, was reacting to stock prices in a way that they label as “destabilizing” towards the stock market and the real economy, since the estimated coefficient was a little higher than -0.3. This stance was abruptly reversed as the bubble burst and the economy began to collapse, making many support the view that such a past behavior had had strong destabilizing effects. The plots of Fig. 3, then, seem supportive to this view.

A deeper investigation of this issue, though, yields an interesting result. As Fig. 5 shows, in fact, within this framework a given negative coefficient on the stock-price gap $\phi_q$ does not yield an indeterminate equilibrium if the coefficient on the output gap $\phi_y$ is positive.
and greater in absolute value. This result tells us that an accommodative stance towards stock prices is not destabilizing per se, rather that its effects on macroeconomic stability must be assessed in conjunction with the stance towards real output stabilization. For a given degree of accommodative stance towards stock prices, in fact, a proportionately more aggressive commitment to a zero-output gap offsets the potentially destabilizing effects of a negative $\phi_q$, producing a determinate equilibrium.

6 Concluding Remarks

In this paper we develop a small structural model for the analysis of monetary policy in a world in which consumers' financial wealth and real activity are affected by the stock market performance and stock prices are subject to exogenous stochastic misalignments.

Aside from a standard supply-side in which nominal rigidities are introduced through a price-setting mechanism following Calvo [9], the demand-side of the economy is modelled using a discrete-time overlapping-generations structure, in which consumers have a finite uncertain lifetime and allocate their savings among riskless bonds and a portfolio of shares issued by the wholesalers. Policy makers are in charge of monetary policy, implemented through the setting of a short-term interest rate.

Solution and linearization of the model provide a structural framework in which to analyze monetary policy along several dimensions. In this paper we limit our focus to the characterization of the conditions for equilibrium determinacy. The model, though, is suitable to address several other issues which are controversial in the literature, and which are tackled in two companion papers. In Nisticò [39] the same framework is used to estimate through Kalman Filter and Maximum Likelihood the structural parameters of a model with endogenous persistence in output and inflation; Nisticò [40], moreover, exploits the baseline model outlined here to derive optimal monetary policy under imperfect information, to assess the informational role of stock-price dynamics in driving optimal monetary policy.

The results achieved here are twofold. Derivation of the Wicksellian natural rate of interest shows that a policy maker pursuing price stability should take active notice of stock-price dynamics, as long as stochastic misalignments are effective. Moreover, while the Taylor principle is confirmed as a necessary condition for determinacy, an accommodating stance towards stock prices does not result as destabilizing in itself, as long as a proportionately stronger commitment to real output stabilization is granted.
References


Figure 1: Regions of equilibrium determinacy (white regions) for different policy rule’s calibrations. Grey regions indicate indeterminacy.
Figure 2: Regions of equilibrium determinacy (white regions) for different degrees of inertia. Grey regions indicate indeterminacy.
Figure 3: Regions of equilibrium determinacy (white regions): lower- and upper-bounds. Grey regions indicate indeterminacy.
Figure 4: Regions of equilibrium determinacy (white regions) for different degrees of inertia: lower- and upper-bounds. Grey regions indicate indeterminacy.
Figure 5: Regions of equilibrium determinacy (white regions) when the CB accommodates the stock-market. Grey regions indicate indeterminacy.